سوال ۵

می دانیم که

$$|A|_F = \sqrt{tr(AA^T)} = 1 \Rightarrow tr(AA^T) = 1$$

$$\Rightarrow \sum |a_{ij}|^2 = 1 \Rightarrow A = uv^T \Rightarrow \|u\|_2 \|v\|_2 = 1$$

$$\Rightarrow \|\tilde{u}\|_2 = 1, \|\tilde{v}\|_2 = 1 \Rightarrow P(\tilde{u}, \tilde{v}) \leq P(u, v) \quad (\tilde{u} \in u, \tilde{v} \in v)$$

$$P(\tilde{u}, \tilde{v}) = card(\tilde{u}) \times card(\tilde{v})$$

$$\log(P(\tilde{u}, \tilde{v})) = \log(card(\tilde{u})) + \log(card(\tilde{v}))$$

$$= cd \log(1/\delta) + cn \log(1/\delta)$$

$$= c(n+d) \log(1/\delta)$$

اثبات برای باند بالا

$$|A' - A|_F < \delta \Rightarrow |u'v'^T - uv^T|_F < \delta$$

$$|u'v'^T - uv^T| \leq |u' - u|_2 + |v' - v|_2 < \delta$$

$\Rightarrow$ packing: کره $S^{n-1} \times S^{d-1}$ $\Rightarrow P \leq (1 + 2/\delta)^{n+d} \Rightarrow \log P \leq (n+d) \log(1 + 2/\delta)$

$$\Rightarrow \log P \leq (n+d) \log(1 + 2/\delta) \quad \delta \to 0$$

$$\leq (n+d) \log(2) \log(1/\delta)$$

$\Rightarrow c < \log(2)$ نیز هست.