

Question 10: Please Be Careful Wild Turkey

Description

The turkey guardians care for all turkeys, not just those in their turkey sanctuary. They often observe turkeys in their natural habitat to make sure they are safe.

Recently, the guardians have come across a new rafter of turkeys (bet you didn't know that was a word for a group of turkeys) and found that their habitat seems quite dangerous.

The turkeys live in a gridded area that is surrounded on all sides by hot lava. At each time step, they move between any of the valid adjacent grid points (up, down, left, or right) with a few constraints.

Unlike like the calm turkeys found in a turkey sanctuary, wild turkeys are quite fast and have a lot of momentum. They are so fast that they cannot immediately begin moving in the opposite direction as the direction they are headed. For example, if a turkey is heading left, to begin heading right they must first go up or down for one time step. In other words, a turkey can never go left then right (or up then down, down then up, etc.) in sequential steps.

There are many holes in the newly found habitat, and if a turkey falls in, they cannot get out (so if a turkey falls in a hole, it remains in the same position for all subsequent time steps). Also, the turkeys can not leave their habitat. If they touch the lava at the edges of the habitat, they get burned and quickly jump back a space (away from the lava), but preserve their previous momentum. For example, if a turkey was heading left from square (1, 1) it could have the following series of moves (1, 1) -> [hit the lava] (2, 1), (2, 2). To clarify, upon hitting the left side of the grid (moving left from space (1, 1)), the turkey jumps back a space (to (2, 1)) but maintains the leftward momentum, so moving left, up, or down are all valid but moving right is not.

A turkey begins with upward momentum.

The benevolent guardians wish to determine the safety of the habitat. To do this, they wish to determine the number of paths, out of all possible paths of length T that a turkey can take, do not involve the turkey falling in a hole. Since the number of paths can be very large, report this value modulo 100000007.

Input Description

The input will be given as follows:

The first line will consist of four integers separated by spaces $2 < R \leq 50$, $2 < C \leq 50$, $1 \leq H < R \cdot C$, $1 \leq T \leq 50$, corresponding to the number of rows in the habitat, the number of columns in the habitat, the number of holes, and the number of time steps to be observed, respectively. The next line will contain two integers, $1 \leq x \leq R$ and $1 \leq y \leq C$, corresponding to the starting location of a turkey. The next H lines will contain two numbers, $1 \leq x \leq R$ and $1 \leq y \leq C$ corresponding to the location of a hole.

The grid is oriented as such: the top left of the grid is position (1, 1), the bottom left is position (R, 1), the top right is (1, C), and the bottom right is (R, C).

The grid is always at least 2×2 so there is always enough space for the turkey to be bounced back if it hits the border. You can assume that all **H** holes are distinct, the starting position is not in a hole, and that the turkeys start with upwards momentum.

Output Description

The output should contain a single integer, the number of paths where the turkey does not fall in a hole out of all possible paths, modulo **100000007**.

Input Example

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5 6 2 3
3 3
3 4
4 3
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Output example

16

Explanation

The habitat is a 5×6 grid with 2 holes at $(3, 4)$ and $(4, 3)$. The turkey starts at position $(3, 3)$ with upwards momentum and can move for at most 3 time steps. Here are all of the possible paths the turkey could take (in no particular order):

1. $(3, 3), (3, 2), (3, 1)$ <- hit the lava, $(3, 2)$
2. $(3, 3), (3, 2), (3, 1), (2, 1)$
3. $(3, 3), (3, 2), (3, 1), (4, 1)$
4. $(3, 3), (3, 2), (2, 2), (1, 2)$
5. $(3, 3), (3, 2), (2, 2), (2, 1)$
6. $(3, 3), (3, 2), (2, 2), (2, 3)$
7. $(3, 3), (3, 2), (4, 2), (5, 2)$
8. $(3, 3), (3, 2), (4, 2), (4, 1)$
9. $(3, 3), (3, 2), (4, 2), (4, 3)$ <- fell in hole
10. $(3, 3), (2, 3), (1, 3)$ <- hit the lava, $(2, 3)$
11. $(3, 3), (2, 3), (1, 3), (1, 2)$
12. $(3, 3), (2, 3), (1, 3), (1, 4)$
13. $(3, 3), (2, 3), (2, 2), (2, 1)$
14. $(3, 3), (2, 3), (2, 2), (1, 2)$
15. $(3, 3), (2, 3), (2, 2), (3, 2)$
16. $(3, 3), (2, 3), (2, 4), (2, 5)$
17. $(3, 3), (2, 3), (2, 4), (1, 4)$
18. $(3, 3), (2, 3), (2, 4), (3, 4)$ <- fell in hole
19. $(3, 3), (4, 3)$ <- fell in hole, $(4, 3), (4, 3)$

In path #10, the turkey moves left to $(2, 3)$, then left again to $(1, 3)$. At $(1, 3)$, the turkey again moves left, hits the lava, jumps back to $(2, 3)$ and maintains its leftward momentum. In path #19, the turkey falls in a hole after the first time step. The turkey is unable to leave the hole for the remainder of the time steps.

Out of the 19 possible paths, 3 involve the turkey falling in a hole (#9, #18, and #19). Thus, the correct answer is 16.