به نام خدا

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$\mu < \nu$ ،

(1)

$$P(\text{Error}) \geq \frac{1}{2}\left(1 - TV(P,Q)\right)$$

$$TV(P,Q) = \frac{1}{2}\int |f_P(x) - f_Q(x)| = \int_{f_P > f_Q} \frac{\tau}{2} e^{-\tau|x-\mu|} - \frac{\tau}{2} e^{-\tau|x-\nu|}\, dx$$

$$= \frac{\tau}{2}\int_{-\infty}^{\mu} \left(e^{\tau(x-\mu)} - e^{\tau(x-\nu)}\right) dx + \frac{\tau}{2}\int_{\mu}^{\frac{\nu+\mu}{2}} e^{-\tau(x-\mu)} - e^{\tau(x-\nu)}\, dx$$

$$= \frac{\tau}{2} \times \frac{1}{\tau} e^{\tau(x-\mu)}\Big|_{-\infty}^{\mu} - \frac{1}{\tau} \times \frac{\tau}{2} e^{\tau(x-\nu)}\Big|_{-\infty}^{\mu} - \frac{1\times\tau}{2\tau} e^{-\tau(x-\mu)}\Big|_{\mu}^{\frac{\mu+\nu}{2}} - \frac{\tau}{2} \times \frac{1}{\tau} e^{\tau(x-\nu)}\Big|_{\mu}^{\frac{\mu+\nu}{2}}$$

$$= \frac{\tau}{2} \times \frac{1}{\tau} - \frac{\tau}{2} \times \frac{1}{\tau} e^{\tau(\mu-\nu)} - \frac{1}{\tau} \times \frac{\tau}{2}\left(e^{-\tau\left(\frac{\nu-\mu}{2}\right)} - 1\right) - \frac{\tau}{2} \times \frac{1}{\tau}\left(e^{\tau\left(\frac{\mu-\nu}{2}\right)} - e^{\tau(\mu-\nu)}\right)$$

$$= 1 - e^{-\tau\left|\frac{\mu-\nu}{2}\right|} \Rightarrow P(\text{Error}) = \frac{1}{2} e^{-\tau\left|\frac{\mu-\nu}{2}\right|} \blacksquare$$

ML ،
$$\begin{cases} H_0 = p \\ H_1 = q \end{cases} \qquad \frac{p(x)}{q(x)} \geq 1 \qquad \frac{p(x)}{q(x)} < 1$$

$$\frac{p(x)}{q(x)} \geq 0 \Rightarrow \frac{\frac{\tau}{2} e^{-\tau|x-\mu|}}{\frac{\tau}{2} e^{-\tau|x-\nu|}} \geq 1$$