CONVEX OPTIMIZATION

Computer Homework 2

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Contents

1 Problem #1

1.1 Part A

The likelihood function is:

$$L(\lambda, x_1, ..., x_2) = \prod_{j=1}^{n} exp(-\lambda_j)\frac{\lambda_j^{x_j}}{x_j!}$$

When we get logarithm:

$$l_i = -\lambda_i + N_i \log \lambda_i - \log N_i!$$

So we have:

$$l = \sum_{i=1}^{24} -\lambda_i + N_i \log \lambda_i - \log N_i!$$

$$min \quad l = \sum_{i=1}^{24} -\lambda_i + N_i \log \lambda_i - \log N_i!$$

$$subject \quad \lambda \geq 0$$

$$\min \quad l = \sum_{i=1}^{24} -\lambda_i + N_i \log \lambda_i - \log N_i! = \sum_{i=1}^{24} \min[-\lambda_i + N_i \log \lambda_i - \log N_i!]$$

there for we have $N_i > 0$:

$$\min[-\lambda_i + N_i \log \lambda_i - \log N_i!] \longrightarrow \lambda_i = N_i$$

if $N_i = 0$:

$$\min \lambda_i \geq 0 \longrightarrow \lambda_i = 0\Box$$

1.2 Part B

For regularization part we have:

$$\min \quad l = \sum_{i=1}^{24} -\lambda_i + N_i \log \lambda_i - \log N_i! + \rho(\sum_{i=1}^{23}(\lambda_{i+1} - \lambda_i)^2 + (\lambda_{24} - \lambda_1)^2)$$

$$subject \quad \lambda_i \geq 0$$

$\Box$

1.3 Part C

when $\rho \to \infty$, the only important part is regularization part.
so basically we have:

$$\min \quad (\lambda_{i+1} - \lambda_i)^2 + (\lambda_{24} - \lambda_1)^2$$

$$\lambda_i = \lambda_2 = ... = \lambda_{24} = \hat{\lambda} = \frac{\sum_i N_i}{24}$$

□

1.4 Part D

1.5 Part E

2 Problem #2

2.1 part A

For this problem we have:

$$\begin{cases} \max_x \sum r_i(x_i) \\ \text{subject to :} Ax \leq c^{\max} \end{cases}$$

hence we have:

$$-r_i x_i = \max\{= p_i x_i, -p_i - p_i^{disc}(x_i - q_i)\}$$

So we can have :

$$\begin{cases} \min_{x,t} \sum t_i \\ \text{subject to: } Ax \geq c^{\max} \\ t_i \geq -p_i x_i \\ t_i \geq -p_i - p_i^{disc}(x_i - q_i) \end{cases}$$

therefor:

$$\begin{cases} \min_{x,t} \quad [0_n^T, 1_n^T] \begin{bmatrix} x \\ t \end{bmatrix} \\ \text{subject to: } \begin{bmatrix} A & 0_{n\times n} \\ -\text{diag}(p) & -I_n \\ -\text{diag}(p^{disc} & -I_n \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix} \geq \begin{bmatrix} c^{max} \\ 0_n \\ [p_i q_i - p_i^{disc} q_i]_{n\times 1} \end{bmatrix} \end{cases}$$

□

2.2 Part B

The output of the codes are here:

■

Figure 1: Caption

3 Problem #3

3.1 Part A

We set the problem like this:

$$\begin{cases} \min \sum_{i=1}^{n} t_i \phi(s_i) \\ s_i^{min} \leq s_i \leq s_i^{max} \\ \tau_i^{min} \leq \tau_i \leq \tau_i^{max} \end{cases}$$

we also know that:

$$\tau_i = \sum_{k=1}^{i} t_k = \sum_{k=1}^{i} \frac{d_i}{s_i}$$

$$\begin{cases} \min \sum_{i=1}^{n} t_i \phi(\frac{d_i}{t_i}) \\ \frac{d_i}{s_i^{max}} \leq \tau_i \leq \frac{d_i}{s_i^{min}} \\ \tau_i^{min} \leq \tau_i \leq \tau_i^{max} \end{cases}$$

the subject of the optimization we also know $t_i \phi(\frac{d_i}{t_i})$ is perspective and we can say the sum of them is also convex. we can also compute $s_i = \frac{d_i}{t_i}$.
□

3.2 Part B

4 Problem #4

In thepython notebook file