Modellierung & Simulation I
WS 2018/2019
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Exercise 6

19. November 2018

Abgabe: 26. November 2018, 12:00:00 Uhr

Briefly discuss your findings. Results without an explanation will not be assessed!

Problem 6.1: *Packet arrivals at a network switch*

This problem considers packets arriving at three interfaces on a network switch. The packet interarrival times at each of the interfaces can be described by a Poisson process with rate λ.

1. **[Bonus]** Calculate the cumulative distribution function (CDF) of the time until a first packet arrives at the switch, no matter from which of the incoming interfaces!

   Hint: Consider the minimum of the three inter-arrival times! +5 Points

2. **[Bonus]** The exponential distribution is memoryless. That means, the distribution of the time until the next arrival is exponentially distributed independently of the time of the last arrival. Given this, according to which random variable are packet inter-arrival times at the switch distributed, no matter from which incoming interface? What is the parameter of this distribution function? +5 Points

3. **[Bonus]** How is the number of packets distributed that arrive at the switch within an interval of duration Δ, no matter from which incoming interface? Provide formula and parameters! +5 Points

Problem 6.2: *Packet Sampling*

We assume a switch sends traffic with exponential inter-arrival times over an outgoing interface. It extracts a fraction of $1/n$ of the packets for the purpose of analysis (see, e.g., *sFlow*[1]). We consider two approaches:

(A) The switch extracts every n-th packet.

(B) The switch extracts every packet with a probability of $p = 1/n$.

1. **[Bonus]** How is the time D_A between two extracted packets distributed in case of A? Provide formula and parameters! +5 Points

2. **[Bonus]** Derive the probability density function (PDF) of the Erlang-2 distribution by applying the appropriate convolution formula! +5 Points

3. **[Bonus]** In case of sampling method (B), the time D_B between consecutive sampled packets is again exponentially distributed with rate $p \cdot \lambda$. Proof it!

   Hint: Calculate the PDF of D_B by applying the law of total probability using the conditional probability that D_B takes k exponential inter-arrival times and the probability for that case! Simplify the resulting expression by substituting the series representation of the exponential function $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$! +10 Points

4. **[Bonus]** What is the coefficient of variation of the random variables D_A and D_B? +5 Points

5. **[Bonus]** Assume sampled data are processed by packet analytics. Which sampling process requires a faster packet processing engine to process the data with only little delay? +5 Points

Previously we assumed exponential inter-arrival times. Now we investigate arbitrary inter-arrival times with mean $E[X]$ and coefficient of variation $c_{var}[X]$ for approach A and B.

6. **[Bonus]** Use the concept of composite distributions to show: +10 Points

$$c_{var}^n[A] = \frac{c_{var}[X]}{\sqrt{n}}$$

$$c_{var}^n[B] = \sqrt{\frac{c_{var}[X]^2 + (1 - \frac{1}{n})}{n}}$$

   Hint: Use the following properties for a composited random variable $Y = \sum_{0<i\leq N} X_i$, where X_i is a discrete or continuous iid random variable and N a discrete random variable.

$$E[Y] = E[N] \cdot E[X]$$

$$Var[Y] = E[N] \cdot Var[X] + Var[N] \cdot E[X]^2$$

7. **[Bonus]** Plot the coefficient of variation for $n \in \{1, ..., 10\}$ and $c_{var}[X] \in \{0, 0.5, 1, 1.5, 2\}$ for both approaches! +10 Points

8. **[Bonus]** Name three distribution functions for non-negative random variables with arbitrary positive mean and coefficient of variation! +5 Points

[1] `https://en.wikipedia.org/wiki/SFlow`