CONVEX OPTIMIZATION

Computer Homework 2

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Contents

1 Problem #1

The optimization problem is about minimization of:

$$\min \sum_{t=1}^{T}(r^T q_t + q_t^T diag(s) q_t) + \sum_{t=1}^{T-1}(\tilde{r}^T |f_t| + f_t^T diag(\tilde{s}) f_t)$$

The objective function is sum of convex functions hence its convex So for cvxpy input we will have:

$$\min \sum_{t=1}^{T}(r^T q_t + s^T q_t^2) + \sum_{t=1}^{T-1}(\tilde{r}^T |f_t| + \tilde{s} f_t^2)$$

$$q_{t+1} = A f_t + q_t$$

$$0 \leq q_t \leq Q$$

$$|f_t| \leq F$$

The codes and results are in the ipynb file.

2 Problem #2

For this problem we know that f is convex function and because of that, due to jensen inequality, we can say:

$$f(w) \leq \sum_{i=1}^{K} \theta_i f(w^{(i)})$$

Moreover, we know that When f is a polynomial function,then we can say $f(\log(e^x))$ is convex too. overall, we have:

$$\log(w) = \sum_{i=1}^{K} \theta_i \log(w^{(i)}) \Rightarrow \log(P(w)) \leq \sum_{i=1}^{K} \theta_i \log(P(w^{(i)}))$$

Therefore the convex problem is:

$$\min \theta$$

$$\sum_{i=1}^{K} \theta_i \log(P(w^{(i)})) \leq \log(P_{spec})$$

$$\sum_{i=1}^{K} \theta_i \log(D(w^{(i)})) \leq \log(D_{spec})$$

$$\sum_{i=1}^{K} \theta_i \log(A(w^{(i)})) \leq \log(A_{spec})$$

$$\mathbf{1}^T \theta = 1$$

$$\theta \geq 0$$

The codes and results are in the ipynb file.

3 Problem #3

3.1 Part A

Consider S and T like this:

$$S = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, T = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Now we have C like this:

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1.5 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

by calculating C's eigenvalues you found that this matrix isn't semi-positive so it's not covariance matrix.

3.2 Part B

The problem is:

$$\min_C ||C^{(1)} - S||_F^2 + ||C^{(2)} - T||_F^2 + ||C_{13}||_F^2$$

that's equal to:

$$\min_C ||C_{11}-S_{11}||_F^2+2||C_{12}-S_{12}||_F^2+||C_{22}-S_{22}||_F^2+||C_{22}-T_{22}||_F^2+2||C_{23}-T_{23}||_F^2+||C_{33}-T_{33}||_F^2+||C_{13}||_F^2$$

We can separate them and we found that $C_{11} = S_{11}$,$C_{12} = S_{12}$,$C_{13} = 0$,$C_{11} = S_{11}$,$C_{23} = T_{23}$,and $C_{33} = T_{33}$. but we know that for this problem:

$$\min_x |x - y|^2 + |x - z|^2$$

we have $x = \frac{x+z}{2}$. hence,

$$C_{22} = \frac{S_{22} + T_{22}}{2}$$

3.3 Part C

The codes and results are in the ipynb file.

4 Problem #4

The codes and results are in the ipynb file.

5 Problem #5

5.1 Part A

$$\begin{aligned} \max \quad & \mu^T w - \gamma \max w^T \Sigma^{[k]} w \\ \text{s.t.} \quad & \mathbf{1}^T w = 1 \end{aligned}$$

The problem is equal to:

$$\begin{aligned} \min \quad & -\mu^T w + \gamma t \\ \text{s.t.} \quad & \mathbf{1}^T w = 1 \\ & w^T \Sigma^{[k]} w \leq t \end{aligned}$$

Due to Lagrangian and KKT we have:

$$\mathcal{L}(w, t, , v) = -\mu^T w + \gamma t + \sum_{i=1}^{M} {}_i (w^T \Sigma^{[i]} w - t) + v(\mathbf{1}^T w - 1)$$

$$\begin{aligned} & -\mu + \sum_{i=1}^{M} 2\lambda_i \Sigma^{[i]} w + v = 0 \\ & \gamma - \sum_{i=1}^{M} 2_i = 0 \\ & \mathbf{1}^T w = 1 \\ & w^T \Sigma^{[k]} w \leq t \\ & \gamma \geq 0 \\ & \lambda_k (w^T \Sigma^{[k]} w - t) \geq 0 \end{aligned}$$

on the other hand we have:

$$\begin{aligned} \max \quad & \mu^T w - \sum_{k=1}^{M} \gamma_k w^T \Sigma^{[k]} w \\ \text{s.t.} \quad & \mathbf{1}^T w = 1 \end{aligned}$$

Which is equal to:

$$\begin{aligned} \min \quad & -\mu^T w + \sum_{k=1}^{M} \gamma_k w^T \Sigma^{[k]} w \\ \text{s.t.} \quad & \mathbf{1}^T w = 1 \end{aligned}$$

Due to Lagrangian and KKT we have:

$$\mathcal{L}(w, \alpha) = -\mu^T w + \sum_{i=1}^{M} \gamma_i w^T \Sigma^{[i]} w + \alpha(\mathbf{1}^T w - 1)$$

$$\begin{aligned} & -\mu + \alpha \mathbf{1} + \sum_{i=1}^{M} 2\gamma_i \Sigma^{[i]} w = 0 \\ & \mathbf{1}^T w = 1 \end{aligned}$$

So in the end we can say by choosing $\gamma = \lambda$ we can say that $(w^*, \alpha^*) = (w^*, v^*)$

5.2 Part B

The codes and results are in the ipynb file.