Lab 2: FREE AND FORCED OSCILLATIONS OF A SINGLE DEGREE OF FREEDOM SYSTEM

Objectives of the Lab:

- Study of Free and Forced Oscillations in an RLC Circuit with a Single Degree of Freedom
- Experimental determination of the pseudoperiod, damping factor, and total resistance of the circuit
- Experimental determination of the resonance frequency of the RLC circuit and comparison with its theoretical value
- Study of the amplitude of the voltage across the capacitor as a function of the input frequency
- Determination of the bandwidth and quality factor

1-FREE AND FORCED OSCILLATIONS OF A SERIES RLC CIRCUIT 1-1-. Free Oscillations of a Series RLC Circuit

We have the RLC circuit shown in *Figure 1*. When switch S is closed, the electrical charges stored in the capacitor discharge into the inductor L and the resistor R, resulting in free oscillations within the circuit. The resistor causes energy dissipation, which leads to a gradual decrease in the amplitude of the oscillations. This energy dissipation is responsible for the damping of the oscillations, reducing their intensity over time until the circuit returns to an equilibrium state.

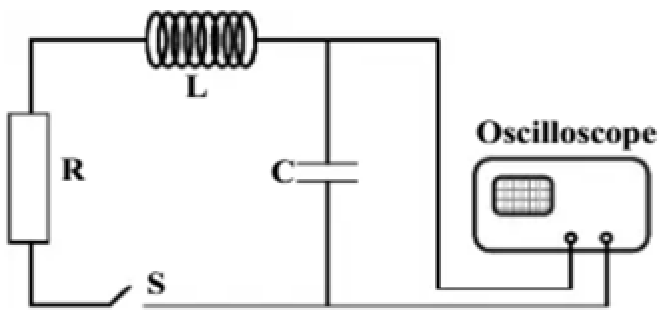

Figure 1: RLC Circuit in Free Oscillations with 1 Degree of Freedom (SDOF)

By applying Kirchhoff's loop law : $U_C+U_L+U_R=0$ If we denote by q(t) the charge as a function of time t, the differential equation describing the RLC circuit in the free oscillation regime is given by:

$$L\frac{d^2q}{dq} + R\frac{dq}{dt} + \frac{1}{c}q = 0 \text{..................(1)}$$

This equation is a second-order linear differential equation with constant coefficients. The terms in the equation correspond respectively to the energy stored in the inductor, the energy dissipation by the resistor, and the energy stored in the capacitor. To analyze the oscillations of the RLC circuit, we start with the characteristic equation associated with the differential equation:

$$Lr^2 + Rr + \frac{1}{C} = 0$$

Where r represents the roots of the characteristic equation. The discriminant of this equation is:

$$\Delta = R^2 - 4\frac{L}{C}$$

Depending on the value of the discriminant, three distinct cases can be observed:

a) Overdamped Response : $\Delta > 0 \rightarrow R > 2\sqrt{\frac{L}{C}}$

The roots of the characteristic equation are real numbers r_1 and r_2 the general solution to equation (1) is:

$$q(t) = Ae^{r_1 t} + Be^{r_2 t}$$

Where A and B are constants.

In this case, the damping is strong enough to prevent continuous oscillations. The system may exhibit a rapid decrease in amplitude without periodic oscillations. The circuit stabilizes with a progressively decreasing exponential amplitude.

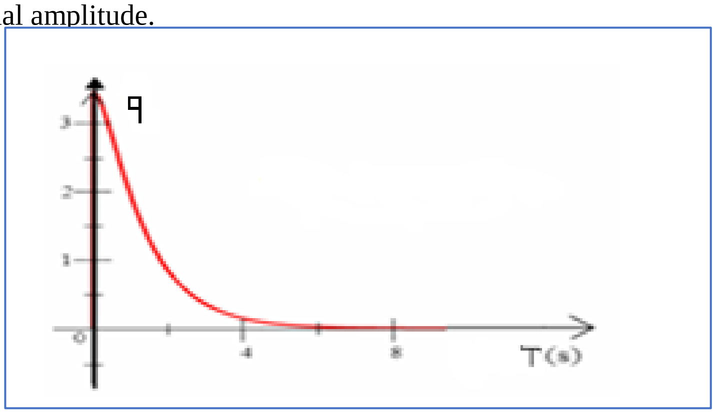

Figure 2: Evolution of the charge in the **overdamped response**

b) Critically Damped Response: $\Delta = 0 \rightarrow R = 2\sqrt{\frac{L}{C}}$

The solution in this case is of the form: $q(t) = (A + Bt)e^{-\frac{R}{2L}t}$

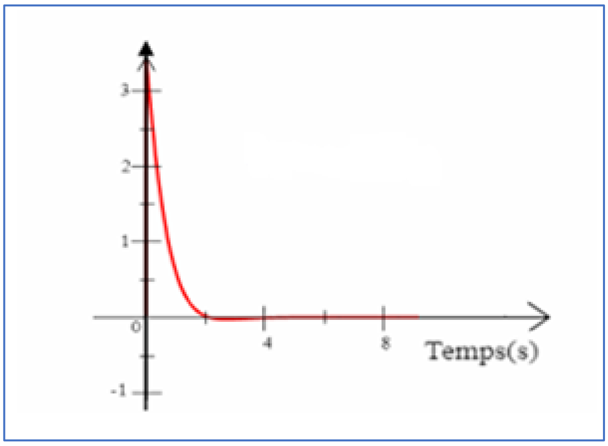

Figure 3: Evolution of the charge for the critically damped response

c) Underdamped Response : $\Delta<0 \rightarrow R< 2\sqrt{\frac{L}{C}}$

The roots of the characteristic equation r_1 and r_2 are complex conjugates. The solution is

$$q(t) = Ae^{r_1 t} + Be^{r_2 t}$$

Where: $r_1 = -\delta + j\omega_d$ et $r_2 = -\delta - j\omega_d$

It can be written in the form :

$$q(t) = q_m e^{-\delta t} \cos(\omega_d t + \varphi)$$

This case indicates that the system exhibits periodic oscillations whose amplitude decreases exponentially due to damping. The oscillations are noticeable, but their intensity gradually diminishes until they disappear.

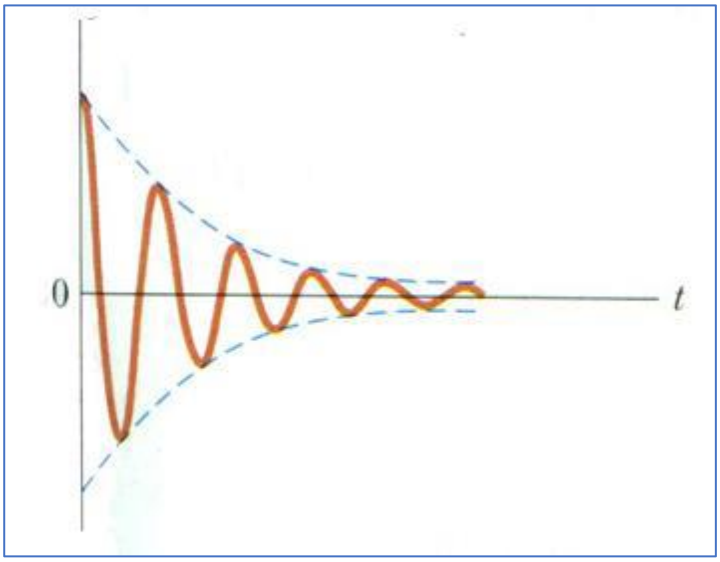

Figure 4: Evolution of the load in the underdamped response

Logarithmic Decrement: The logarithmic decrement is an essential tool for quantifying and understanding the damping of oscillations in a dynamic system.

Definition

The logarithmic decrement D is the natural logarithm of the ratio between two successive amplitudes of an oscillation cycle. Formally, it is given by:

$$D = \frac{1}{n} \ln(\frac{q(t)}{q(t + nT_d)}$$

$$T_d = \frac{2\pi}{\omega_d}$$

q(t) is the initial amplitude of the oscillation, $q(t+nT_d)$ is the amplitude after n pseudo-periods, n is the number of Underdamped-periods after which the amplitude $q(t+nT_d)$ is measured.

1-2- Forced Oscillations of a Series RLC Circuit

When the switch S is replaced by a generator that delivers a time-varying voltage, forced oscillations occur in the circuit. Specifically, if the generator provides a sinusoidal voltage, the RLC circuit will be subjected to forced oscillations with the same frequency as the applied voltage. (Figure 5)

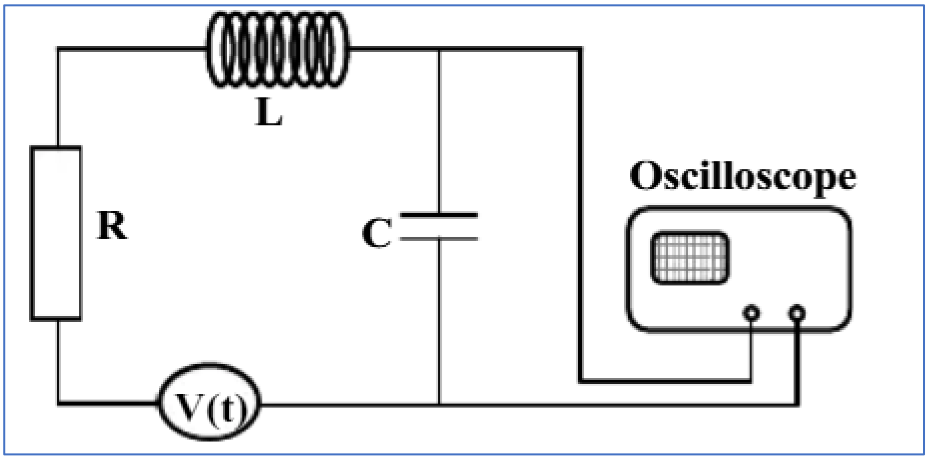

Figure 5: RLC Circuit in Forced Oscillations

The equation of the circuit is:

$$Lr^2 + Rr + \frac{1}{C} = V_m e^{j\omega t}$$

$V_m e^{j\omega t}$: Is the applied voltage. The general solution to this equation is of the form: $q(t) = q_h(t) + q_p(t)$, $q_h(t)$ is the solution to the homogeneous equation, and $q_p(t)$ is the particular solution with the non-homogeneous term. It can be written in the form: $q(t) = q_0 e^{j(\omega t+\varphi)}$

- **Study of the Amplitude** The amplitude of the potential difference across the capacitor is:

$$V_c = \frac{V_m}{\sqrt{(1-LC\omega^2)^2+R^2C^2\omega^2}}$$

The derivative $dV/d\omega$ is zero for two values of ω:

a- ω=0 (direct current, solution to be rejected)

b- $\omega_m^2 = \frac{2LC-R^2C^2}{2L^2C^2}$

This last value is only possible if the numerator is positive, which gives the condition : $R< \sqrt{\frac{2L}{C}}$ then presents a maximum: this is the resonance phenomenon If $R> \sqrt{\frac{2L}{C}}$, the resonance phenomenon disappears, as shown in Figure 6.

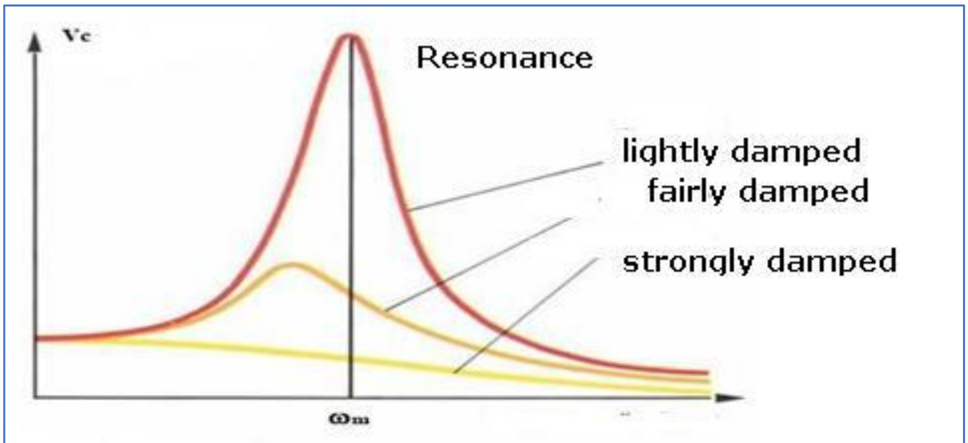

Figure 6: Evolution of V_c for an RLC circuit in forced oscillations

Resonance If the frequency of the generator matches the natural frequency of the circuit, the phenomenon of resonance will be observed, where the oscillations will be strongest. It's as if the circuit is "in tune" with the frequency of the generator. Outside of resonance, the oscillations are less significant because the impedance of the circuit increases, resisting the current more. In the case where $R< \sqrt{\frac{2L}{C}}$, we define the bandwidth of the circuit: it is the range of values of the driving angular frequency Ω, within the interval $[\omega_2, \omega_1]$, such that the amplitude $V_m(\Omega)$ of the output voltage takes values between $\frac{V_m(\Omega)}{\sqrt{2}}$ and $V_m(\Omega)$ is the resonance angular frequency

ω_1 and ω_2 are the cutoff frequencies (the corresponding cutoff frequencies can also be used: f_1 and f_2 . Thus, we have: $B=\omega_2-\omega_1$ It can be demonstrated that $B=\omega_2-\omega_1=2\delta$, where δ is the damping factor. We also define the quality factor Q of the circuit as $Q=\frac{\omega_0}{B}=\frac{\omega_0}{2\delta}$. A system is more heavily damped if the quality factor is small.
Figure 7 illustrates the concept of determining the bandwidth.

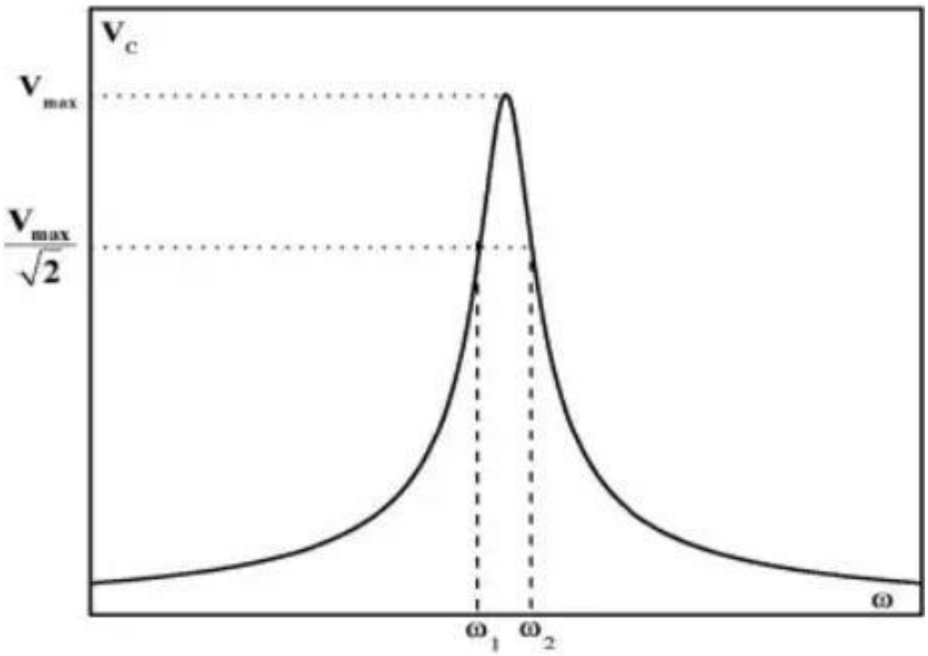

Figure 7: Experimental determination of the bandwidth

EXPERIMENTATION

• **Equipment used :**

- Low-frequency generator
- Variable resistance box
- Capacitor
- Inductor
- Oscilloscope
- Electrical wires

1. Oscillations of a Series RLC Circuit

Set up the circuit as shown in Figure 1. Use an oscilloscope to measure the potential difference. Replace switch S with a low-frequency generator (LFG) producing a square signal. Choose a relatively low frequency, between 100 and 1000 Hz, and use an inductance of 5 mH and a capacitor of 22 nF.
Note: Before powering the circuit, ensure that your setup is verified by your professor.

Underdamped response Set $R=0\Omega$. In this case, damping is ensured by other resistances present in the circuit, such as those from the generator, Inductor, and Electrical wires.
Measure the *Underdamped -period* experimentally and the damping factor δ, then calculate the total sum of the resistances in the circuit.

Critically Damped Response: : The Critically Damped Response is experimentally observed when damping is low enough that the circuit's response becomes purely exponential. To determine this critical regime:

- Start with a relatively high resistance (e.g 10 kΩ) and gradually reduce it.
- Note the resistance value where the response becomes exponential, and compare this to the theoretical value. **Conclusion?**

2. Forced Oscillations of a Series RLC Circuit

Set up the circuit according to Figure 5, using an inductance L=5 mH and a capacitor C=22nF.

- Plot the curves representing the amplitude of the voltage across the capacitor C as a function of the input signal frequency e(t).
- Determine the experimental resonance frequencies and compare them with the theoretical values.

For accurate results, vary the input signal frequency by reducing the intervals around resonance, taking at least four measurements on each side of the maximum amplitude V. Measure the amplitude of the response across C relative to e(t) from the oscilloscope. Ensure that the input signal amplitude e(t) remains constant, readjusting it if necessary.

- From the amplitude curves, determine the bandwidth and deduce the corresponding quality factors. Finally, draw a conclusion based on your results.