

June 14, 2020

Reply to reviewer 2

Dear reviewer,

First of all, we truly appreciate your careful review and valuable suggestions. We have studied your comments carefully and made a lot of corrections. We hope that the drawbacks pointed out in the original manuscript are overcome in this revised one. The details are presented below, as well as in the revised manuscript.

Great appreciation for your comments and suggestions as follows.

1-The following references are related to this subject. Would you please check them and in expressing the novelty it should be mention the connections with these papers if exactly have connection, anyway I propose a check.

1-Generalized Ordered Propositions Fusion Based on Belief Entropy, Y. Li, Y. Deng INTERNATIONAL JOURNAL OF COMPUTERS COMMUNICATIONS & CONTROL

2- An Improved Belief Entropy in Evidence Theory, HANGYU YAN^{1,2}, YONG DENG¹ 2020, IEEE Access (Volume: 8)

3- Belief Reliability Distribution Based on Maximum Entropy Principle TIAN-PEI ZU, RUI KANG, MEILIN WEN , AND QINGYUAN ZHANG 2017- IEEE Access (Volume: 6)

4- Using Maximum Entropy and Generalized Belief Propagation in Estimation of Distribution Algorithms, Robin Höns, Markov Networks in Evolutionary Computation pp 175-190 , 2012.

5- New definition of the cross entropy based on the Dempster-Shafer theory and its application in a decision-making process, Mehran KhalajORCID Icon, Reza Tavakkoli-MoghaddamORCID Icon, Fereshteh KhalajORCID Icon & Ali Siadat, Communications in Statistics - Theory and Methods. Volume 49, 2020 - Issue 4.

*6- A new definition of entropy of belief functions in the Dempster-Shafer theory(Article) * Jiroušek, R. Shenoy, P.P International Journal of Approximate Reasoning Volume 92, January 2018, Pages 49-65*

2-The format of references should have one style, for example [14] is not its style like others. Also many misprint can be find in the references and the article text.

3-Renyi is the correct name in Renyi entropy.

4-Names should start with capital letters such as Tsallis.

5-The numerical example should specify more with motivation of the generalized maximum belief.

6-The novelty of the paper need more care, it should be clear to reader in conclusion.

Our corresponding revisions are detailed as follows

1. For your comment and suggestion as follow:

The following references are related to this subject. Would you please check them and in expressing the novelty it should be mention the connections with these papers if exactly have connection, anyway I propose a check.

Thank you very much for your valuable suggestion. We carefully checked these references and found that there is a connection between these references and our work. So we introduced these references into our article and introduced their work in the Introduction section.

Both evidence theory and information entropy can be used to process information. How to apply classic information entropy to evidence theory has been the work of some scholars. In this process, many entropies has been proposed [40–49], Renyi proposed the Renyi entropy [50], which is widely used in statistics [51]. As a kind of extension of Boltzmann entropy, Tsallis entropy [52] is a non-scalable entropy that also has a wide range of applications [53]. Recently, Deng entropy [54] was proposed as a new kind of entropy used to measure the uncertainty of BPA, Deng entropy better solves the problem of uncertainty measurement of multi-element sets. ~~The greater the Deng entropy, the higher the degree of uncertainty.~~ Based on Deng entropy, the Generalized Ordered Propositions Fusion is presented in [55]. This article obtains the generalized ordered proposition by extending the basic support function on the power set of the ordered proposition, and proposes a fusion method of the generalized ordered proposition. Some scholars have proposed an improved belief entropy based on Shannon entropy and Deng entropy [56], the improved belief entropy fully considers the relationship between the subsets, which makes it have a better effect in the measurement of uncertainty. Different with Deng entropy, a new belief entropy is proposed in [57], which consists of two parts, the first part is the Shannon entropy with plausibility transform, the second part is Dubois-Prade's definition of entropy of basic probability assignments in the DS theory. What is important is that it also satisfies the six expected properties of the entropy of the DS belief function theory. We often discuss entropy based

Applying the principle of maximum entropy and Generalized Belief Propagation, [58] presents an algorithm for estimating the distribution by taking into account the dependencies among variables, so that a larger distribution can be estimated

from a smaller subset distribution using the principle of maximum entropy. It is often used in statistics [59–62], especially parameter estimation [63, 64] and some aspects of biochemistry [65], some theories [66–68] are also improved based on it. The principle of maximum entropy can also be used to determine BPA. [69] proposes a new model for determining BPA. This model has certain significance for determining BPA when the available information is limited. The principle

2. For your comment and suggestion as follow:

The format of references should have one style, for example [14] is not its style like others. Also many misprint can be find in the references and the article text.

Thank you very much for your careful review and valuable suggestion. We have revised all the style of references in the original manuscript, part of the details are presented as follows:

Entropy is a measure of uncertainty, and many studies have focused on the essence of entropy [1]. When the classical thermodynamic entropy is evolving [2, 3], the concept of entropy is also widely used in various disciplines [4–10] and industries [11–13]. In the field of information theory, Shannon proposed information entropy [14], classical information entropy is often used to compare with the newly proposed entropy [15, 16], literature [17] followed up on the aforementioned literature [15] and made some notes.

3. For your comment and suggestion as follow:

Renyi is the correct name in Renyi entropy.

Thank you very much for your careful review and valuable suggestion. We carefully checked all the corresponding places and revised them. The details can be showed as follows:

In evidence theory, the generalized belief entropy model unifies ~~Renyi~~ Renyi entropy, Tsallis entropy, and Deng entropy. In order to further unify the maximum values of ~~Renyi~~ Renyi entropy, Tsallis entropy, and Deng entropy, this paper proposes a maximum model of generalized belief entropy by analyzing the generalized belief entropy model, this model shows that the size of the maximum generalized belief entropy is not related to the specific mass value, but is related to the size of each propositional space, and the maximum values of R-D entropy and T-D entropy are obtained through this model. In addition, the applicability of the proposed model is obtained through verification tests and sensitivity analysis of the model.

Keywords: Entropy; Uncertainty measure; Tsallis entropy; ~~Renyi~~ Renyi entropy; Shannon entropy; Deng entropy

According to the previous ideas, a natural question will be asked: what is the maximum value of the generalized belief entropy model? And under what conditions does the generalized belief entropy reaches this maximum, this subject has not been covered by too many scholars. In this paper, we will discuss the conditions when the generalized belief entropy reaches a maximum value, and give the maximum generalized belief entropy based on the results of the discussion. Through the work of this paper, the maximum Shannon entropy, the maximum ~~Renyi-Deng~~ Renyi-Deng entropy, and the maximum Tsallis-Deng entropy can be unified.

4. For your comment and suggestion as follow:

Names should start with capital letters such as Tsallis.

Thank you very much for your careful review and valuable suggestion. We have checked all the names in the original manuscript and revised names which not start with capital letter, part of the details are showed as follows:

Given a discrete random variable Y and its probability distribution is $\{p_i | i = 1, 2, 3, \dots\}$, then the Tsallis entropy [52] can be expressed as:

$$S_q(p_i) = \frac{k}{q-1} (1 - \sum_{i=1}^n p_i^q)$$

where k, q are parameters, and k is often set to 1 in statistics. And q reflects the degree of non-extensiveness. In particular, when $q \rightarrow 1$, Tsallis entropy degenerates to Shannon entropy:

$$\lim_{q \rightarrow 1} S_q(p_i) = S_1(p_i) \equiv - \sum_{i=1}^n p_i \log_2 p_i$$

It should be noted that the literature [73] suggests that the Tsallis entropy can be used to replace Shannon entropy when $q = 2$, so that logarithmic operations can be avoided. In the latter part of this article, the situation of $q = 2$ will also be included in the discussion, so $q = 2$ is a special case of this article.

According to the previous ideas, a natural question will be asked: what is the maximum value of the generalized belief entropy model? And under what conditions does the generalized belief entropy reaches this maximum, this subject has not been covered by too many scholars. In this paper, we will discuss the conditions when the generalized belief entropy reaches a maximum value, and give the maximum generalized belief entropy based on the results of the discussion. Through the work of this paper, the maximum Shannon entropy, the maximum ~~Renyi-Deng~~ Renyi-Deng entropy, and the maximum Tsallis-Deng entropy can be unified.

The structure of this paper is as follows: The Preliminaries in the second section introduce the related concepts of evidence theory, Deng entropy, Renyi entropy, Tsallis entropy, generalized belief entropy (R-D entropy, T-D entropy). In section 3, the generalized maximum belief entropy model is proposed, and the maximum values of R-D entropy and T-D entropy are obtained according to this model. The fourth section is the conclusion of this paper.

5. For your comment and suggestion as follow:

The numerical example should specify more with motivation of the generalized maximum belief.

Thank you very much for your valuable suggestion. As for the motivation of the generalized maximum entropy, we added instructions at the beginning of example 3 to formulate the significance of this example:

4.3. Example 3

In this example, we mainly discuss the motivation of maximum generalized belief entropy. We will explain how to adapt this model to the maximum R-D entropy model, the maximum T-D entropy model, and the maximum Deng entropy model by modifying the parameters in the maximum generalized entropy model.

And we enlarged the size of FOD on the basis of original, and the result showed that the performance of our model under the enlarged FOD is consistent with that of smaller FOD, that is to say our model can be applied to the situation under more larger FOD. In this way, our model can be transformed into other models under more larger FOD.

The details are presented as follows:

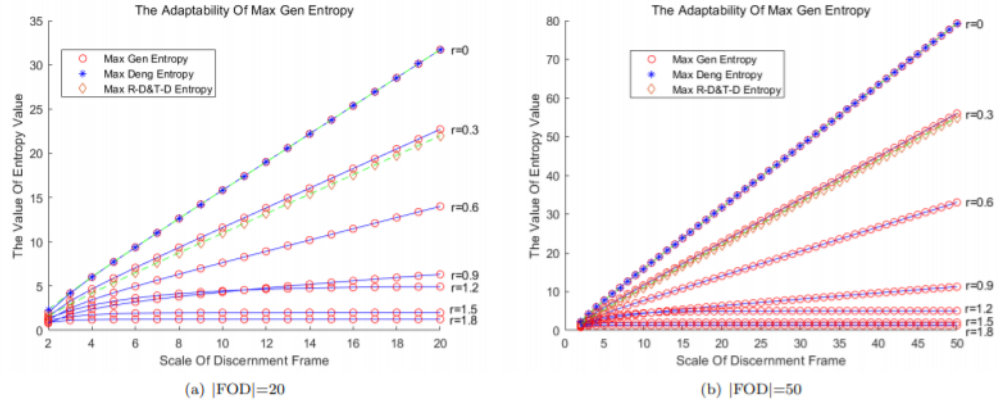


Figure 5: More Larger FOD

Fig.5 shows the performance of our model under a larger FOD. When the maximum generalized belief entropy holds, the distribution of BPA is determined for a certain identification framework, namely:

$$m(A_i) = \frac{2^{|A_i|} - 1}{\sum_j 2^{|A_j|} - 1}$$

so it can be said that in this case, the distribution of BPA is fixed. From Fig.5 we can see that the performance of our model under larger FOD is consistent with Fig.4. It is worth mentioning that in Fig.5, the step size of the r iteration has changed. We can see that when $r = 0.3$, our model is very close to the maximum R-D entropy and the maximum T-D entropy. Referring to the case where the limit value is obtained when $r=1$ mentioned above, maybe in this case ($r = 0.3$) a non-limiting model that is sufficiently close to the maximum R-D entropy and maximum T-D entropy models can be obtained.

Furthermore, we added interpretation to the figure 1 to make it more clear:

As shown in Fig.1, we can see that:

- 1) When the value of r tends to 1, the maximum generalized belief entropy model degenerates into the maximum R-D entropy model.
- 2) When the value of r tends to q , the maximum generalized belief entropy model degenerates into the maximum T-D entropy model.
- 3) The maximum R-D entropy model and the maximum T-D entropy model are consistent in expression [72] with the maximum Deng entropy.
- 4) The maximum Deng entropy degenerates to the maximum Shannon entropy in the case of a single element subset. [70]

6. For your comment and suggestion as follow:

The novelty of the paper need more care, it should be clear to reader in conclusion.

Thank you very much for your valuable suggestion. In the conclusion, we explain the novelty of our work according to your suggestion:

The novelty of our work lies in the unification of the maximum R-D entropy model, the maximum T-D entropy model and the maximum Deng entropy model through the maximum generalized entropy model we have obtained. The unification process is demonstrated in Section 4.3.

Above are our detailed revision according to your comments. Thanks again for your careful review and valuable suggestions to improve our manuscript. If there are still any problems in the revised manuscript, please point out. And we will make efforts to revise the manuscript according to your suggestions. We are looking forward to your positive response.

Sincerely yours

Siran Li, RuiCai