清华大学电子工程系

媒体与认知 课堂 2

2023-2024 学年春季学期

作业 1

毕嘉仪 2024 年 3 月 20 日

理论部分

1 单选题（15 分）

1.1 B

1.2 A

1.3 B

1.4 A

1.5 B

2 计算题（15 分）

2.1 设隐含层为 $\mathbf{z} = \mathbf{W}^T\mathbf{x} + \mathbf{b}$，其中 $\mathbf{x} \in R^{(m\times 1)}$，$\mathbf{z} \in R^{(n\times 1)}$，$\mathbf{W} \in R^{(m\times n)}$，$\mathbf{b} \in R^{(n\times 1)}$ 均为已知，其激活函数如下：

$$\mathbf{y} = \delta(\mathbf{z}) = tanh(\mathbf{z})$$

tanh 表示双曲正切函数。若训练过程中的目标函数为 L，且已知 L 对 y 的导数 $\frac{\partial L}{\partial \mathbf{y}} = [\frac{\partial L}{\partial y_1}, \frac{\partial L}{\partial y_2}, ..., \frac{\partial L}{\partial y_n}]^T$ 和 $\mathbf{y} = [y_1, y_2, ..., y_n]^T$ 的值。

2.1.1 请使用 y 表示出 $\frac{\partial \mathbf{y}^T}{\partial \mathbf{z}}$，这里的 $\mathbf{y}^T$ 为行向量。

解.

$$\frac{\partial \boldsymbol{y}^{\mathrm{T}}}{\partial \boldsymbol{z}}_{n\times n} = \begin{bmatrix} \frac{\partial y_1}{\partial z_1} & \cdots & \frac{\partial y_n}{\partial z_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_1}{\partial z_n} & \cdots & \frac{\partial y_n}{\partial z_n} \end{bmatrix}$$

当 $i \neq j$, 易知 $\frac{\partial y_i}{\partial z_i} = 0$

当 $i = j$,

$$\tanh' z_i = 1 - \tanh^2 z_i$$

$$z_i = \operatorname{arctanh} y_i$$

$$\therefore \tanh' z_i = 1 - y_i^2$$

$$\therefore \frac{\partial \boldsymbol{y}^{\mathrm{T}}}{\partial \boldsymbol{z}}_{n\times n} = \operatorname{diag}(1-y_i^2) = \begin{bmatrix} 1-y_1^2 & & & \\ & 1-y_2^2 & & \\ & & \ddots & \\ & & & 1-y_n^2 \end{bmatrix}$$

□

2.1.2 请使用 y 和 $\frac{\partial L}{\partial \mathbf{y}}$ 表示 $\frac{\partial L}{\partial \mathbf{x}}$，$\frac{\partial L}{\partial \mathbf{W}}$，$\frac{\partial L}{\partial \mathbf{b}}$。

提示:$\frac{\partial L}{\partial \mathbf{x}}$，$\frac{\partial L}{\partial \mathbf{W}}$，$\frac{\partial L}{\partial \mathbf{b}}$ 与 x,W,b 具有相同维度。

解. 对 $\frac{\partial L}{\partial \boldsymbol{x}}$，由链式法则：

$$\frac{\partial L}{\partial \boldsymbol{x}}_{m\times 1} = \frac{\partial \boldsymbol{z}^{\mathrm{T}}}{\partial \boldsymbol{x}}_{m\times n} \frac{\partial \boldsymbol{y}^{\mathrm{T}}}{\partial \boldsymbol{z}}_{n\times n} \frac{\partial L}{\partial \boldsymbol{y}}_{n\times 1} = W \operatorname{diag}(1-y_i^2) \frac{\partial L}{\partial \boldsymbol{y}}$$

对于 $\frac{\partial L}{\partial W}$，先计算

$$\frac{\partial \boldsymbol{z}_i}{\partial W}_{m\times n} = \begin{bmatrix} \frac{\partial z_i}{\partial W_{11}} & \cdots & \frac{\partial z_i}{\partial W_{1n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial z_i}{\partial W_{m1}} & \cdots & \frac{\partial z_i}{\partial W_{mn}} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & \frac{\partial z_i}{\partial W_{1i}} & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & \frac{\partial z_i}{\partial W_{mi}} & \cdots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} & x_1 & \\ & \vdots & \\ & x_m & \end{bmatrix} = \begin{bmatrix} 0 & \dots & \boldsymbol{x} & \dots & 0 \end{bmatrix}$$

$$\therefore \frac{\partial L}{\partial W}_{m\times n} = \left(\frac{\partial \boldsymbol{z}_1}{\partial W}_{m\times n} \frac{\partial \boldsymbol{y}^{\mathrm{t}}}{\partial \boldsymbol{z}}_{n\times n} \frac{\partial L}{\partial \boldsymbol{y}}_{n\times 1}, \frac{\partial \boldsymbol{z}_2}{\partial W} \frac{\partial \boldsymbol{y}^{\mathrm{T}}}{\partial \boldsymbol{z}} \frac{\partial L}{\partial \boldsymbol{y}}, \dots, \frac{\partial \boldsymbol{z}_n}{\partial W} \frac{\partial \boldsymbol{y}^{\mathrm{T}}}{\partial \boldsymbol{z}} \frac{\partial L}{\partial \boldsymbol{y}} \right)_{m\times n}$$
$$= \left(\boldsymbol{x} \operatorname{diag}(1-y_i^2) \frac{\partial L}{\partial \boldsymbol{y}}, \dots, \boldsymbol{x} \operatorname{diag}(1-y_i^2) \frac{\partial L}{\partial \boldsymbol{y}} \right)$$

对于 $\frac{\partial L}{\partial \boldsymbol{b}}$，先计算

$$\frac{\partial \boldsymbol{z}^{\mathrm{T}}}{\partial \boldsymbol{b}}_{n\times n} = \begin{bmatrix} \frac{\partial z_1}{\partial b_1} & \cdots & \frac{\partial z_n}{\partial b_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial z_1}{\partial b_n} & \cdots & \frac{\partial z_n}{\partial z_n} \end{bmatrix} = I_n$$

$$\therefore \frac{\partial L}{\partial \boldsymbol{b}}_{n\times 1} = \frac{\partial \boldsymbol{z}^{\mathrm{T}}}{\partial \boldsymbol{b}}_{n\times n} \frac{\partial \boldsymbol{y}^{\mathrm{T}}}{\partial \boldsymbol{z}}_{n\times n} \frac{\partial L}{\partial \boldsymbol{y}}_{n\times 1} = \operatorname{diag}(1-y_i^2) \frac{\partial L}{\partial \boldsymbol{y}}$$

□