Master's degree in Artificial Intelligence and Computer Science

OPTIMIZATION FOR MACHINE LEARNING - 6 CFU

Project N. 79

Let **dataset79MIL.mat** be a two-dimensional Multiple Instance Learning (MIL) dataset, where

- X is the matrix whose rows contain the instances of the bags to be classified;
- y is the array of the class labels of the bags;
- *instanceBag* is the array containing the bags to which the instances belong.

1. Indicating by m be the number of positive bags, by k the number of negative bags, and by J_i^+ and J_i^- the corresponding index sets, implement the following algorithm, named **SemiProximal Support Vector Machine (mi-SPSVM)**:

   Algorithm mi-SPSVM

   - Step 0 (Initialization). Set $J^+ := \{j \in J_i^+,\ i = 1, \ldots, m\}$ and $J^- := \{j \in J_i^-,\ i = 1, \ldots, k\}$. Set $l := 0$.
   - Step 1 (Solving the optimization problem). Solve the following semiproximal problem for $C = 1$:

$$\begin{cases} \min\limits_{v,\gamma,\xi} & \frac{1}{2}\left\| \begin{matrix} v \\ \gamma \end{matrix} \right\|^2 + \frac{C}{2}\sum\limits_{j\in J^+}\xi_j^2 + C\sum\limits_{j\in J^-}\xi_j \\ & \xi_j = 1 - (v^T x_j - \gamma) \quad j \in J^+ \\ & \xi_j \geq 1 + (v^T x_j - \gamma) \quad j \in J^- \\ & \xi_j \geq 0 \quad j \in J^-, \end{cases}$$

and let $(v^{(l)}, \gamma^{(l)}, \xi^{(l)})$ be its optimal solution.

- Step 2 (Stopping criterion). For any $i \in \{1, \ldots, m\}$ compute

$$j_i^* \stackrel{\triangle}{=} \arg \max_{j \in (J_i^+ \cap J^+)} \{v^{(l)T} x_j - \gamma^{(l)}\}.$$

  Let $J^* = \{j_i^*,\ i = 1, \ldots, m \mid v^{(l)T} x_{j_i^*} - \gamma^{(l)} \leq -1\}$ and $\bar{J} = \{j \in J^+ \setminus J^* \mid v^{(l)T} x_j - \gamma^{(l)} \leq -1\}$. If $\bar{J} = \emptyset$, STOP.
- Step 3 (Updating J^+ and J^-). Set $J^+ := J^+ \setminus \bar{J}$ and $J^- := J^- \cup \bar{J}$.
- Step 4 (New iteration) Set $l := l + 1$ and go to Step 1.

2. Draw a picture containing the following objects:

   - the instances (suggestion: represent the instances of the positive bags by filled circles and the instances of the negative bags by unfilled circles. For each bag, use different colors.)
   - the separating hyperplane $H(v, \gamma)$;
   - the hyperplane

$$H^+(v, \gamma) \stackrel{\triangle}{=} \{x \in R^n \mid v^T x = \gamma + 1\};$$

   - the hyperplane

$$H^-(v, \gamma) \stackrel{\triangle}{=} \{x \in R^n \mid v^T x = \gamma - 1\}.$$

3. Compute the training correctness.