Tobias Hille tobias.hille@student.uni-tuebingen.de 3905597
Robin Schmidt rob.schmidt@student.uni-tuebingen.de 4255055

Modellierung & Simulation I

Serie 02

Problem 2.2.1

The discrete event simulation (DES) has various components, which work together to successfully simulate discrete events. These components include the simulation clock, events, statistic counters and the main program.

The simulation clock shows the current system time. Here the real time needs to be converted to the corresponding simulation time. Events are associated with a discrete simulation time, at which the event runs his corresponding event routine. An event routine leads to an action in the system which can influence for example the system state. All events are managed in an event chain (event queue). Statistic counters contain information about the system, which can be for example packet sizes or queue lengths.

The first step in the main program is to initialize the system state, containing variables, lists and queues. After everything is initialized the event chain gets executed. During this process the next event in the event chain gets selected, the simulation time gets forwarded to the event time and finally the event routine gets executed. While the event chain is being executed, the size of the chain is always greater than 1 because of the terminating stop event. Once the termination event is reached, the simulation process terminates.

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2.2 2.3

Problem 2.2.4

Results of simulation 1 with an interarrival time of customers $10s$, service time $9s$ and simulation time $10000s$:

	Simulation 1	
	mean	c_{var}
occupancy time	0.0	0.0
service time	9.0	0.0
utilization time	0.9	0.3333333333333326
waiting time	0.0	0.0

Results of simulation 2 with an interarrival time of customers $10s$, service time $10s$ and simulation time $10000s$:

	Simulation 2	
	mean	c_{var}
occupancy time	0.0	0.0
service time	10.0	0.0
utilization time	1.0	0.0
waiting time	0.0	0.0

Results of simulation 3 with an interarrival time of customers $10s$, service time $11s$ and simulation time $10000s$:

	Simulation 3	
	mean	c_{var}
occupancy time	44.865	0.5849120747543387
service time	11.0	0.0
utilization time	1.0	0.0
waiting time	454.0	0.5783039542709295

Results of simulation 4 with an interarrival time of customers $10s$, service time $9s$ and simulation time $100000s$:

	Simulation 4	
	mean	c_{var}
occupancy time	0.0	0.0
service time	9.0	0.0
utilization time	0.9	0.33333333333333326
waiting time	0.0	0.0

Results of simulation 5 with an interarrival time of customers $10s$, service time $10s$ and simulation time $100000s$:

	Simulation 5	
	mean	c_{var}
occupancy time	0.0	0.0
service time	10.0	0.0
utilization time	1.0	0.0
waiting time	0.0	0.0

Results of simulation 6 with an interarrival time of customers $10s$, service time $11s$ and simulation time $100000s$:

	Simulation 6	
	mean	c_{var}
occupancy time	453.9546	0.5781011743810226
service time	11.0	0.0
utilization time	1.0	0.0
waiting time	4544.5	0.5774455511199658

Discussion? - 5

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Problem 2.2.5

Histogram for waiting and service time per customer for simulation 1:

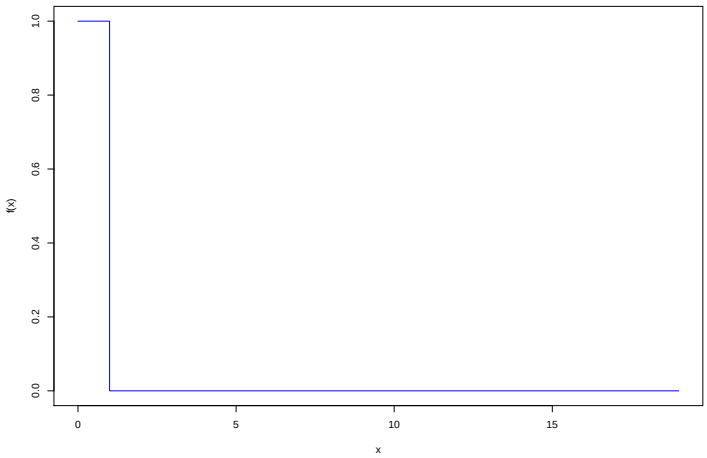

Figure 1: Simulation 1 - Waiting Time

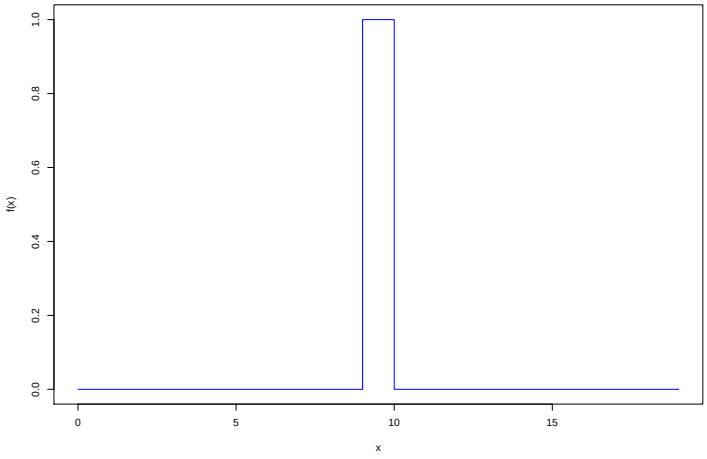

Figure 2: Simulation 1 - Service Time

Histogram for waiting and service time per customer for simulation 2:

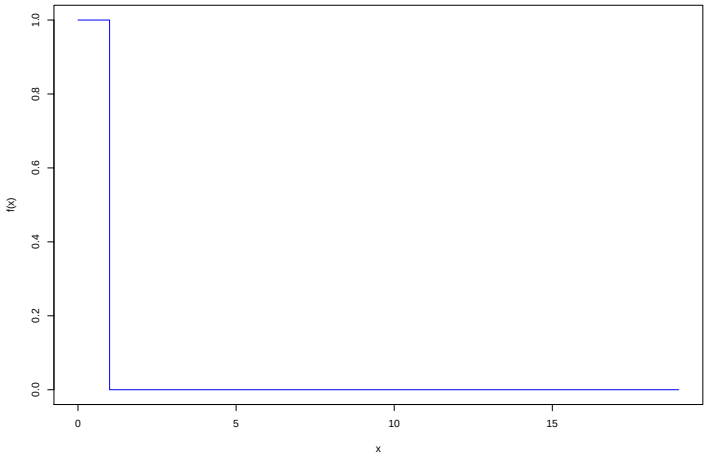

Figure 3: Simulation 2 - Waiting Time

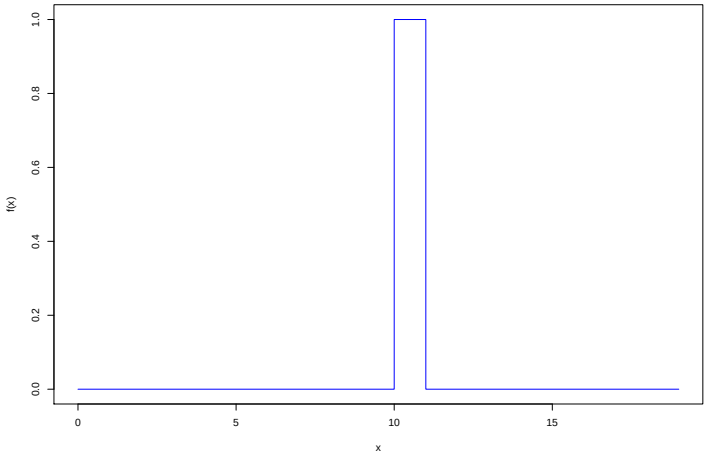

Figure 4: Simulation 2 - Service Time

Histogram for waiting and service time per customer for simulation 3:

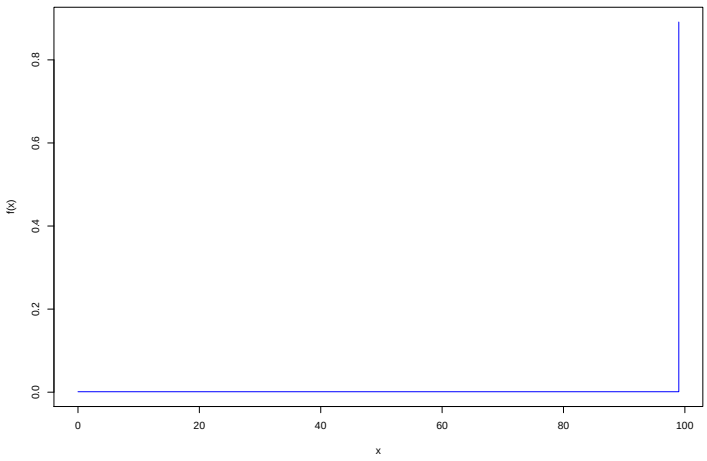

Figure 5: Simulation 3 - Waiting Time

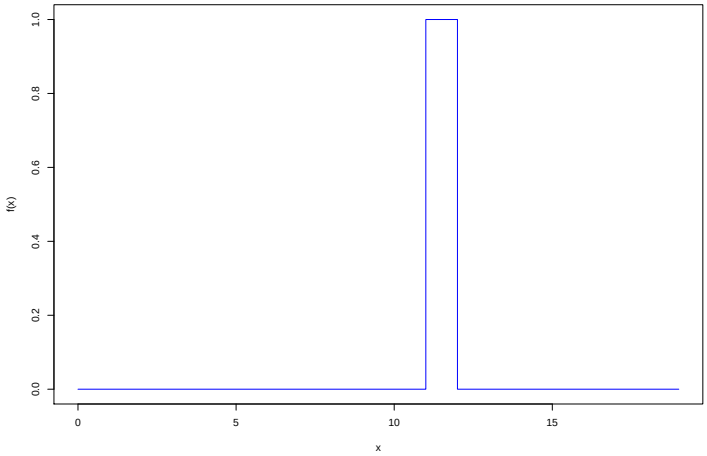

Figure 6: Simulation 3 - Service Time

Histogram for waiting and service time per customer for simulation 4:

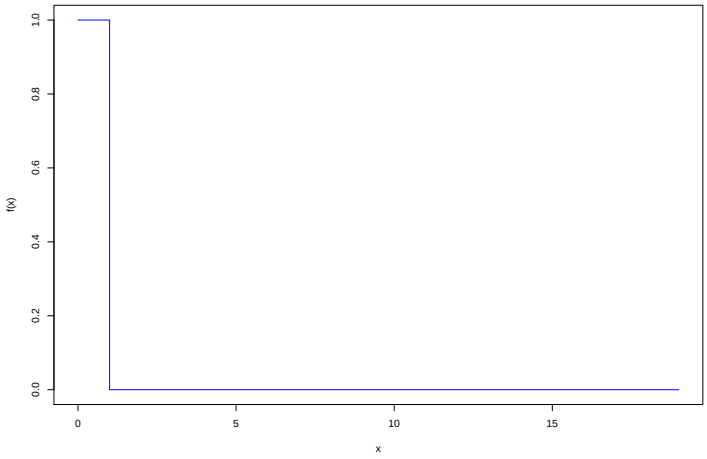

Figure 7: Simulation 4 - Waiting Time

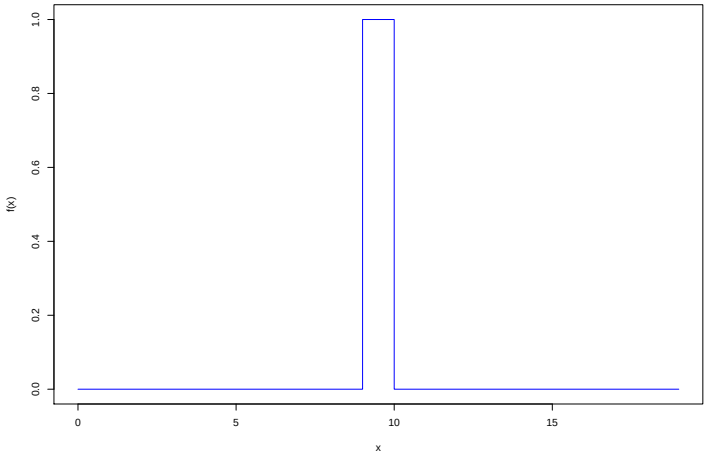

Figure 8: Simulation 4 - Service Time

Histogram for waiting and service time per customer for simulation 5:

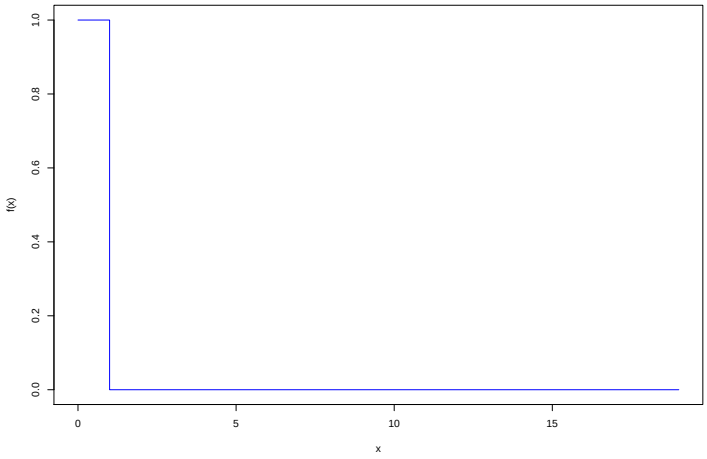

Figure 9: Simulation 5 - Waiting Time

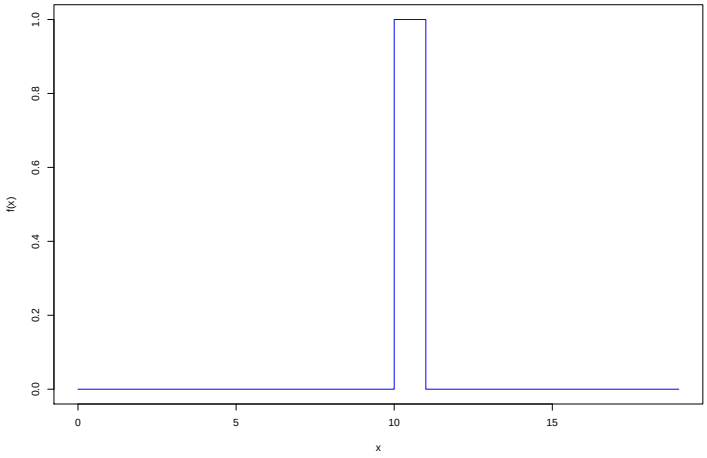

Figure 10: Simulation 5 - Service Time

Histogram for waiting and service time per customer for simulation 6:

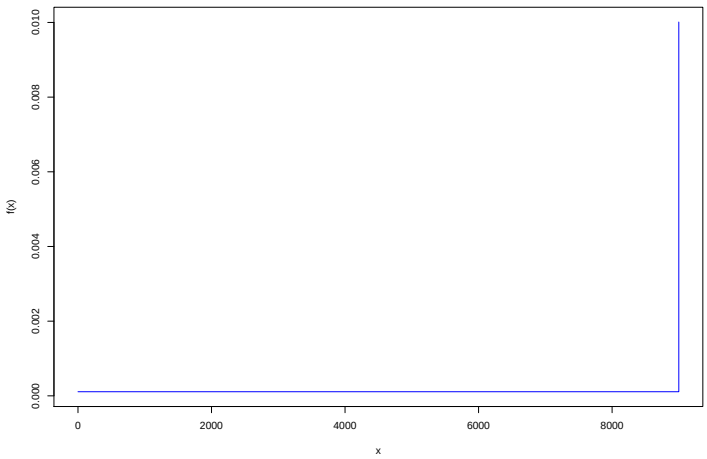

Figure 11: Simulation 6 - Waiting Time

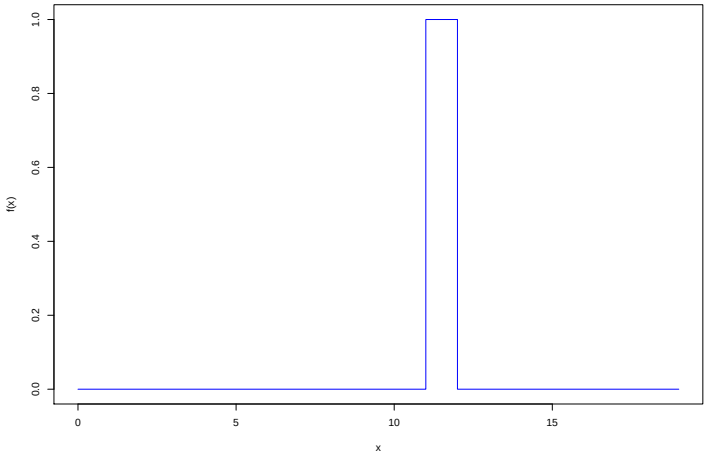

Figure 12: Simulation 6 - Service Time

Problem 2.2.6

Using a discrete counter instead of continous counter for queue occupancy and server utilization with interrival time of customers $10s$, service time $9s$ and simulation time $10000s$:

	Simulation 7	
	mean	c_{var}
occupancy time	0.0	0.0
utilization time	0.5002498750624688	0.9997502185548647

Comparison of mean values and the coefficient of the variation with results from simulation 1:

The results for the occupancy time stay the same, but the results for the utilization time change. The *mean* now has a smaller value and the c_{var} now has a higher value.

Problem 2.3.1

please see code

Problem 2.3.2

please see code

Problem 2.3.3

The Exponential distributions with $mean = 1$, $c_{var} = 0.1$ and $mean = 1$, $c_{var} = 2$ cannot be instantiated. This is because the relation $c_{var} = \frac{\sigma}{mean} = \frac{\sqrt{\frac{1}{\lambda^2}}}{\frac{1}{\lambda}} = 1$ is not fulfilled.

The ErlangK distribution with $mean = 1$,$c_{var} = 2$ cannot be instantiated. This is because the relation $c_{var} = \frac{\sigma}{mean} = \frac{\frac{\sqrt{k}}{\lambda}}{\frac{k}{\lambda}} = \frac{1}{\sqrt{k}}$ is not true for any integer k.

The HyperExponential distribution H2 with $mean = 1$, $c_{var} = 0.1$ cannot be instantiated. This is because $c_{var}^2 - 1$ has to be greater or equal 0.

The HyperExponential distribution H2 with $mean = 1$, $c_{var} = 1$ can be instantiated, but because the two resulting lambda's are the same, the two probabilities p are the same too. It follows, that this is a degenerated hyperexponential distribution and behaves just like a "normal" exponential distribution.

The following pages contain the console output for a specific run of the RandVarTest.java file.

```
analytical properties: Uniform
        mean: 1.0
        cvar: 0.09999999999999996
        standard deviation: 0.09999999999999996
        variance: 0.009999999999999993

hyperparameters:
        low: 0.8267949192431123
        high: 1.1732050807568877

observed random variable: u(1/0.1)
        counter type: discrete-time counter
        number of samples: 1000000
        mean: 0.9999339261755062
        variance: 0.009988439713883829
        standard deviation: 0.09994218185472953
        coefficient of variation: 0.09994878585326436
        minimum: 0.8267955791053156
        maximum: 1.1732049888411293

analytical properties: Uniform
        mean: 1.0
        cvar: 0.9999999999999999
        standard deviation: 0.9999999999999999
        variance: 0.9999999999999999

hyperparameters:
        low: -0.7320508075688772
```

```
        high: 2.732050807568877

observed random variable: u(1/1)
        counter type: discrete-time counter
        number of samples: 1000000
        mean: 1.0009451761584902
        variance: 0.999562640641644
        standard deviation: 0.9997812964051909
        coefficient of variation: 0.9988372192793153
        minimum: -0.7320505527403145
        maximum: 2.7320475527485084

analytical properties: Uniform
        mean: 1.0
        cvar: 1.9999999999999998
        standard deviation: 1.9999999999999998
        variance: 3.9999999999999996

hyperparameters:
        low: -2.4641016151377544
        high: 4.464101615137754

observed random variable: u(1/2)
        counter type: discrete-time counter
        number of samples: 1000000
        mean: 0.9985852814415288
        variance: 3.9979063626309275
        standard deviation: 1.9994765221504671
        coefficient of variation: 2.0023092261725317
        minimum: -2.4641008274808156
        maximum: 4.464098555505992

java.lang.IllegalArgumentException: no exponential distribution can ful

analytical properties: Exponential
        mean: 1.0
        cvar: 1.0
        standard deviation: 1.0
```

```
        variance: 1.0

hyperparameters:
        lambda: 1.0

observed random variable: e(1/1)
        counter type: discrete-time counter
        number of samples: 1000000
        mean: 0.9999244242179334
        variance: 1.0005224835713675
        standard deviation: 1.00026120767096
        coefficient of variation: 1.0003368089076232
        minimum: 5.087969042600845E-7
        maximum: 14.901709321775272

java.lang.IllegalArgumentException: no exponential distribution can ful

analytical properties: ErlangK
        mean: 1.0000000000000002
        cvar: 0.09999999999999999
        standard deviation: 0.10000000000000002
        variance: 0.010000000000000004

hyperparameters:
        k: 100
        lambda: 99.99999999999999

observed random variable: k(1/0.1)
        counter type: discrete-time counter
        number of samples: 1000000
        mean: 1.0000091665363684
        variance: 0.00999800732006829
        standard deviation: 0.09999003610394533
        coefficient of variation: 0.09998911955004453
        minimum: 0.5619249658793721
        maximum: 1.5924694607913528

analytical properties: ErlangK
```

```
        mean: 1.0
        cvar: 1.0
        standard deviation: 1.0
        variance: 1.0

hyperparameters:
        k: 1
        lambda: 1.0

observed random variable: k(1/1)
        counter type: discrete-time counter
        number of samples: 1000000
        mean: 1.0001891637870386
        variance: 0.9985986936338268
        standard deviation: 0.9992991011873406
        coefficient of variation: 0.999110105736071
        minimum: 6.5180126533125E-7
        maximum: 15.033974577307747

java.lang.IllegalArgumentException: no erlang distribution can fulfill

java.lang.IllegalArgumentException: no hyperexponential distribution can

java.lang.IllegalArgumentException: lambda1 == lambda2 -> degenerated h

analytical properties: HyperExponential
        mean: 1.0
        cvar: 2.0
        standard deviation: 2.0
        variance: 4.0

hyperparameters:
        p1: 0.1127016653792583
        lambda1: 0.2254033307585166
        p2: 0.8872983346207417
        lambda2: 1.7745966692414834

observed random variable: h(1/2)
```

```
counter type: discrete-time counter
number of samples: 1000000
mean: 2.0961089668213866
variance: 20.964138207185904
standard deviation: 4.578661180649416
coefficient of variation: 2.184362193532648
minimum: 4.161550920427093E-7
maximum: 73.34553838167366
```