1	Σ
63	63/100

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Modellierung & Simulation I

Serie 08

Problem 8.1.1

We get a set of states $X = \{0, 1, 2, 3, 4\}$, which represent the following system states:

State 0: The system is empty

State 1: One service unit is currently working, zero waiting slots are occupied

State 2: Two service units are currently working, zero waiting slots are occupied

State 3: Two service units are currently working, one waiting slot is occupied

State 4: Two service units are currently working, two waiting slots are occupied

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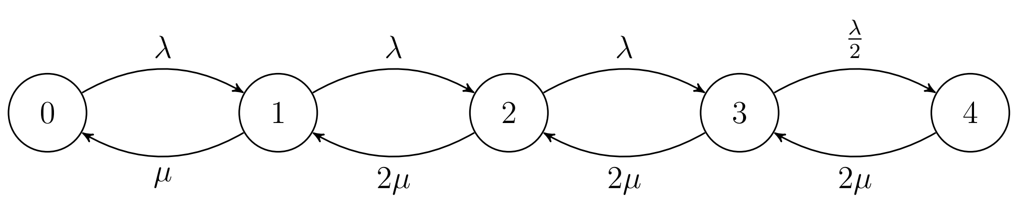
So the states represent the number of customers currently in the system.

Problem 8.1.2



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Problem 8.1.3

Using the formulas from course script chapter 9.4 and the corresponding macro states being a set of neighboring states gives:

$$x(i) = x(0) \cdot \frac{\prod_{0<k\leq i} \lambda_{k-1}}{\prod_{0<k\leq i} \mu_k}$$

$$x(0) = \left(1 + \sum_{0<i\leq n} \frac{\Pi_{0<k\leq i}\lambda_{k-1}}{\Pi_{0<k\leq i}\mu_k}\right)^{-1} = \left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2} + \frac{\lambda^3}{4\mu^3} + \frac{\lambda^4}{16\mu^4}\right)^{-1}$$ ✓

$$x(1) = x(0) \cdot \frac{\lambda}{\mu}$$

$$x(2) = x(0) \cdot \frac{\lambda^2}{2\mu^2}$$

$$x(3) = x(0) \cdot \frac{\lambda^3}{4\mu^3}$$

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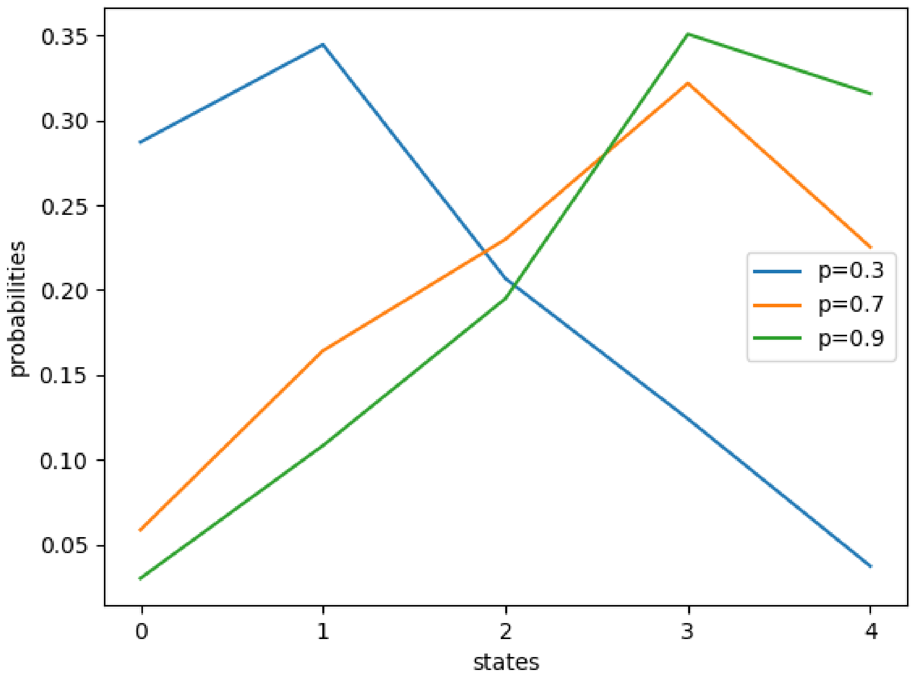
$$x(4) = x(0) \cdot \frac{\lambda^4}{16\mu^4}$$ ✓

Problem 8.1.4

We use the equations from 8.1.3. Please see provided python code for this diagramm.

n = 2



Diskussion?
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Figure 1: Diagram for Problem 8.1.4

Problem 8.1.5 & Problem 8.1.6

The questions are very similar.
The blocking probability is calculated using the formula given the section 9.5.2 in the lecture notes.

$$p_b = \frac{\frac{\lambda}{2}x(4)}{\lambda(x(0) + x(1) + x(2) + x(3)) + \frac{\lambda}{2}x(4)}$$

Analogously we calculate the waiting probability with:

$$p_w = \frac{x(3) + x(4)}{x(0) + x(1) + x(2) + x(3) + x(4)}$$

For the different values of ρ this leads to the following table:

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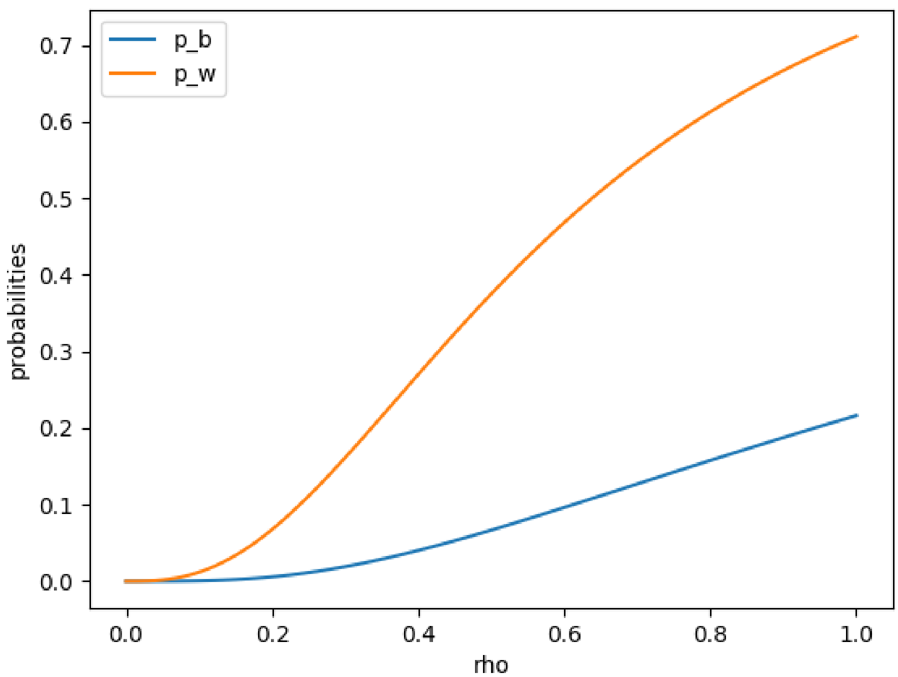
ρ	0.3	0.7	0.9
blocking probability	0.01896511	0.12696314	0.18750536
waiting probability	0.16130515	0.54720345	0.66668271

These values (and subsequently those from the next diagram) seem odd, because for a relative offered load of 0.9 the blocking probability appears to be too low (But on the other side there are two service units and the intuition can be wrong).

Problem 8.1.7

please see provided code.



(✓)

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Figure 2: Diagram for Problem 8.1.7

Problem 8.1.8

Only for the states 3 and 4 a queue exists and the length is 1 and 2 respectively. Therefor the mean waiting length is calculated as:

$$E(Q) = 1 * x(3) + 2 * x(4)$$ √

The server utilization is calculated similarly. No server is active in state 0, only one is active in state 1 and both are active in the other states. This leads to:

$$E(S) = 1 * x(1) + 2 * (x(2) + x(3) + x(4))$$ f

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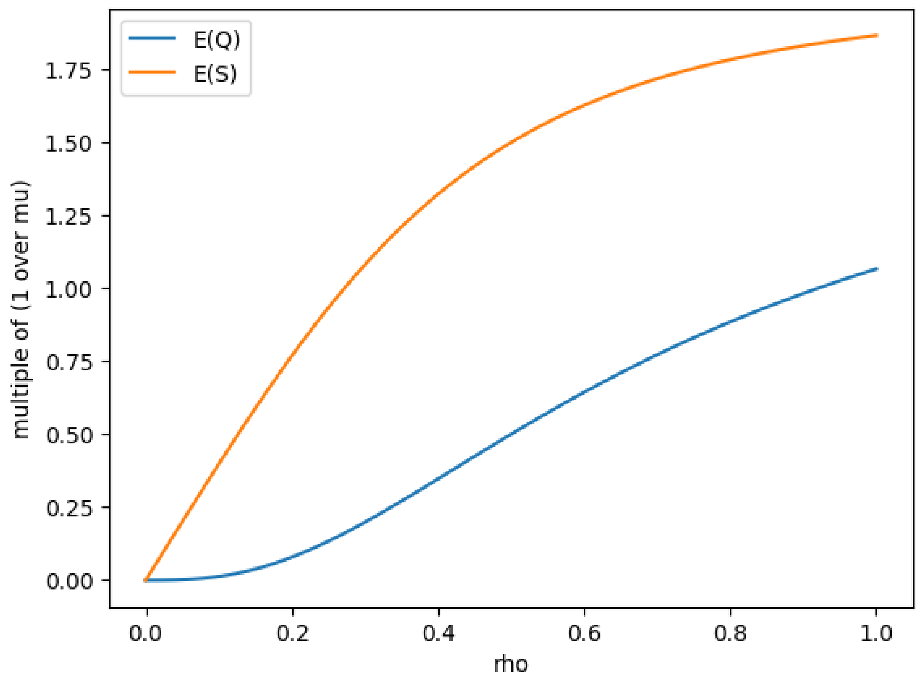
For the different values of ρ this leads to the following table:

ρ	0.3	0.7	0.9
mean waiting length	0.19852941	0.77252252	0.98247978
mean server utilization	1.08088235	1.71846847	1.83153639

Problem 8.1.9



Discussion?

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Figure 3: Diagram for Problem 8.1.9

Problem 8.1.10 & Problem 8.1.11

After the theorem from Little we can write for the mean waiting time of all customers:

$$E(A) = \frac{E(Q)}{\lambda}$$

Similarly the following expression holds for the mean waiting time for all waiting customers:

$$E(W) = \frac{E(Q}{\lambda * p(w)}$$

f

Obviously this is not the answer and the lecture notes contain the solution for a system with infinite waiting slots at page 167, but we do not know how to apply this to the given system.