Lab 2: FORCED OSCILLATIONS OF A TWO-DEGREE-OF-FREEDOM SYSTEM

Objectives of the Lab:

- To study the response of a 2DOF (two degrees of freedom) electrical circuit to a periodic excitation.
- To analyze the evolution of voltage variations in different parts of the circuit as a function of the frequency of the applied signal.
- To plot response curves in order to identify the characteristics of the circuit, such as resonance peaks.
- To study the phenomena of resonance and attenuation and their effects on circuit behavior.
- To examine the impact of the coupling coefficient on the natural frequencies of the system.

1- Theoretical Background

Study of the response to an external sinusoidal excitation: We examine symmetrically coupled circuits, as illustrated in Figure III-1, which are now powered by a sinusoidal voltage source.

$$e(t) = e_0 e^{j\omega t}$$

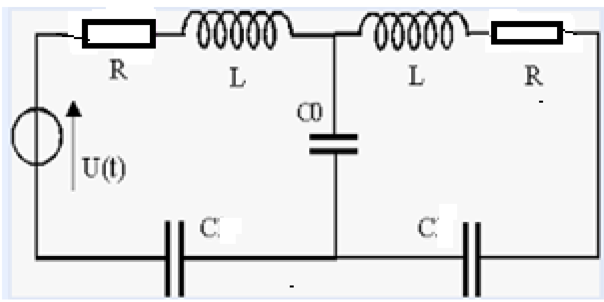

Figure 1: Symmetrically Coupled RLC Circuits (2DOF)

The coupling coefficient K between the two loops is defined by: $K = \frac{C}{C+C_0}$ From which it follows $\frac{C_0}{C} = \frac{1-K}{K}$ We can also study the oscillations of the system by analyzing the loop currents $i_1(t)$ and $i_2(t)$, which, through integration, will provide the charges $q_1(t)$ and $q_2(t)$, and consequently the desired voltages $V_1(t)$ and $V_2(t)$

We define $Z=R+j\ (L\omega-1c\omega)$ as the impedance of each uncoupled loop.

We have

$$i_1+i_2=eZ$$
$$i_1-i_2=jZC_0i_2$$

The solutions to this system are as follows:

$$i_1 = \frac{1+jZC_0\omega}{Z(2+jZC_0\omega)}e(t)$$

$$i_2 = \frac{1}{Z(2+jZC_0\omega)}e(t)$$

We deduce that:

$$V_1(t) = \frac{\frac{K}{1-K} + \left((1-x^2)+2\,j\beta x\right)}{\left((1-x^2)+2j\beta x\right)\left(\frac{2K}{1-K}+(1-x^2)+2j\beta x\right)}e(t)$$

$$V_2(t) = \frac{\frac{K}{1-K}}{\left((1-x^2)+2j\beta x\right)\left(\frac{2K}{1-K}+(1-x^2)+2j\beta x\right)}e(t)$$

By setting $X=\omega/\omega_0$ et $\beta=\delta/\omega_0$ the amplitudes of $V_1(t)$ and $V_2(t)$ tend respectively toward $e_0/(1+K)$ et $Ke_0/(1+K)$ as ω tends to 0

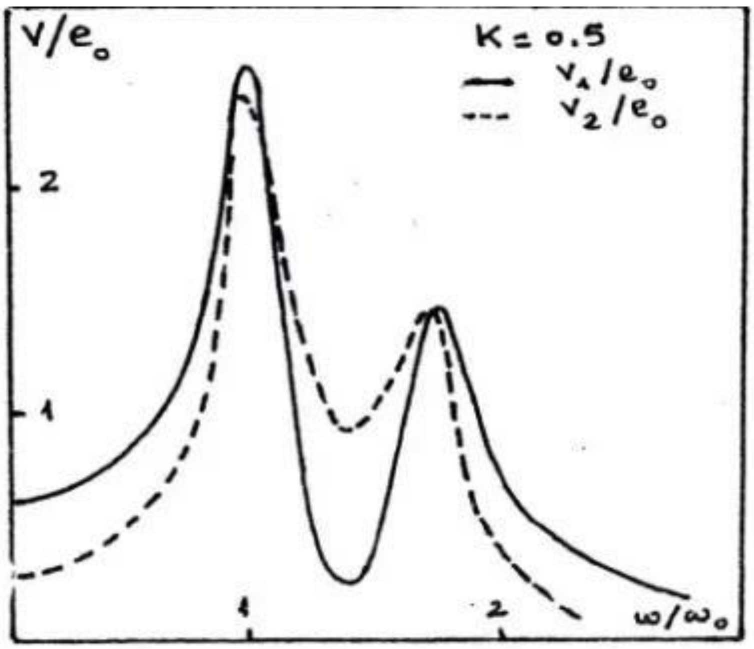

Figure I2 illustrates the variation of the relative amplitudes $\mathbf{V_1/e_0}$ and $\mathbf{V_2/e_0}$ as a function of the ratio ω/ω_0 for a coupling coefficient K=0.5. It shows the presence of two distinct resonances for each amplitude.

2. EXPERIMENTAL PROCEDURE Equipment Used:

- Low-frequency generator providing a sinusoidal signal
- Capacitors
- Two inductors
- An oscilloscope
- Wires

Procedure:

Initially, observe the evolution of the voltages V1(t) on the oscilloscope as a function of the excitation frequency, for a given coupling coefficient. Then plot the curve showing how their amplitudes vary with ω. This will highlight the resonance phenomena, where the amplitudes reach their maximum, as well as the vibration attenuation phenomena

In the next step, use these observations to study how the system's natural frequencies change with the coupling coefficient K. You will need to analyze the effect of Kon the resonance frequencies and the system's dynamics.

Use C=22 nF and C_0=22 nF. The inductances can be 3.3 mH . Observe the signal e(t) on one of the oscilloscope channels. Adjust and record its amplitude e_0, which should be chosen to be less than 0.5 Volt.

3. Study of Voltage Response Curves

a) Observation of the response $V_1(t)$

Connect the oscilloscope so that it is possible to alternately visualize the voltages $V_1(t)$ Vary the frequency of the input signal e(t) over a wide range while observing the shape of $V_1(t)$ In particular, highlight the resonances and attenuation of V1. Measure the frequencies and amplitudes at resonance.

b) Plotting of Response Curves:

The plotting should be done simultaneously with the measurements. Since the maximum amplitude values have already been measured, the vertical scale in V1 can therefore be defined in advance. Moreover, the angular frequency ω will be varied from approximately 0 up to $3\omega_0$.

Reminder: ω_0 represents the natural angular frequency of each of the two decoupled loops. The horizontal scale in ω can therefore also be defined in advance. Record and plot, for each value of the angular frequency ω.

c) Comparison with Predictions: From the graph, deduce the resonance angular frequencies Ω1 et Ω2 as well as ω_{min}, the anti-resonance angular frequency of the oscillation V1(t), then estimate the amplitude limits as ω tends to 0. Calculate the coupling coefficient using the various relations introduced in the theoretical section. Calculate the theoretical values of these same quantities based on the circuit elements. Compare both sets of values.

4. Influence of Coupling on Natural Frequencies:

The results obtained in the previous section have shown that a forced system with two degrees of freedom resonates whenever the frequency of the external excitation is equal to, or very close to, one of the two natural frequencies of the system.

In this section, we aim to use this resonance phenomenon to measure the natural angular frequencies $\Omega_1 et\ \Omega_2$ of a two-degree-of-freedom system by varying the coupling coefficient K. The goal is to study the evolution of $\Omega_1 et\ \Omega_2$ as a function of K. We keep the previous experimental setup with the additional possibility of modifying the capacitance C_0 using various capacitors (10 nF, 22 nF, 32 nF, 47 nF, 57 nF) connected in series or parallel. This provides different values of the coupling coefficient K. For each of these values of K, we will determine the two resonance angular frequencies $\Omega_1 et\ \Omega_2$ Plot on the same graph the curves of $\Omega_1^2\ et\ \Omega_2^2$ as a function of K. Compare with the theoretical predictions and discuss.