June 14, 2020

Reply to reviewer 1

Dear reviewer,

First of all, we sincerely thank you for your encouragement, which greatly encourages us to continue further research:

This manuscript investigates Generalized belief entropy. Its main result is the derivation of the Dempster-Shafer function with maximum entropy. This work is interesting and the results obtained are technically sound.

And we truly appreciate your careful review and valuable suggestions. We have studied your comments carefully and made a lot of corrections.We hope that the drawbacks pointed out in the original manuscript are overcome in this revised one. The details are presented below, as well as in the revised manuscript.

Great appreciation for your comments and suggestions as follows.

1. In Abstract, it is suggested to newly add main results obtaining from the analytic solution of the maximum belief entropy.

2.The subject matter is presented reasonably comprehensively, although serious English language editing is required.

3. It is suggested to discuss the future works in the Conclusion section.

4.There are many methods in the D-S theory framework for uncertainty measure, how to apply the proposed method to the other measures?

5. Example 3 seems to be special cases. If examples are used for validation, then the examples should cover a large range of cases. E.g. how about a change FOD with achanging belief distribution and more larger?

6.The description of Table 1 is misplaced. Please place it at the head of the Table.

7.In section 3.3, some demonstrations and declaration could be improved and more significant.

8.please check and revise all the typos and grammar mistakes.

9. About maximum belief entropy. Does this property is consistent with the required property of the "maximum" of uncertainty measure mentioned in the other papers?

10.There are some minor problems: 1)On Page 1, "In addition,the applicability of the proposed model is obtained" —¿"In addition, the applicability of the proposed model is obtained" (A space is required after all punctuation, please check

the whole paper.)

2)On Page 1, "Entropy is a measure of uncertainty, and many studies have focused on the essence of entropy[1]."—¿"Entropy is a measure of uncertainty, and many studies have focused on the essence of entropy [1]." (A space is needed between the word and the reference, please check the whole paper.)

3)On Page 7, The font in Figure 1 is too small.

Our corresponding revisions are detailed as follows

1. For your comment and suggestion as follow:

In Abstract, it is suggested to newly add main results obtaining from the analytic solution of the maximum belief entropy.

Thank you very much for your valuable suggestion. We added the main results of the maximum belief entropy to the abstract:

> In evidence theory, the generalized belief entropy model unifies Renyi entropy, Tsallis entropy, and Deng entropy. In order to further unify the maximum values of Renyi entropy, Tsallis entropy, and Deng entropy, this paper proposes a maximum model of generalized belief entropy by analyzing the generalized belief entropy model, this model shows that the size of the maximum generalized belief entropy is not related to the specific mass value, but is related to the size of each propositional space, and the maximum values of R-D entropy and T-D entropy are obtained through this model. In addition, the applicability of the proposed model is obtained through verification tests and sensitivity analysis of the model.

2. For your comment and suggestion as follow:

The subject matter is presented reasonably comprehensively, although serious English language editing is required.

Thank you very much for your valuable suggestion. We have revised the English grammar expression and other terms in the manuscript.

3. For your comment and suggestion as follow:

It is suggested to discuss the future works in the Conclusion section.

Thank you very much for your valuable suggestion. Based on your suggestions, we introduced the future work direction in the final conclusion:

> We have not discussed the practical significance of r in the generalized maximum belief entropy model, which can become the direction of future work, and we will also try to apply the maximum generalized entropy model to information conflict handling.

4. For your comment and suggestion as follow:

There are many methods in the D-S theory framework for uncertainty measure, how to apply the proposed method to the other measures?

Thank you very much for your valuable suggestion. We explained the entropy transformation graph in the original manuscript. Our model can get other models through the change of parameters, so that it can be applied to the

situation where other models are satisfied.

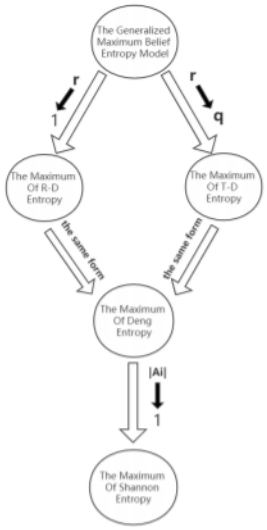


Figure 1: Relationship between several maximum entropy models.

As shown in Fig.1, we can see that:

1) When the value of r tends to 1, the maximum generalized belief entropy model degenerates into the maximum R-D entropy model.

2) When the value of r tends to q, the maximum generalized belief entropy model degenerates into the maximum T-D entropy model.

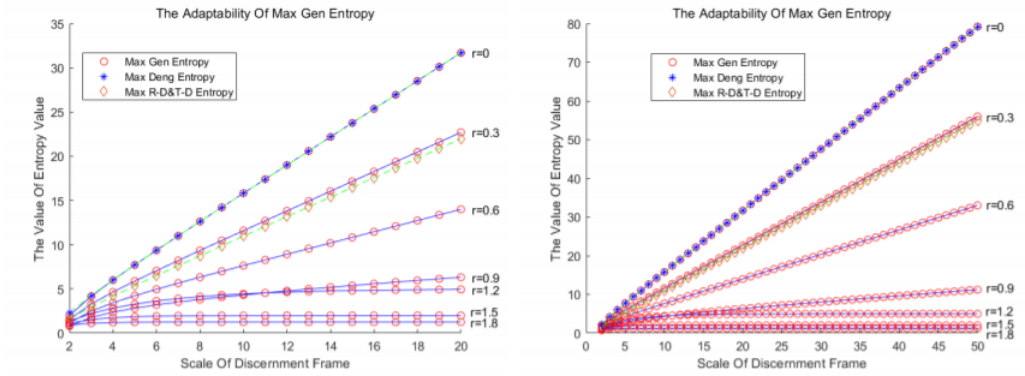
3) The maximum R-D entropy model and the maximum T-D entropy model are consistent in expression [72] with the maximum Deng entropy.

4) The maximum Deng entropy degenerates to the maximum Shannon entropy in the case of a single element subset. [70]

5. For your comment and suggestion as follow:

Example 3 seems to be special cases. If examples are used for validation, then the examples should cover a large range of cases. E.g. how about a change FOD with achanging belief distribution and more larger?

Thank you very much for your valuable suggestion. Since our model is discussed under the condition of maximum entropy, so the belief distribution under this condition is determined by the size of the frame of discernment(FOD). So according to your suggestion, we discussed the performance of our model under the larger FOD:



(a) |FOD|=20 (b) |FOD|=50

Figure 5: More Larger FOD

Fig.5 shows the performance of our model under a larger FOD. When the maximum generalized belief entropy holds, the distribution of BPA is determined for a certain identification framework, namely:

$$m(A_i) = \frac{2^{|A_i|} - 1}{\sum_j 2^{|A_j|} - 1}$$

so it can be said that in this case, the distribution of BPA is fixed. From Fig.5 we can see that the performance of our model under larger FOD is consistent with Fig.4. It is worth mentioning that in Fig.5, the step size of the r iteration has changed. We can see that when $r = 0.3$, our model is very close to the maximum R-D entropy and the maximum T-D entropy. Referring to the case where the limit value is obtained when r=1 mentioned above, maybe in this case ($r = 0.3$) a non-limiting model that is sufficiently close to the maximum R-D entropy and maximum T-D entropy models can be obtained.

6. For your comment and suggestion as follow:

The description of Table 1 is misplaced. Please place it at the head of the Table.

Thank you very much for your careful review and valuable suggestion. We have revised the position of the description of the table in the manuscript:

Table 2: The quantitative relationship between entropy

Scale of FOD	R-D&T-D Entropy	Deng Entropy	R-D&T-D/Deng
2	1.6094	2.3219	1.4427
3	2.9444	4.2479	1.4427
4	4.1744	6.0224	1.4427
5	5.3519	7.7211	1.4427
6	6.4998	9.3772	1.4427
7	7.6300	11.0077	1.4427
8	8.7491	12.6223	1.4427
9	9.8612	14.2266	1.4427
10	10.9686	15.8244	1.4427

7. For your comment and suggestion as follow:

In section 3.3, some demonstrations and declaration could be improved and more

significant.

Thank you very much for your valuable suggestion. We have revised the expression of the fomula and some declarations in the original manuscript:

The proposition A_i in the generalized maximum belief entropy model proposed above is obejective, so we mainly discuss the value of parameter r to obtain the maximum value of the generalized maximum belief entropy model. We find the partial derivative about r for the generalized maximum belief entropy model obtained, that is:

$$\frac{\partial E_r(A_i)_{max}}{\partial r} = \frac{\left(\sum_j 2^{|A_j|}-1\right)^{1-r}\left[-\ln\left(\sum_j 2^{|A_j|}-1\right)(1-r)-1\right]}{(1-r)^2} = 0$$

then we perform power series expansion on equation above to get:

$$(2) = \frac{1}{1-q}\left[1+(1-q)\ln\left(\sum_j 2^{|A_j|}-1\right)+\sum_{i=2}^{n}\frac{(1-q)^n\left[\ln\left(\sum_j 2^{|A_j|}-1\right)\right]^n}{n!}-1\right]$$

1) when $0 \le q < 2$, that is $1-q \in [-1,1)$, and for a definite frame of discernment Ω, $\left(\sum_j 2^{|A_j|}-1\right)$ is also definite, so we can get:

$$\lim_{n\to\infty}\sum_{i=2}^{n}\frac{(1-q)^n\left[\ln\left(\sum_j 2^{|A_j|}-1\right)\right]^n}{n!} = 0$$

so Eq.3 can be rewritten as : $\ln\left(\sum_j 2^{|A_j|}-1\right)+0$, therefore, in this situation, the maximum value of T-D entropy is: $\ln\left(\sum_j 2^{|A_j|}-1\right)$.

8. For your comment and suggestion as follow:

please check and revise all the typos and grammar mistakes.

Thank you very much for your valuable suggestion. We have revised all the typos and grammar mistakes, and some of them are showed as follows:

the maximum ~~Renyii-Deng~~ Renyi-Deng entropy, and the maximum

From the data in the ~~above table~~ table above, we can see that the mutiple between maximum R-D entropy, t

uncertainty measurement of multi-element sets. ~~The greater the Deng entropy, the higher the degree of uncertainty.~~ I

9. For your comment and suggestion as follow:

About maximum belief entropy. Does this property is consistent with the required property of the "maximum" of uncertainty measure mentioned in the other papers?

Thank you very much for your valuable suggestion. In order to clarify this problem, we have listed a table. In the table, we have compared the conditions for obtaining the maximum value in several uncertain measurement methods and their maximum values. The results show that our model is consistent with the recently proposed model in terms of conditions and maximum values:

We compared the entropy mentioned in the article, as shown in Table.1. It can be seen that the conditions for the maximum value of Renyi entropy and Tsallis entropy proposed earlier are the same, that is, at this time, each p_i occupies the same possibility, and the system is most uncertain. The maximum value of Tsallis entropy is consistent with the maximum value of Renyi entropy when $q \to 1$. Deng entropy generalizes the individual in question to multi-element subsets. The condition for Deng entropy to obtain the maximum value can be understood as that for each multi-element subset, all its possible values (power set) account for all possible elements of all multi-element subsets. Value ratio, this idea is in line with Renyi entropy and Tsallis entropy. Our model is based on a multi-element subset, and the condition and maximum form of our maximum value are consistent with Deng entropy, R-D entropy and T-D entropy.

Table 1: Uncertainty measures

Name	Expression	Conditions	Maximum value
Renyi Entropy	$H_\alpha(p_i) = \frac{1}{1-\alpha}\log_2(\sum_{k=1}^{N} p_k^\alpha)$	$p_k = \frac{1}{N}$	$\log_2(N)$
Tsallis Entropy	$S_q(p_i) = \frac{k}{q-1}(1 - \sum_{i=1}^{n} p_i^q)$	$p_i = \frac{1}{N}$	$\frac{1-N^{1-q}}{q-1}$
Deng Entropy	$E_d = -\sum_{A_i \subseteq \Omega} m(A_i) \times \log_2 \frac{m(A_i)}{2^{\lvert A_i\rvert}-1}$	$m(A_i) = \frac{2^{\lvert A_i\rvert}-1}{\sum_j 2^{\lvert A_j\rvert}-1}$	$\log_2\left(\sum_j 2^{\lvert A_j\rvert} - 1\right)$
R-D Entropy	$E_\alpha(m(A_i)) = \frac{1}{1-\alpha}\ln\left[\sum_i (\frac{m(A_i)}{2^{\lvert A_i\rvert}-1})^\alpha (2^{\lvert A_i\rvert} - 1)\right]$	$m(A_i) = \frac{2^{\lvert A_i\rvert}-1}{\sum_j 2^{\lvert A_j\rvert}-1}$	$\log_2\left(\sum_j 2^{\lvert A_j\rvert} - 1\right)$
T-D Entropy	$E_q(m(A_i)) = \frac{1}{q-1}\left[1 - \sum_i (\frac{m(A_i)}{2^{\lvert A_i\rvert}-1})^q (2^{\lvert A_i\rvert} - 1)\right]$	$m(A_i) = \frac{2^{\lvert A_i\rvert}-1}{\sum_j 2^{\lvert A_j\rvert}-1}$	$\log_2\left(\sum_j 2^{\lvert A_j\rvert} - 1\right)$
Gen Entropy	$E_{t,r}(m(A_i)) = \frac{1}{1-r}\left[\left[\sum_i (\frac{m(A_i)}{2^{\lvert A_i\rvert}-1})^t (2^{\lvert A_i\rvert} - 1)\right]^{\frac{1-r}{1-t}} - 1\right]$	$m(A_i) = \frac{2^{\lvert A_i\rvert}-1}{\sum_j 2^{\lvert A_j\rvert}-1}$	$\log_2\left(\sum_j 2^{\lvert A_j\rvert} - 1\right)$

In the framework of D-S evidence theory, there are many ways to measure uncertainty. We can degenerate our model into other measures of uncertainty by changing the parameters of the maximum generalized entropy model, thus adapting our model to situations that are met by other measures:

10. For your comment and suggestion as follow:

1) On Page 1, "In addition,the applicability of the proposed model is obtained" —>"In addition, the applicability of the proposed model is obtained" (A space is required after all punctuation, please check the whole paper.)

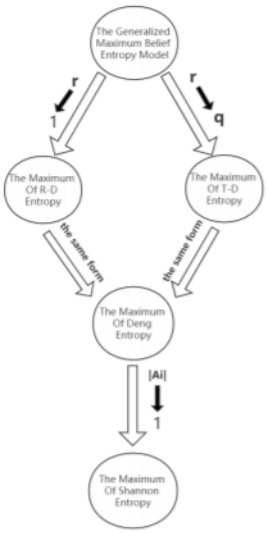
Thank you very much for your careful revise and valuable suggestion. According to your suggestion, we have checked the whole manuscript and revised the place without spaces after all punctuation.

2) On Page 1, "Entropy is a measure of uncertainty, and many studies have focused on the essence of entropy[1]."—>"Entropy is a measure of uncertainty, and many studies have focused on the essence of entropy [1]." (A space is needed between the word and the reference, please check the whole paper.)

Thank you very much for your careful revise and valuable suggestion. We have corrected all the palces without a space between the word and the reference.

3) On Page 7, The font in Figure 1 is too small.

Thank you very much for your valuable suggestion. We have redrawed the figure in the corresponding position. The figure is showed as follows:



Above are our detailed revision according to your comments. Thanks again for your careful review and valuable suggestions to improve our manuscript.If there are still any problems in the revised manuscript, please point out. And we will make efforts to revise the manuscript according to your suggestions. We are looking forward to your positive response.

Sincerely yours

Siran Li, RuiCai