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Modellierung & Simulation I
WS 2018/2019
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Exercise 7

26. November 2018

Abgabe: 3. Dezember 2018, 12:00:00 Uhr

Briefly discuss your findings. Results without an explanation will not be assessed!

All implementations of the following problems should be done in R. You're **not** allowed to use the package *markovchain* (or a similar package) within your submitted solution.

You may use *knitr* (R markdown) to generate a final report containing all calculations, text answers and R scripts for handing in your solution. However, this is not mandatory.

If you are using RStudio, you just need to select `File` $\rightarrow$ `New File` $\rightarrow$ `R Markdown`. Insert a title, your name and select PDF as output format to get an example with markdown formatted text content and R code blocks. If all software requirements are fulfilled, selecting `Knit PDF` should generate a PDF document.

Problem 7.1: *Discrete time markov chains*

Given is a clocked system with the states 0, 1, 2, and 3. After each clock cycle a state transition possibly occurs. A transition from state 0 to the states 0, 1, 2, and 3 happens with probability 0.1, 0.2, 0.3, and 0.4. Transitions from states $n \in \{1, 2, 3\}$ to state $(n + 1) \mod 4$ happen with probability 1.0.

1. Give the state transition matrix and the state transition diagram showing only transitions with positive probability. 10 Points
2. Calculate the state distribution vectors x_n for $n \in [0; 10]$ if the systems starts in state 0! 10 Points
3. What is the probability for the system to be in state 0, 1, 2, and 3 in the long run? 5 Points

Remove the possibility for the transitions from state 0 to state 0, and from state 0 to state 2 by setting the probabilities for the transitions from state 0 to state 1 and from state 0 to state 3 to 0.5!

5. Give the state transition matrix and the state transition diagram! 5 Points
6. Calculate the state distribution vectors x_n for $n \in [0; 10]$ if the systems starts in state 0! 5 Points
7. Explain the observed effects!
   Hint: see Section 8.9 Example 3! 5 Points
8. What is the probability for the system to be in state 0, 1, 2, and 3 in the long run? 5 Points

9. Would it have been sufficient to remove only one state transition to observe the effect? 5 Points

Problem 7.2: *Repeated coin flipping*

We take a look at repeated coin flipping. This time, «head» gives one point, «tail» gives two points. The game ends when at least five points are reached. Calculate the distribution of the number of coin flippings needed to finish the game and the distribution of the points at the end of the game using markov chains with stopping times.

1. Give the state space and the set of terminating states! 5 Points
2. Give the state transition equations including the probabilities for the transitions! 10 Points
3. Give the state transition diagram!

   Hint: You may omit labeling the probabilities for the transitions for the sake of clarity! 5 Points
4. Implement a program based on a transition matrix to calculate
   (a) the distribution that the game ends after N coin flippings with X points.
   (b) the distribution of the number of coin flippings needed to end the game.
   (c) the distribution of the number of points at the end of the game. 30 Points

Total: 100 Points