به نام خدا

سوال ۳)

a)

$$E(\gamma \hat{z}) = E(\gamma \hat{z} \mid Z=z)\, E(Z)$$

$$= E(\gamma\, \mathrm{sgn}(u+z) \mid Z=z)\, E(Z)$$

$$= \left(\gamma \times 1 \times \frac{2\gamma-(\gamma-z)}{2\gamma} + \gamma \times (-1) \times \frac{\gamma-z}{2\gamma}\right) E(Z)$$

$$= \underline{E(Z)} \checkmark$$

b)

طبق تست الف) برای اینکه و شرطی تمامی بیانش منطبق شود $\gamma = |\gamma| > \max_i |y_i|$

حال داریم که:

$$P(\max_{i \in [1,n]} |y_i| > \gamma) \leq 2n e^{-\frac{\gamma^2}{2\sigma^2}} \qquad y_i \sim \text{sub-gaussian}(\sigma^2)$$

$$P(\max |y_i| \leq \gamma) = 1 - P(\max |y_i| > \gamma) \geq 1 - 2n e^{-\gamma^2/2\sigma^2}$$

$$\Rightarrow P(\max |y_i| \leq \gamma) \geq 1 - 2n e^{-\gamma^2} \simeq 1 - \underset{\varepsilon \to 0}{\varepsilon} \qquad \sigma = 1 \Rightarrow O(2\sigma^2)$$

$$\Rightarrow -2n e^{-\gamma^2} = -\varepsilon \Rightarrow 2n e^{-\gamma^2} = \varepsilon \Rightarrow e^{-\gamma^2} = \frac{\varepsilon}{2n} \Rightarrow -\gamma^2 = \log(\varepsilon) \times 1 - \log(2n)$$

$$\gamma^2 = \log(\varepsilon) \log(2n) \Rightarrow \gamma^2 = \log(2n) \times k \Rightarrow \gamma = c\sqrt{\log(n)} \checkmark$$

نکته قابل توجه این است که $\Rightarrow$ بی نهایت مقدار ... $\sqrt{\log n}$ باشد.

$$Y = \text{subG}(1) \qquad = \theta^T$$

$$\hat{Y}_{k,j} := \text{sgn}(Y_k + U_{k,j})$$

$$\tilde{Y}_k = \sum_{j=1}^{f(n)} \gamma \hat{Y}_{k,j} = \theta^T X_k + \xi_k$$

$$E(\tilde{Y}_k) = E(Y_k) = c \times O(1) = c$$

$$\tilde{Y}_k - Y_k = \theta^T X_k + \xi_k - \theta^T X_k - \varepsilon_k$$

$$= \xi_k - \varepsilon_k \Rightarrow \text{subG}(1) = \text{subG}(\tilde{Y}_k) - \text{subG}(1)$$

$$\Rightarrow \sum \gamma c^j \sqrt{\frac{j}{r}} = o(1)$$

$$= \sum_{j=1}^{f(n)} \gamma \sqrt{\frac{j}{r}} = o(1)$$

$$\Rightarrow \frac{\gamma}{\sqrt{r}} \sum \sqrt{j} = o(1) \Rightarrow \frac{\gamma}{r}\left(f(n)\sqrt{f(n)} + \frac{1}{2}\sqrt{f(n)} + o(\sqrt{f(n)})\right) = O(1)$$

$$\frac{\gamma}{r} f(n)\sqrt{f(n)^{\mu}/r} + o(\sqrt{f(n)}) = o(1)$$

$$\approx \gamma f(n)\sqrt{f(n)} = o(1)$$

$$f(n)^{\mu}/r = \frac{1}{\gamma} \Rightarrow O(f(n)) = O\left(\left(\frac{1}{\sqrt{\log n}}\right)^{2/3}\right)$$

$$= O(\log n^{1/\mu}) \checkmark$$