EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Modellierung & Simulation I
WS 2018/2019
Prof. Dr. M. Menth
D. Merling

Exercise 5

12. November 2018

Abgabe: 19. November 2018, 12:00:00 Uhr

Briefly discuss your findings. Results without an explanation will not be assessed!

Problem 5.1: *Batch-means method, simulation of* $GI/GI/1-\infty$

If simulation data are correlated, the sample variance is underestimated and so are the sizes of confidence intervals. To cope with this problem, the batch-means method may be applied. The simulation data are grouped into sufficiently large batches of samples such that the means of these batches are uncorrelated. Then, the estimation theory for independent samples can be applied to construct confidence intervals for these uncorrelated values. Please download the sourcecode `Exc05_Assignment.zip` from Moodle and use it as basis for your own implementation!

1. Implement a simulation study (*study.SimulationStudy*) for the $GI/GI/1-\infty$ queuing system ! Use the batch-means method to calculate the confidence intervals for the mean waiting time in order to control the duration of the simulation! 15 Points
   - The initial transient phase must not be considered for the data collection. To cope with that problem, provide a configurable number n_{init} of initial samples (waiting times) that are discarded before statistics are collected by counters!
   - The length of a batch is l_{batch} and it should be configurable by the user.
   - The simulation stops if the relative error of the 90% confidence interval for the estimated mean is lower than 5% or if its absolute error is smaller than 0.0001 s.
2. The Erlang-k, Exponential or Hyperexponential distribution may be used to model a desired mean and variance of the service time. The Normal distribution also models a desired mean and variance. Why is the Normal distribution not an adequate substitute for the other distributions in this context? 5 Points
3. The system utilization $\rho = {E[ST]}/{E[IAT]}$ is the ratio of the mean service time (ST) and the mean customer inter-arrival time (IAT). Provide features to keep $E[ST] = 1$ and to vary $E[IAT]$ for the simulation of the system utilization of $\rho \in [0.05; 0.95]$ with a granularity of 0.05! 10 Points
4. Provide features to calculate 20 Points
   - the mean waiting time and
   - the probability that the waiting time exceeds five times the mean service time $E[ST]$

and provide confidence intervals

- based on individual waiting times of a single run assuming that waiting times of consecutive customers are not correlated and
- using the batch-means method knowing that waiting times of consecutive customers are correlated!

5. The batch-means method must be configured appropriately so that the mean values of the waiting times of consecutive batches are not correlated. 10 Points
   - Use a *DiscreteAutocorrelationCounter* to control the autocorrelation of the batch-means of the waiting times!
   - How does the autocorrelation depend on the system utilization ρ and the coefficient of variation $c_{var}[IAT]$? For which combination of $\rho \in [0.05, 0.95]$ and $c_{var}[IAT] \in \{0.5, 1, 2\}$ do you observe the strongest autocorrelation of the batch-means?
   - Choose an appropriate l_{batch} so that the autocorrelation of the batch-means is sufficiently small for utilization values ρ between 0.05 and 0.95 and for coefficients of variation of the inter-arrival time $c_{var}[IAT]$ between 0.5 and 2.0! Use this value l_{batch} in the remainder of this exercise!
6. Calculate the mean waiting time and the waiting probability including appropriate confidence intervals for $0.05 \leq \rho \leq 0.95$ with a step width of 0.05 for $c_{var}[IAT] = 0.5, 1, 2$ using the batch-means method! 10 Points
7. Simulate $\rho = 0.95$ and $c_{var}[IAT] = 2.0$ using the batch-means method. What is the size of the confidence interval when it is calculated based on the batch-means or on individual samples, respectively? Why do you observe a difference? What is the simulated mean value? How many samples of customer waiting time are needed? 10 Points
8. Now simulate $\rho = 0.95$ and $c_{var}[IAT] = 2.0$ without the batch-means method. Just use $l_{batch} = 1$ for this purpose! What is the simulated mean value? How many samples for the customer waiting time are needed? Explain the difference to the results for the appropriate l_{batch}! Which values are correct? 10 Points
9. The overall number of collected samples in the simulation is $n_{batches} \cdot l_{batch}$. Explain by formulae why the width of the confidence interval depends on l_{batch} for a given overall number of samples! 5 Points
10. Is there an alternative method to the batch-means approach that avoids the choice of a proper batch length l_{batch} to produce a mean value? What is the drawback of that method? 5 Points

Total: 100 Points