1	2	Σ
45	55	100/100

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Modellierung & Simulation I

Serie 04

35 /35

Problem 4.1.1

please see provided code

Problem 4.1.2

Using the first two testsets from the exercise sheet and coming up with four additional testsets leads to the following six testsets:

$$\text{Testset } 1 = \{2, 2, 2, 2, 2, 2, 2, 2, 2, 2\}$$
$$\text{Testset } 2 = \{2, -2, 2, -2, 2, -2, 2, -2, 2, -2\}$$
$$\text{Testset } 3 = \{2, 2, -2, 2, 2, -2, 2, 2, -2, 2\}$$
$$\text{Testset } 4 = \{2, 2, 3, 2, 3, 2, 3, 2, 2, 2\}$$
$$\text{Testset } 5 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$
$$\text{Testset } 6 = \{1, 4, 8, 0, -1, -12, 13, 20, 5, 4\}$$

For the testsets stated above, the following autocovariance and autocorrelation values were observed. For example purposes we only show the values for a lag of 2:

Testset	1	2	3	4	5	6
Autocovariance	0.0	4.0	-1.56	0.1150	4.25	-32.61
Autocorrelation	1.0	0.899999	-0.417857	0.4928	0.4636	-0.4449

✓

For testset 1 the autocovariance and autocorrelation are straight forward, because all numbers are the same so they are highly correlated to each other with a value of 1. For the testset 2 the autocorrelation is still pretty high because every two numbers are the same. With a lag of 2 testset 3 is negatively corelated due to the distances in which -2 and 2 vary. Testset 4 and testset 5 are positively correlated with the given lag and testset 6 is negatively correlated. This is because the distances between samples in testset 4 and 5 is smallly increasing compared to those in testset 6, which causes the negative correlation. ✓ 10/10

Problem 4.2.1

please see provided code ✓ 15/15

Problem 4.2.2

For a mean utilization of the system of 80 % the mean time of the exponential distributed service time has to be 0.8 . ✓ 5/5
A simple simulation run can verify this statement with sufficient precision:

```
Simulation time in seconds: 1000000.0

observed random variable: server utilization/time
        counter type: continuous-time counter
        number of samples: 1996657
        mean: 0.8000858062149303
        variance: 0.15994850890833534
        standard deviation: 0.39993563095620194
        coefficient of variation: 0.49986592419159304
        minimum: 0.0
        maximum: 1.0

observed random variable: service time/customer
        counter type: discrete-time counter
        number of samples: 998328
        mean: 0.8014256506879233
        variance: 0.6425255021325139
        standard deviation: 0.801576884729415
        coefficient of variation: 1.0001887062653434
        minimum: 0.001
        maximum: 10.982

observed random variable: queue occupancy/time
```

```
        counter type: continuous-time counter
        number of samples: 998329
        mean: 3.2050899006856426
        variance: 18.707393944042067
        standard deviation: 4.3252044973668085
        coefficient of variation: 1.3494799307943117
        minimum: 0.0
        maximum: 56.0

observed random variable: waiting time/customer
        counter type: discrete-time counter
        number of samples: 998328
        mean: 3.211894302273389
        variance: 15.52748910278651
        standard deviation: 3.9404935100551186
        coefficient of variation: 1.2268440799144682
        minimum: 0.0
        maximum: 46.107
```

Problem 4.2.3 & 4.2.4

Number of Customers	10	99928
Waiting Time Mean	0.2293	3.0832312765190646

10/10

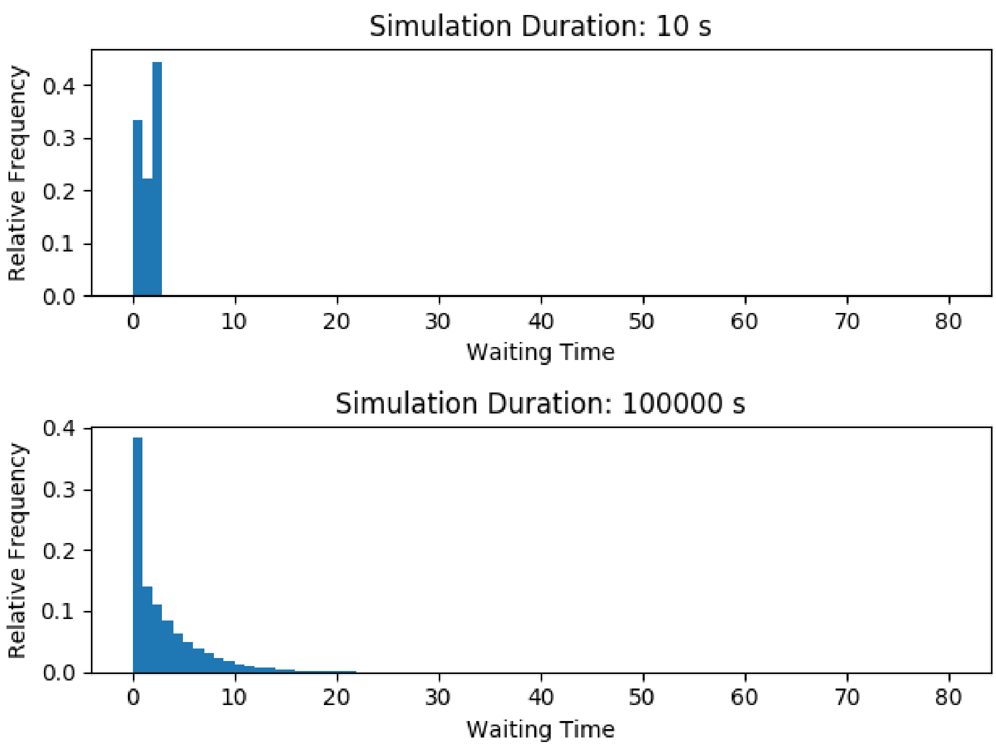

Figure 1: Histograms

Discussion:
The first 10 customers make a small sample size. Because of this the observed distribution of the waiting time is quite different from the true one, which is an (hyper-)exponential. This causes the estimate of the mean to be too small and in contrary the observed mean/distribution for the run with almost 100000 customers is a way better approximation of the underlying process.
The following pages contain the detailed statistics for the two compared sim-

ulation runs:

```
Simulation time in seconds: 10.0

observed random variable: server utilization/time
        counter type: continuous-time counter
        number of samples: 21
        mean: 0.799240480344863
        variance: 0.1604551349229757
        standard deviation: 0.4005685146425961
        coefficient of variation: 0.5011864695213578
        minimum: 0.0
        maximum: 1.0

observed random variable: service time/customer
        counter type: discrete-time counter
        number of samples: 10
        mean: 0.7787
        variance: 0.20966134444444446
        standard deviation: 0.45788791690155417
        coefficient of variation: 0.588015817261531
        minimum: 0.164
        maximum: 1.827

observed random variable: queue occupancy/time
        counter type: continuous-time counter
        number of samples: 11
        mean: 0.0
        variance: 0.0
        standard deviation: 0.0
        coefficient of variation: 0.0
        minimum: 0.0
        maximum: 1.0

observed random variable: waiting time/customer
        counter type: discrete-time counter
        number of samples: 10
```

```
        mean: 0.2293
        variance: 0.09162889999999999
        standard deviation: 0.30270265938706253
        coefficient of variation: 1.3201162642261777
        minimum: 0.0
        maximum: 0.898

Simulation time in seconds: 100000.0

observed random variable: server utilization/time
        counter type: continuous-time counter
        number of samples: 199858
        mean: 0.7978519206174655
        variance: 0.16128423338448705
        standard deviation: 0.4016020833916167
        coefficient of variation: 0.5033541601063176
        minimum: 0.0
        maximum: 1.0

observed random variable: service time/customer
        counter type: discrete-time counter
        number of samples: 99928
        mean: 0.7984155491954235
        variance: 0.6401180433048178
        standard deviation: 0.8000737736639152
        coefficient of variation: 1.0020768940060885
        minimum: 0.001
        maximum: 10.0

observed random variable: queue occupancy/time
        counter type: continuous-time counter
        number of samples: 99930
        mean: 3.077889485251493
        variance: 16.737584281033318
        standard deviation: 4.091159283263525
        coefficient of variation: 1.3292092854104662
        minimum: 0.0
        maximum: 37.0
```

```
observed random variable: waiting time/customer
        counter type: discrete-time counter
        number of samples: 99928
        mean: 3.0832312765190646
        variance: 14.024372162058013
        standard deviation: 3.7449128377117153
        coefficient of variation: 1.2146065286220378
        minimum: 0.0
        maximum: 32.23
```

√

10/10

Problem 4.2.5

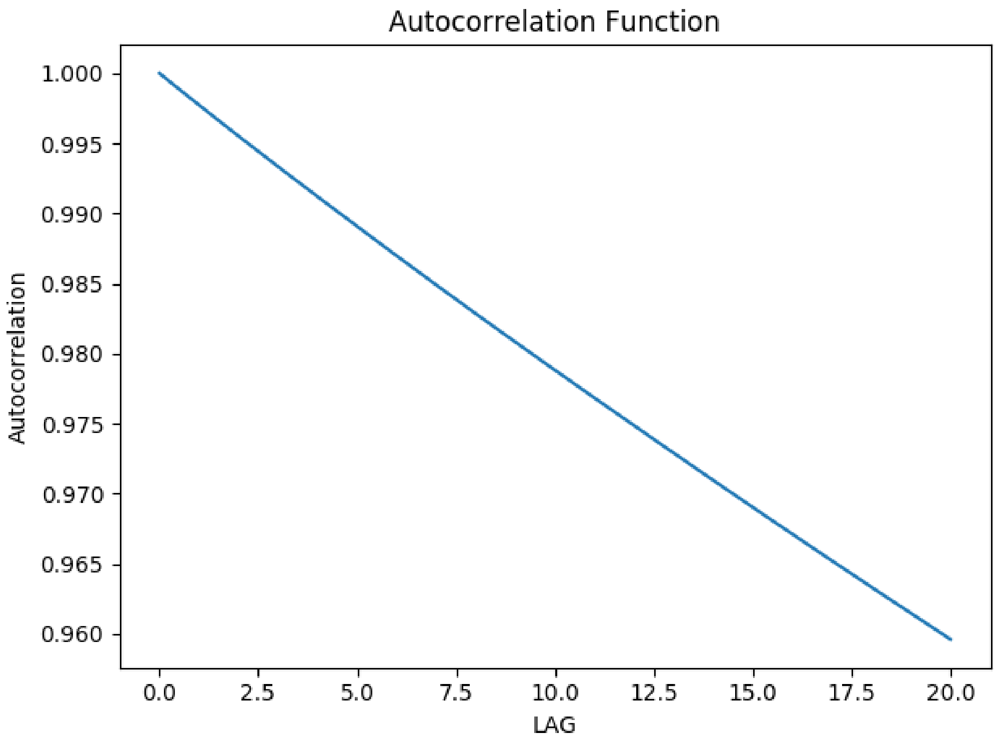

Figure 2: Autocorrelation

Because the system is almost always busy, the correlation between the waiting time of two customers who enter the system is very high. So if the first customer waited x amount of time, it is very likely, that the second customer, who arrives later, has to wait a comparable amount of time too. ✓ 15/15