Large-Scale Distributed Second-Order Optimization

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Motivation and Related Work

- **Optimization**: Any branch of ML has optimization problems (RL, Graphics & Vision, DL, etc.)
- **Problem**: Increasing Data sizes $\Rightarrow$ Faster Convergence $\Rightarrow$ Better Optimizers, Parallel computing
- **First-Order Optimization Methods**: *SGD* [RM51], *Adam* [KB14], *AdamW* [LH17], *AMSGrad* [RKK19], *AdaBound* [LXLS19], *AMSBound* [LXLS19], *RAdam* [LJH^{+}19], *LookAhead* [ZLHB19]
- **Second-Order-Optimization Methods**: *Gauss-Newton-Method* [Sch02], *Natural Gradient Descent* (NGD) [Ama98], *Kronecker-factored Approximate Curvature* (K-FAC) [MG15]

- **Problem in Parallel Optimization:** Increasing Mini-Batch Size decreases validation accuracy

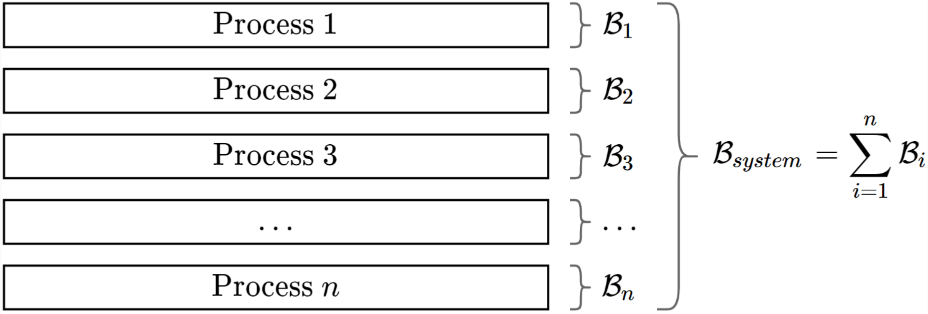

Figure: Increasing Mini-Batch size $\mathcal{B}_{system}$ for Parallel Computing

Update Rules Second-Order Optimization and FIM

Loss Term: $\mathcal{L}(\boldsymbol{\theta}; \mathcal{T}) = \sum_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathcal{T}} \ell\left(\hat{\mathbf{y}}_i, \mathbf{y}_i\right) = \sum_{(\mathbf{x}_i, \mathbf{y}_i) \in \mathcal{T}} \ell\left(F\left(\mathbf{x}_i; \boldsymbol{\theta}\right), \mathbf{y}_i\right)$

SGD Update Rule:

NGD Update Rule:

Fisher Information matrix:

Figure: Introduction to Notation and Update Rules

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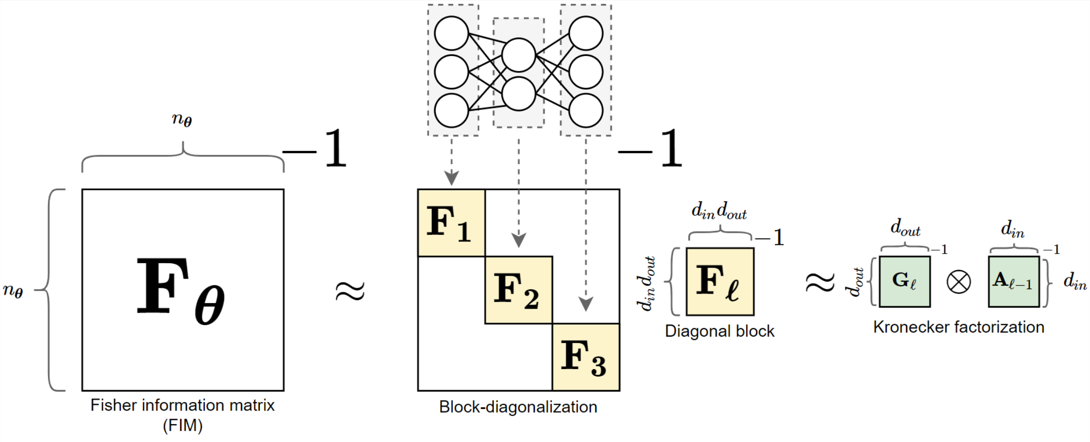


Figure: Approximation of the Fisher Information matrix alternated from: [Osa18]

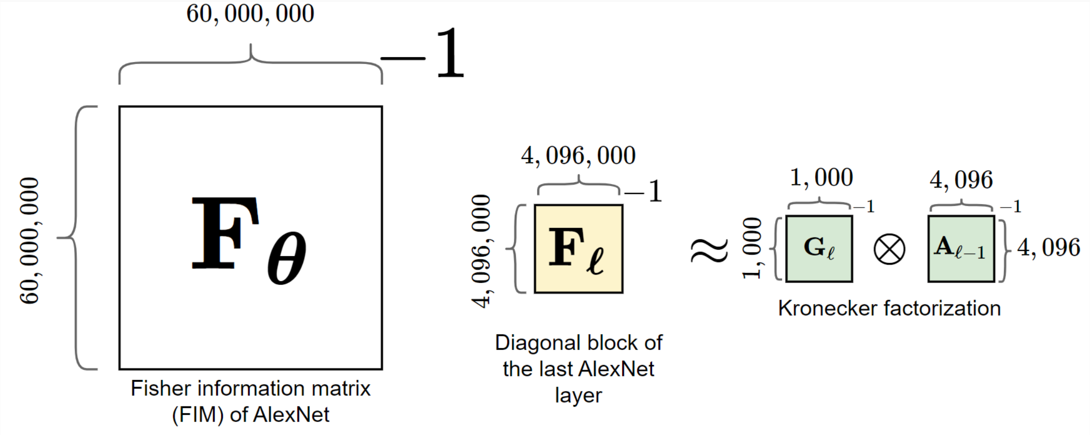
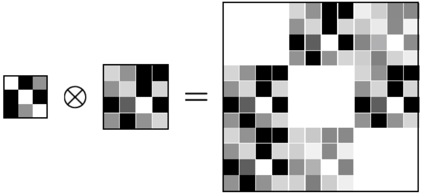


Figure: Approximation of the Fisher Information matrix for AlexNet alternated from: [Osa18]

Kronecker Product: [MG15]

$$\mathbf{A} \otimes \mathbf{B} := \begin{pmatrix} [\mathbf{A}]_{1,1}\,\mathbf{B} & \cdots & [\mathbf{A}]_{1,n}\,\mathbf{B} \\ \vdots & \ddots & \vdots \\ [\mathbf{A}]_{m,1}\,\mathbf{B} & \cdots & [\mathbf{A}]_{m,n}\,\mathbf{B} \end{pmatrix} \in \mathbb{R}^{ma \times nb}$$



$\mathbf{A} \in \mathbb{R}^{m \times n}, \mathbf{B} \in \mathbb{R}^{a \times b}$: Kronecker factors

Figure: Visualized Kronecker Product alternated from: [Osa18]

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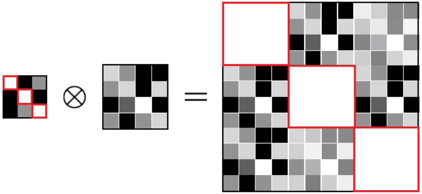

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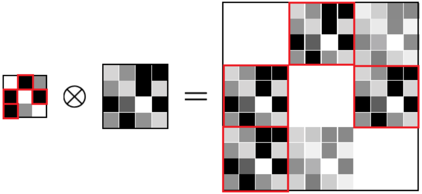

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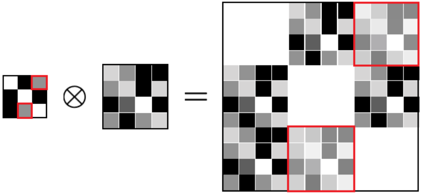

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K-FAC Comparison: [Osa18]

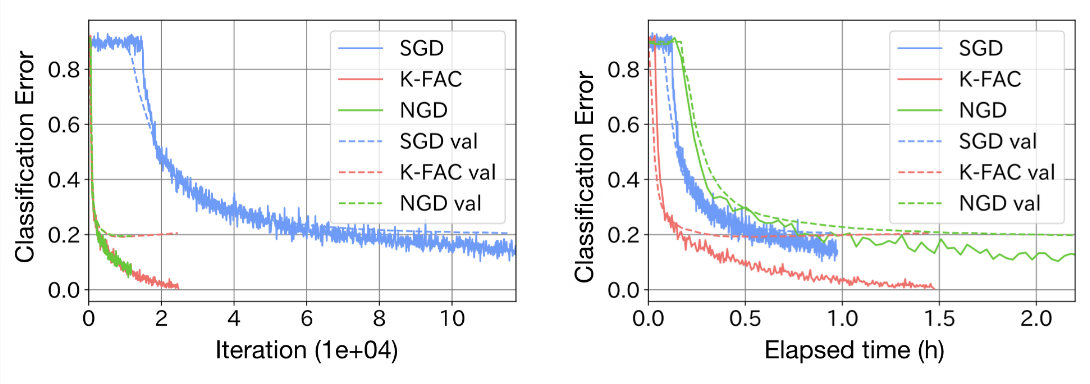

Figure: Comparison of training of ConvNet for CIFAR-10 dataset. Solid line - train, dashed line - validation: [Osa18]

Proposed Parallelized K-FAC Overview: [OTU+18]

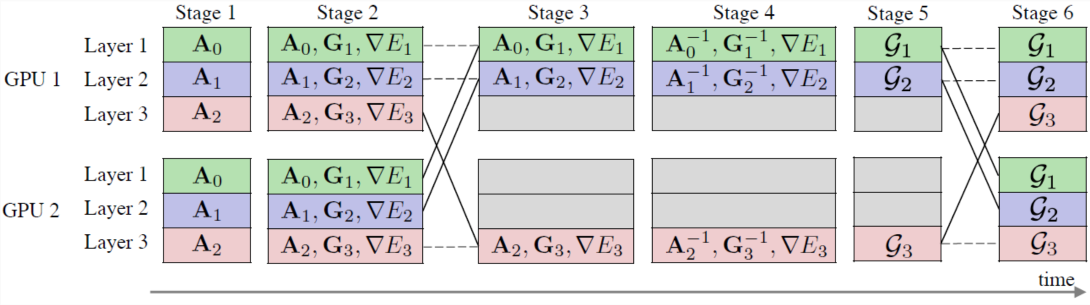

Figure: Proposed Parallelized K-FAC Overview: [OTU+18]

Results: [OTU+18]

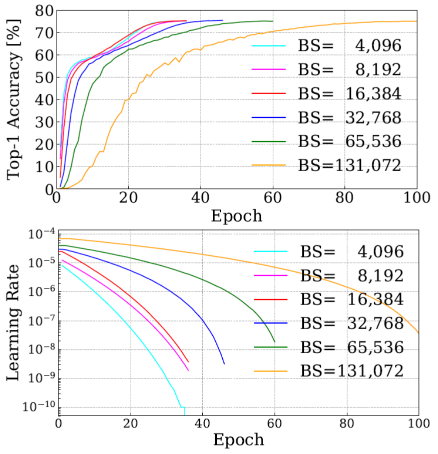

Figure: Accuracy & Learning rate of Parallelized K-FAC with different Batch sizes: [OTU+18]

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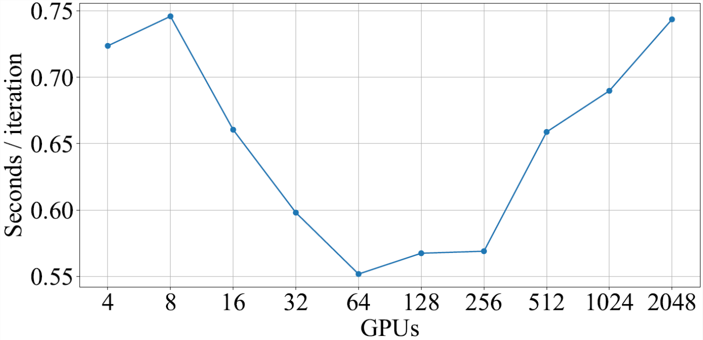

Figure: Iteration cost of Parallelized K-FAC with different amount of GPUs: [OTU$^+$18]

Results: [OTU$^+$18]

	Hardware	Software	Mini-batch size	Optimizer	Iteration	Time	Accuracy
Goyal *et al.* [9]	Tesla P100 × 256	Caffe2	8,192	SGD	14,076	1 hr	76.3%
You *et al.* [29]	KNL × 2048	Intel Caffe	32,768	SGD	3,519	20 min	75.4%
Akiba *et al.* [3]	Tesla P100 × 1024	Chainer	32,768	RMSprop/SGD	3,519	15 min	74.9%
You *et al.* [29]	KNL × 2048	Intel Caffe	32,768	SGD	2,503	14 min	74.9%
Jia *et al.* [15]	Tesla P40 × 2048	TensorFlow	65,536	SGD	1,800	6.6 min	75.8%
Ying *et al.* [28]	TPU v3 × 1024	TensorFlow	32,768	SGD	3,519	2.2 min	76.3%
Mikami *et al.* [22]	Tesla V100 × 3456	NNL	55,296	SGD	2,086	2.0 min	75.3%
This work (Sec. 5.4)	Tesla V100 × 1024	Chainer	32,768	K-FAC	1,760	10 min	74.9%
This work (Sec. 5.3)	-	Chainer	131,072	K-FAC	978	-	75.0%

Figure: Training iterations (time) and top-1 single-crop validation accuracy of ResNet-50 for ImageNet reported by related work: [OTU$^+$18]

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