Modellierung & Simulation I - Exercise 1

1) rel. Häufigkeit

$$h(A_i) = \frac{n_i}{n} = \frac{283\,789}{2\,300\,000} \approx 0{,}12339 \mathrel{\hat=} \underline{\underline{12{,}339\,\%}}$$

$$p(A_i) = \frac{m_i}{m} = \frac{3^3}{6^3} = \frac{27}{216} = 0{,}125 \mathrel{\hat=} \underline{\underline{12{,}5\,\%}}$$

2)

a)

$X_{sum} \in \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$X_{min} \in \{1, 2, 3, 4, 5, 6\}$

$X_{max} \in \{1, 2, 3, 4, 5, 6\}$

$X_{diff1} \in \{0, 1, 2, 3, 4, 5\}$

$X_{diff2} \in \{0, 1, 2, 3, 4, 5\}$

b)

$P(X_{sum} = 2) = \frac{1}{36}$

$P(X_{sum} = 3) = \frac{2}{36}$

$P(X_{sum} = 4) = \frac{3}{36}$

$P(X_{sum} = 5) = \frac{4}{36}$

$P(X_{sum} = 6) = \frac{5}{36}$

$P(X_{sum} = 7) = \frac{6}{36}$

$P(X_{sum} = 8) = \frac{5}{36}$

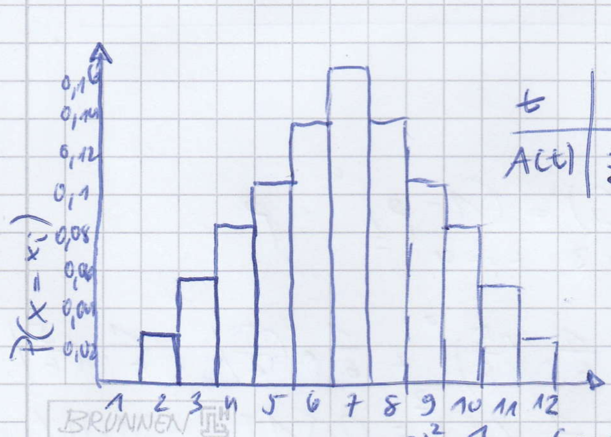
$P(X_{sum} = 9) = \frac{4}{36}$

$P(X_{sum} = 10) = \frac{3}{36}$

$P(X_{sum} = 11) = \frac{2}{36}$

$P(X_{sum} = 12) = \frac{1}{36}$

x_i	2	3	4	5	6	7	8	9	10	11	12
$P(X_{sum} = x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



t	2	3	4	5	6	7	8	9	10	11	12
$A(t)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{15}{36}$	$\frac{21}{36}$	$\frac{26}{36}$	$\frac{30}{36}$	$\frac{33}{36}$	$\frac{35}{36}$	$\frac{36}{36}$

~~Var~~

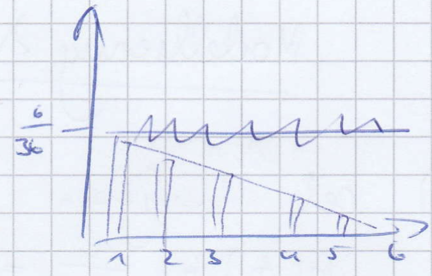
$$E(X_{sum}) = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} = 7$$

5,6

$$\text{Varianz } \sigma^2 = (2-7)^2 \cdot \frac{1}{36} + (3-7)^2 \cdot \frac{2}{36} + (4-7)^2 \cdot \frac{3}{36} \ldots$$

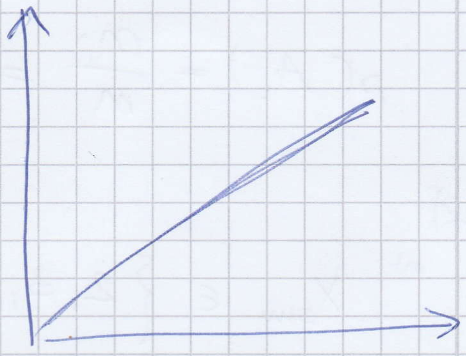
$=$

x_i	1	2	3	4	5	6
$P(X_{min} = x_i)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$
$A(t)$	$\frac{11}{36}$	$\frac{20}{36}$	$\frac{27}{36}$	$\frac{32}{36}$	$\frac{35}{36}$	$\frac{36}{36}$



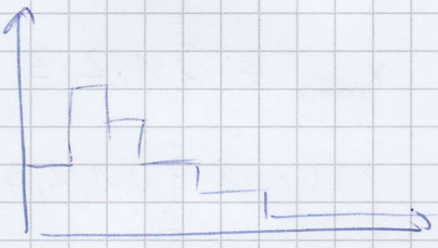
$E(x) = 2,5278$

x_i	1	2	3	4	5	6
$P(x_{max} = x_i)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$
$A(t)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{9}{36}$	$\frac{16}{36}$	$\frac{25}{36}$	$\frac{36}{36}$



$E(x) = \underline{4,47}$

x_i	0	1	2	3	4	5
$P(x_{diff_1} = x_i)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$
$A(t)$	$\frac{6}{36}$	$\frac{16}{36}$	$\frac{24}{36}$	$\frac{30}{36}$	$\frac{34}{36}$	$\frac{36}{36}$



$E(x) = \underline{1,94}$

s.o.

x_i	0	1	2	3	4	5
$P(x_{diff_2} = x_i)$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

→ abklären

Verteilungsfunktion:

$$F_X(t) = P(X \leq t) = \sum_{i: x_i \leq t} P(X = x_i)$$

③ $X \in \{1 \leq i \leq \infty\}$

geometrische Verteilung $x(i) = (1-p)^i \cdot p$

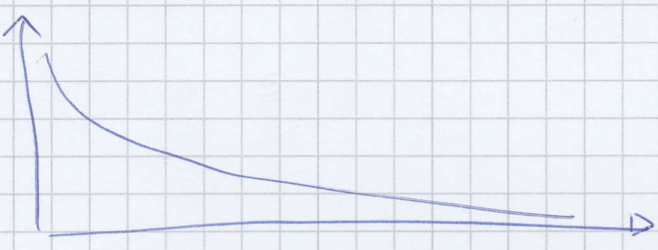
$i = 1 \quad + i = 2 \quad + i = 3 \quad + i = 4$

$$x(1) + x(2) + x(3) + x(4) = \frac{1}{6} \cdot \left(\frac{5}{6}\right)^0 + \frac{1}{6} \cdot \left(\frac{5}{6}\right)^1 + \frac{1}{6} \cdot \left(\frac{5}{6}\right)^2 + \frac{1}{6} \cdot \left(\frac{5}{6}\right)^3 = 0,5177 \triangleq \underline{51,77\%}$$

1.2

1/

geometrische Verteilung je nach p unterschiedlich



⇒ Kontinuierliche Verteilungsfunktionen wie die exponentielle Verteilung haben für alle x-Werte einen y-Wert; diskrete nicht

3/

geo

$$P(X \leq 1) = p \cdot (p-1)^0 = p$$

expon.

$$P(X \leq 1) = 1 - e^{-\lambda \cdot 1} + 1 - e^{-\lambda \cdot 0} = 1 - e^{-\lambda}$$