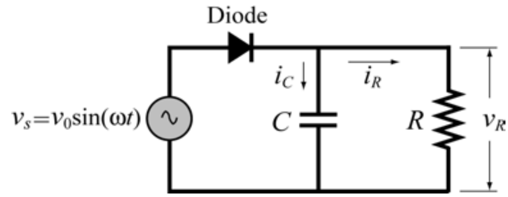
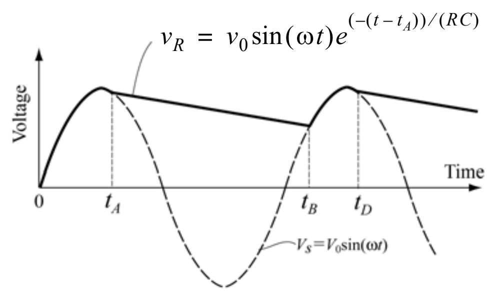
Sample Problem 6-12: AC to DC converter



A half-wave diode rectifier is an electrical circuit that converts AC voltage to DC voltage. A rectifier circuit that consists of an AC voltage source, a diode, a capacitor, and a load (resistor) is shown in the figure. The voltage of the source is $v_s = v_0 \sin(\omega t)$, where $\omega = 2\pi f$, in which f is the frequency. The operation of the circuit is illustrated in the lower diagram where the dashed line shows the source voltage and the solid line shows the voltage across the resistor. In the first cycle, the diode is on (conducting current) from $t = 0$ until $t = t_A$. At this time the diode turns off and the power to the resistor is supplied by the discharging capacitor. At $t = t_B$ the diode turns on again and continues to conduct current until $t = t_D$. The cycle continues as long as the voltage source is on. In this simplified analysis of this circuit, the diode is assumed to be ideal and the capacitor is assumed to have no charge initially (at $t = 0$). When the diode is on, the resistor's voltage and current are given by:



$$v_R = v_0 \sin(\omega t) \text{ and } i_R = v_0 \sin(\omega t)/R$$

The current in the capacitor is:

$$i_C = \omega C v_0 \cos(\omega t)$$

When the diode is off, the voltage across the resistor is given by:

$$v_R = v_0 \sin(\omega t_A) e^{(-(t - t_A))/(RC)}$$

The times when the diode switches off (t_A, t_D, and so on) are calculated from the condition $i_R = -i_C$. The diode switches on again when the voltage of the source reaches the voltage across the resistor (time t_B in the figure).

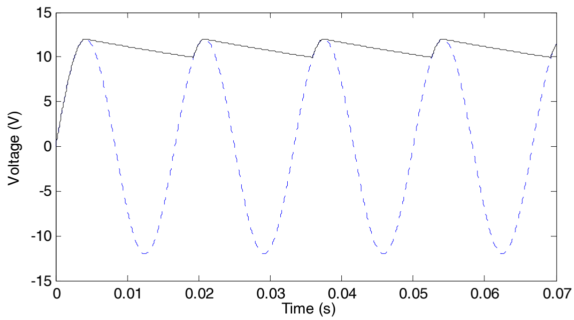
Write a MATLAB program that plots the voltage across the resistor v_R and the voltage of the source v_s as a function of time for $0 \le t \le 70$ ms. The resistance of the load is 1,800 Ω, the voltage source $v_0 = 12$ V, and $f = 60$ Hz. To examine the effect of capacitor size on the voltage across the load, execute the program twice, once with $C = 45$ μF and once with $C = 10$ μF.

Solution

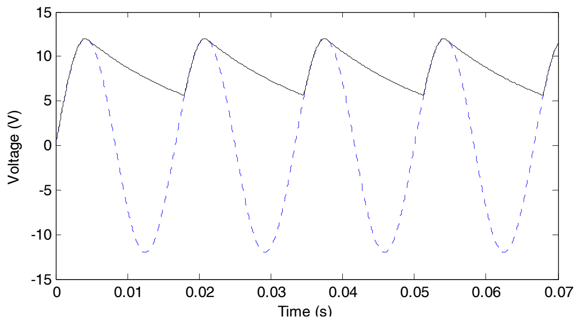
A program that solves the problem is presented below. The program has two parts—one that calculates the voltage v_R when the diode is on, and the other when the diode is off. The `switch` command is used for switching between the two parts. The calculations start with the diode on (the variable `state='on'`), and when $i_R - i_C \leq 0$ the value of `state` is changed to `'off'`, and the program switches to the commands that calculate v_R for this state. These calculations continue until $v_s \geq v_R$, when the program switches back to the equations that are valid when the diode is on.

```
V0=12; C=45e-6; R=1800; f=60;
Tf=70e-3; w=2*pi*f;
clear t VR Vs
t=0:0.05e-3:Tf;
n=length(t);
state='on'                                    Assign 'on' to the variable state.
for i=1:n
   Vs(i)=V0*sin(w*t(i));                      Calculate the voltage of the source at time t.
   switch state
      case 'on'                               Diode is on.
      VR(i)=Vs(i);
      iR=Vs(i)/R;
      iC=w*C*V0*cos(w*t(i));
      sumI=iR+iC;
      if sumI <= 0                            Check if iR - iC <= 0.
          state='off';                        If true, assign 'off' to state.
          tA=t(i);                            Assign a value to tA.
      end
      case 'off'                              Diode is off.
      VR(i)=V0*sin(w*tA)*exp(-(t(i)-tA)/(R*C));
      if Vs(i) >= VR(i)                       Check if vs >= vR.
         state='on';                          If true, assign 'on' to the variable state.
      end
   end
end
plot(t,Vs,':',t,VR,'k','linewidth',1)
xlabel('Time (s)'); ylabel('Voltage (V)')
```

The two plots generated by the program are shown below. One plot shows the result with $C = 45$ μF and the other with $C = 10$ μF. It can be observed that with a larger capacitor the DC voltage is smoother (smaller ripple in the wave).



$C = 45$ μF



$C = 10$ μF

6.8 PROBLEMS

1. Evaluate the following expressions without using MATLAB. Check the answer with MATLAB.

   (*a*) $5 + 3 > 32/4$ (*b*) $y = 2 \times 3 > 10/5 + 1 > 2^2$

   (*c*) $y = 2 \times (3 > 10/5) + (1 > 2)^2$ (*d*) $5 \times 3 - 4 \times 4 < = \sim 2 \times 4 - 2 + \sim 0$

2. Given: $a = 6$, $b = 2$, $c = -5$. Evaluate the following expressions without using MATLAB. Check the answer with MATLAB.

   (*a*) $y = a + b > a - b < c$ (*b*) $y = -6 < c < -2$

   (*c*) $y = b + c > \; = c > a/b$ (*d*) $y = a + c == \sim(c + a \sim= a/b - b)$

3. Given: $v = [4\ \ -2\ \ -1\ \ 5\ \ 0\ \ 1\ \ -3\ \ 8\ \ 2]$ and $w = [0\ \ 2\ \ 1\ \ -1\ \ 0\ \ -2\ \ 4\ \ 3\ \ 2]$. Evaluate the following expressions without using MATLAB. Check the answer with MATLAB.

   (*a*) $\sim(\sim v)$ (*b*) $u == v$

   (*c*) $u - v < u$ (*d*) $u - (v < u)$