① $f(x) = \frac{1}{x} + 2\arctan x$ Zjistěte asymptoty

PREDTERMÍN

22.12.14

$D_f = \mathbb{R} \setminus \{0\}$

- $\lim_{x \to 0+} f(x) = \infty + 2 \cdot 0 = \infty$ $\underline{\underline{x = 0}}$ svislá

- $\lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{1}{x^2} + \frac{2\arctan x}{x} = 0 + 0 = 0$

$\lim_{x \to \pm\infty} (f(x) - 0x) = 0 \pm \pi = \pm\pi$

Asymptoty v $+\infty$ $\underline{\underline{y = \pi}}$ } vodorovné

v $-\infty$ $\underline{\underline{y = -\pi}}$

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② $f(x) = |4x - 2| - x^3 + 2x^2 - 1$ / Nalezněte max. a min. f na int $[-1, 3]$

→ spojitá → musí max. a min. existovat

$4x - 2 = 0 \Rightarrow x = \frac{1}{2}$

1) $x \in (-1; \frac{1}{2})$

$f(x) = -4x + 2 - x^3 + 2x^2 - 1$

$f'(x) = -3x^2 + 4x - 4 = 0$

$D = 16 - 4 \cdot (-3)(-4)$

$D < 0$

$f'(x) \neq 0$ na $(-1; \frac{1}{2})$

2) $x \in (\frac{1}{2}, 3)$

$f(x) = 4x - 2 - x^3 + 2x^2 - 1$

$f'(x) = -3x^2 + 4x + 4 = 0$

$D = 16 - 16 \cdot (-3) = 64$

$x_{1,2} = \frac{-4 \pm 8}{-6} < \begin{matrix} -\frac{2}{6} \\ 2 \end{matrix}$

Podezřelé body: $-1, 3, \frac{1}{2}, 2$

$f(-1) = 8$

$f(3) = 10 - 27 + 18 - 1 = 0$

$f(\frac{1}{2}) = -\frac{1}{8} + \frac{1}{2} - 1 = -\frac{5}{8}$

$f(2) = 6 - 8 + 8 - 1 = 5$

$\max f = f(-1) = 8$

$\min f = f(\frac{1}{2}) = -\frac{5}{8}$

③ $\int \frac{\ln^2 x + 2\ln x + 4}{x + x\cdot\ln^2 x}\,dx \overset{t=\ln x}{=} \int \frac{t^2+2t+4}{(1+t^2)\cancel{x}}\,dt\,\cancel{x} = \int 1 + \frac{2t+3}{1+t^2}\,dt$

$t = \ln x$
$d\cancel{x} = dt\,x$

$= \int dt + \int \frac{2x}{1+y^2} + 3\cdot\int \frac{1}{1+y^2} = t + \ln|1+t^2| + 3\arctan(t) + c$

$= \underline{\ln x + \ln|1+\ln^2 x| + 3\arctan(\ln x) + c}$

pro $x > 0$!

④ $\lim_{n\to\infty} \frac{(-2)^{n+1}+4^n}{2^{2n-1}+3^n} = \lim_{n\to\infty} \frac{(-2)^{n+1}+4^n}{\frac{1}{2}\cdot 4^n + 3^n}$ dominant $= \lim_{n\to\infty} \frac{-2(-2)^n + 4^n}{\frac{1}{2}\cdot 4^n + 3^n} =$

$= \lim_{n\to\infty} \frac{-2\left(-\frac{2}{4}\right)^n + 1}{\frac{1}{2} + \left(\frac{3}{4}\right)^n} = \frac{0+1}{\frac{1}{2}+0}$ dominuji $= \underline{\underline{2}}$

⑤ $\int_{-\infty}^{0} e^{2x} \sin\left(\frac{\pi}{2} e^x\right)dx = \left| \begin{array}{l} dt = \frac{\pi}{2}e^x dx \\ t = \frac{\pi}{2}e^x \end{array} \right| = \frac{2^2}{\pi^2}\int_0^{\frac{\pi}{2}} t\cdot\sin t = \frac{4}{\pi^2} =$

$\frac{4}{\pi^2}\int_{-\infty}^{0} \frac{\pi^2}{4} e^{2x}\sin\left(e^x \frac{\pi}{2}\right)dx$

$\left| \begin{array}{ll} f(x) = t & f'(x) = 1 \\ g(x) = \sin t & G(x) = (-\cos t) \end{array} \right.$

$= \left\{ \underbrace{\left[t(-\cos t)\right]_0^{\frac{\pi}{2}}}_{0} - \int_0^{\frac{\pi}{2}} (-\cos t)\cdot 1\,dt \right\} = \frac{4}{\pi^2}\int_0^{\frac{\pi}{2}} \cos t\,dt =$

$= \frac{4}{\pi^2}\left[\sin t\right]_0^{\frac{\pi}{2}} = \frac{4}{\pi^2}(1-0) = \underline{\underline{\frac{4}{\pi^2}}}$

(6.) Laplaceova transformace

$$y'' - y' - 2y = te^t \qquad y(0) = 0, \quad y'(0) = 1$$

$$p^2 \cdot \mathcal{L}(y) - p \cdot 0 - 1 - p \cdot \mathcal{L}(y) + 0 - 2\mathcal{L}(y) = \frac{1}{(p-1)^2}$$

$$\mathcal{L}(y) \cdot \underbrace{(p^2 - p - 2)}_{(p-2)(p+1)} - 1 = \frac{1}{(p-1)^2}$$

$$\mathcal{L}(y) = \frac{1 + (p-1)^2}{(p-1)^2 (p-2)(p+1)}$$

$$\mathcal{L}(y) = \frac{A}{p-1} + \frac{B}{(p-1)^2} + \frac{C}{p-2} + \frac{D}{p+1}$$

$$\mathcal{L}(y) = \frac{-\frac{1}{4}}{p-1} + \frac{-\frac{1}{2}}{(p-1)^2} + \frac{\frac{2}{3}}{p-2} + \frac{-\frac{5}{12}}{p+1}$$

$$B = -\frac{1}{2}, \quad C = \frac{2}{3}, \quad D = \frac{5}{-12}$$

$$p^2 - 2p + 2 = A \cdot (p-1)(p-2)(p+1) + B(p-2)(p+1) + C \cdot (p-1)^2 (p+1) + D \cdot (p-1)^2 (p-2)$$

$$p^3: \quad 0 = A + C + D \Rightarrow A = -C - D = -\frac{2}{3} + \frac{5}{12} = \frac{-8+5}{12} = -\frac{1}{4}$$

$$\mathcal{L}^{-1}(y) = -\frac{1}{4}e^t - \frac{1}{2}te^t + \frac{2}{3}e^{2t} - \frac{5}{12}e^{-t}$$

$t \geq 0$

(7) Posloupnost $(a_n)_{n=1}^{\infty}$ je omez., jestliže ex $K \in \mathbb{R}: |a_n| \leq K$ $\forall n$

Posloupnost $(a_n)_{n=1}^{\infty}$ je rostoucí jestliže $a_{n+1} > a_n$ $\forall n \in \mathbb{N}$

Ukažte, že

$$\left(\frac{3n-1}{n+1}\right)_{n=1}^{\infty}$$ je rost. & omez. ?

Rostoucí: $a_{n+1} - a_n = \frac{3(n+1)-1}{(n+1)+1} - \frac{3n-1}{n+1} = \frac{3n+2}{n+2} - \frac{3n-1}{n+1}$

$= \frac{(3n+2)(n+1) - (3n-1)(n+2)}{(n+2)(n+1)}$ → vždy + $= \frac{2+2}{(n+2)(n+1)} = \frac{+}{+}$ → rostoucí

OMEZENÁ

$$\left|\frac{3n-1}{n+1}\right| = \frac{3n-1}{n+1} \leq \frac{3n}{n+1} = \frac{3}{\frac{n}{n} + \frac{1}{n}} = \frac{3}{1 + \frac{1}{n}} \leq 3$$

← je vždy kladné

⇒ je omezená