Computer Engineering
วิศวกรรมคอมพิวเตอร์

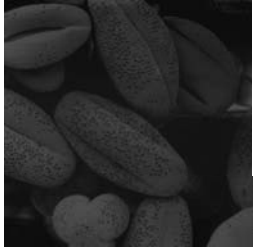
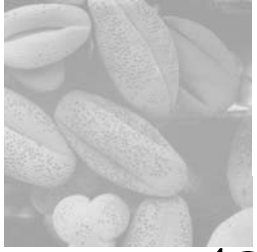
Image Enhancement in the Spatial Domain
(Histogram Processing)

เอกสารประกอบการเรียนการสอน

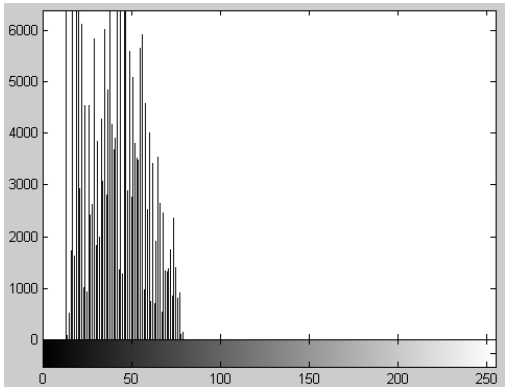
วิชา 01074402 IMAGE PROCESSING

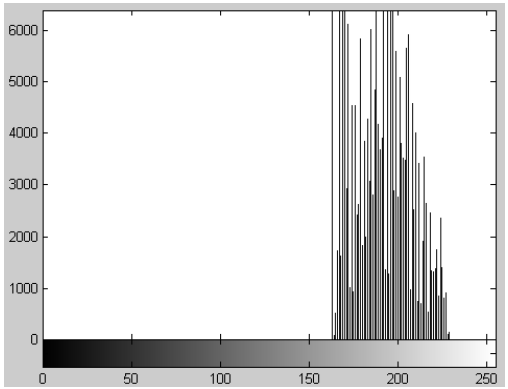
อ.จิระศักดิ์ สิทธิกร (ksjirasa@hotmail.com)

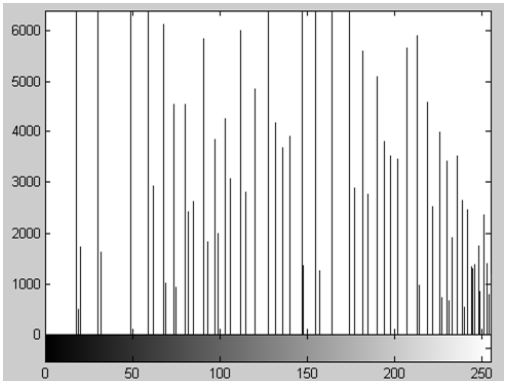
Histogram Equalization


- ใช้ Histogram ของภาพกำหนดฟังก์ชันการเปลี่ยนแปลงระดับของความเข้มแสงให้เป็นแบบ Non-linear
- โดยทำให้ฟังก์ชันการกระจายความน่าจะเป็น (probability density function: PDF) ของค่าความเข้มแสงของภาพมีการกระจายออก คือ ทำให้ความสูงของ PDF แบนราบลง โดยให้มีการกระจายตัวแบบรูปแบบ uniform







Histogram Equalization

- สมการการเปลี่ยนแปลงค่าแบบ Histogram Equalization

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

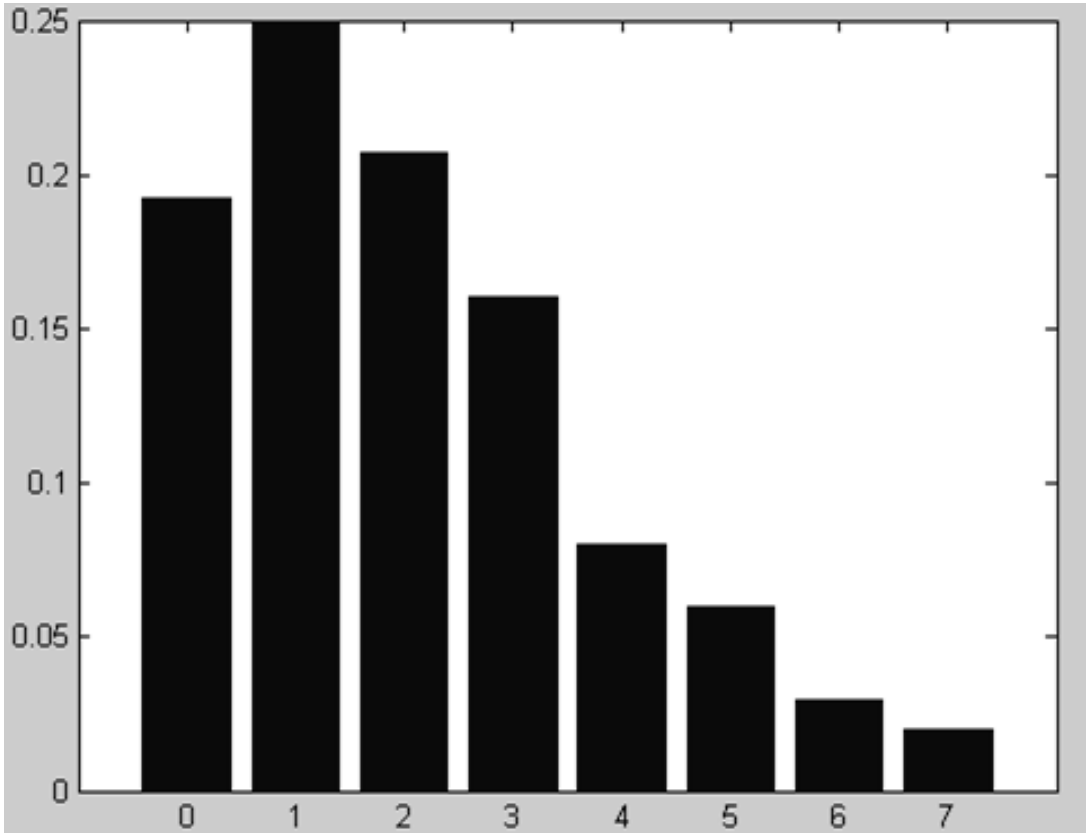
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0,1,2,\ldots,L-1$$

EXAMPLE : Histogram Equalization

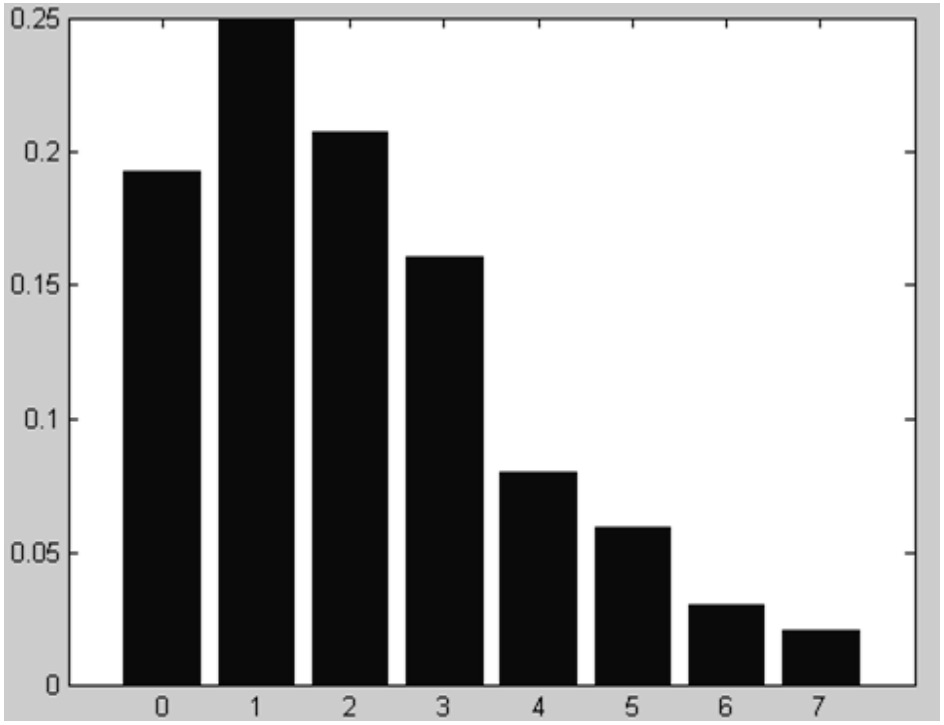
- EXAMPLE : Intensity distribution and histogram values for a 3-bit, 64 × 64 digital image

r_k	N_k	$p_k(r_k) = n_k/MN$
$r_0 = 0$	790	0.1929
$r_1 = 1$	1023	0.2498
$r_2 = 2$	850	0.2075
$r_3 = 3$	656	0.1602
$r_4 = 4$	329	0.0803
$r_5 = 5$	245	0.0598
$r_6 = 6$	122	0.0298
$r_7 = 7$	81	0.0198

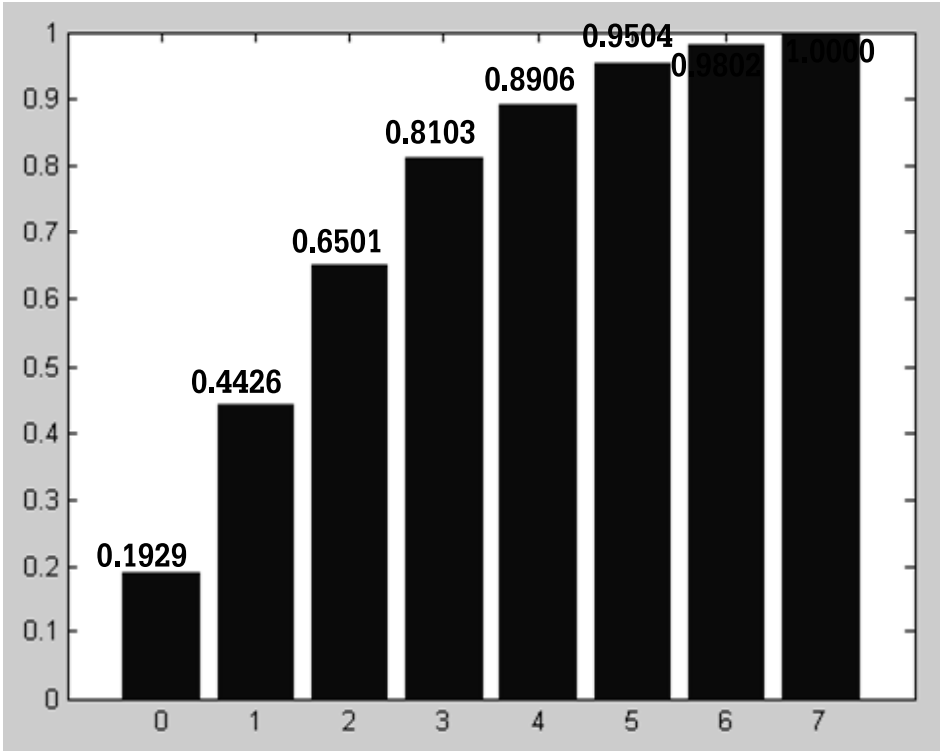


EXAMPLE

$p_k(r_k)$



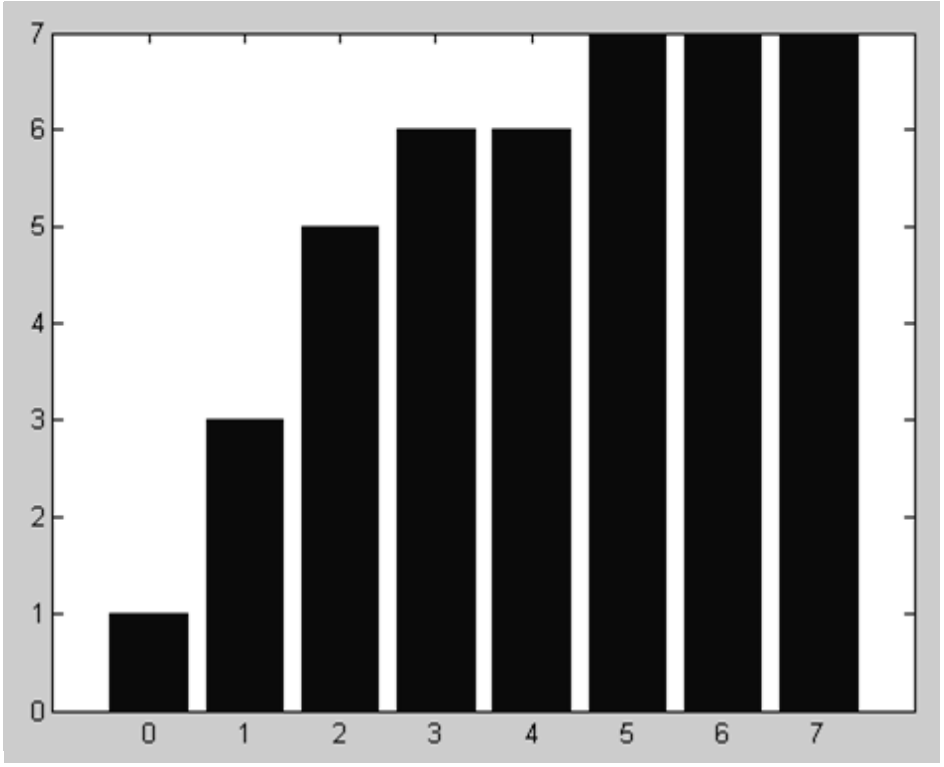
$c_k(r_k)$



EXAMPLE

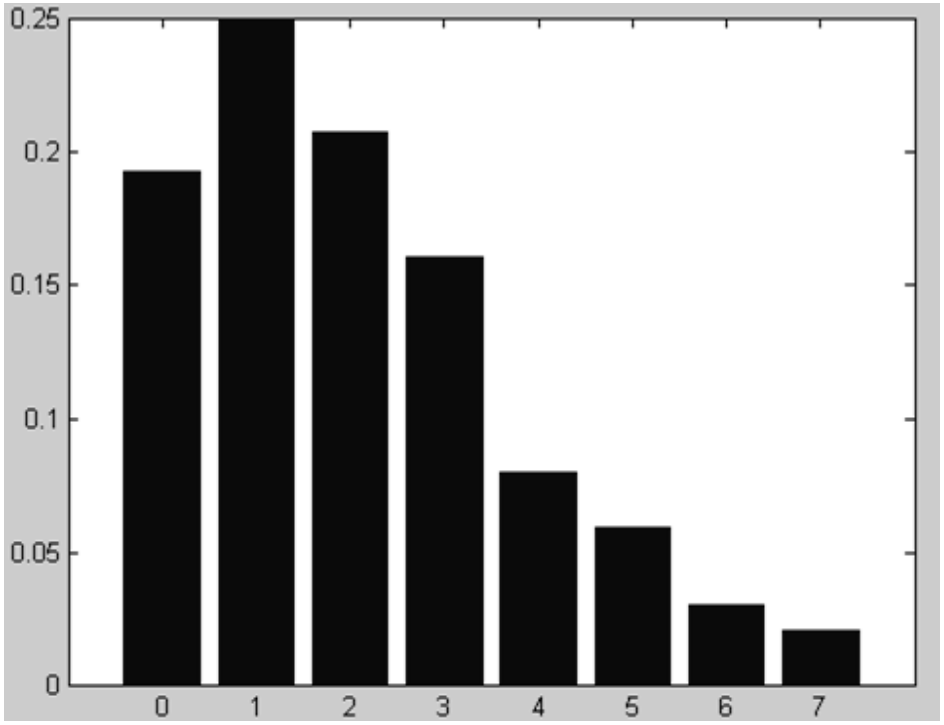
r_k	$c_k(r_k)$	s_k
$r_0 = 0$	0.1929	1.3501→1
$r_1 = 1$	0.4426	3.0984→3
$r_2 = 2$	0.6501	4.5510→5
$r_3 = 3$	0.8103	5.6721→6
$r_4 = 4$	0.8906	6.2344→6
$r_5 = 5$	0.9504	6.6531→7
$r_6 = 6$	0.9802	6.8616→7
$r_7 = 7$	1.0000	7.000→7

s_k

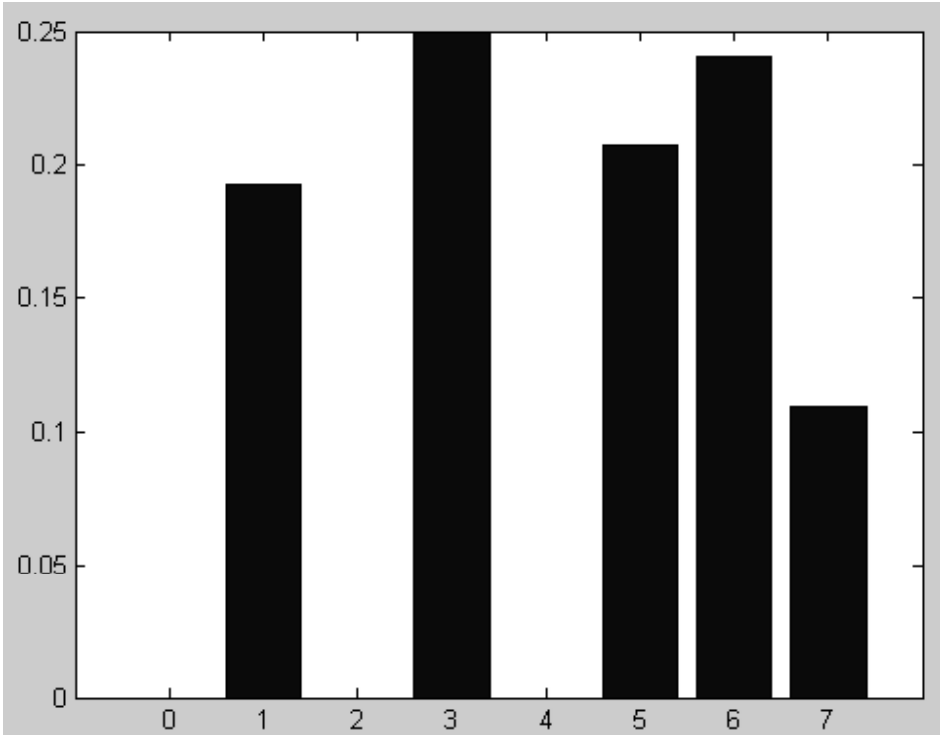


EXAMPLE

$p_k(r_k)$



$p_k(s_k)$

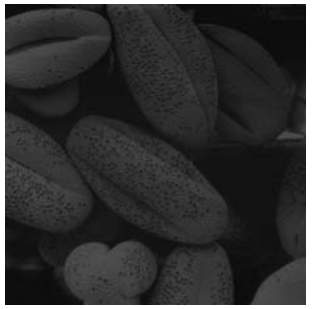


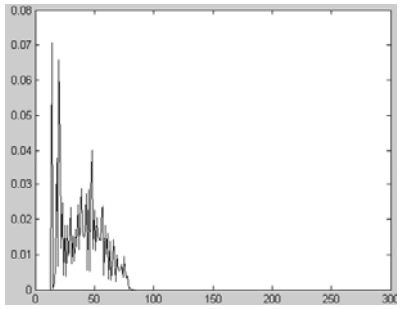
Pseudo code Histogram Equalization

$$s_k = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

- คำนวณ Normalized Histogram

```
loop : k = 0 to L-1
        p[k] = h[k] / all piexl
end
```





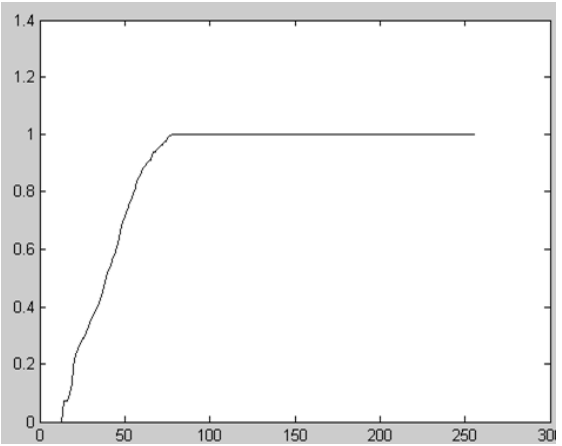
- กำหนดค่าเริ่มต้นของฟังก์ชันการเปลี่ยนแปลง

```
c[0] = h[0]
```

$$s_k = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

- สร้างฟังก์ชันการเปลี่ยนแปลง

```
loop : k = 1 to L-1
        c[k] = c[k-1] + h[k]
end
```



Pseudo code Histogram Equalization

$$s_k = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

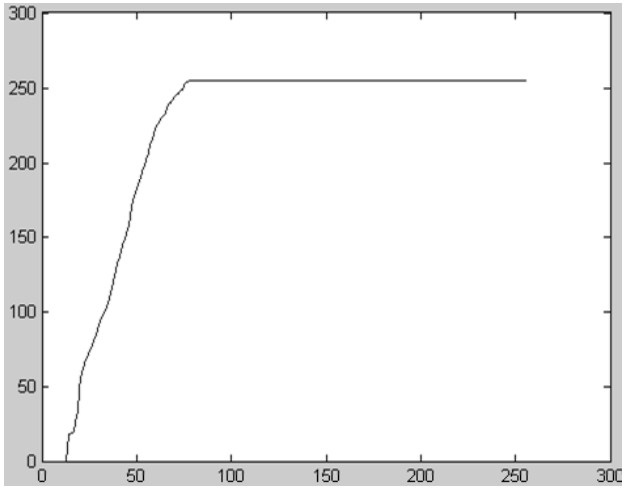
- ปรับค่าฟังก์ชันการจาก [0 - 1] $\longrightarrow$ [0 – L-1]

loop : k = 0 to L-1

s[k] = c[k] × (L-1)

end

s[al] = round s[all]

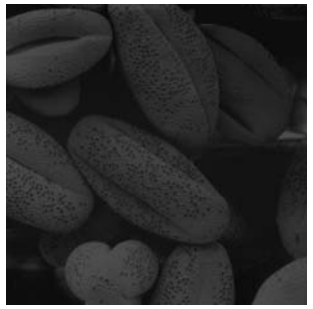


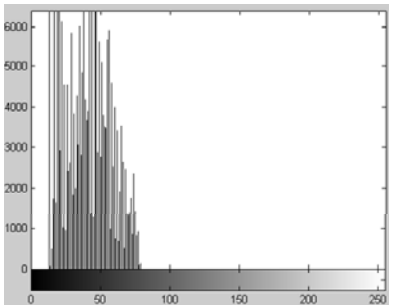
- กำหนด

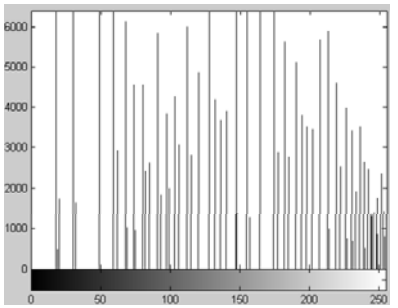
loop : All pixel

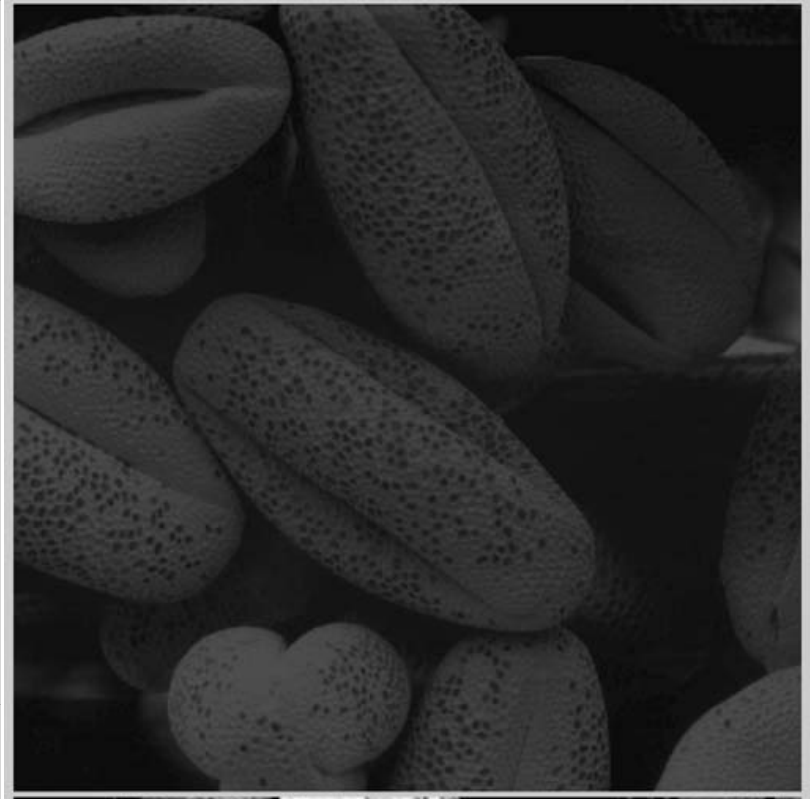
g[x, y] = s[f[x,y]]

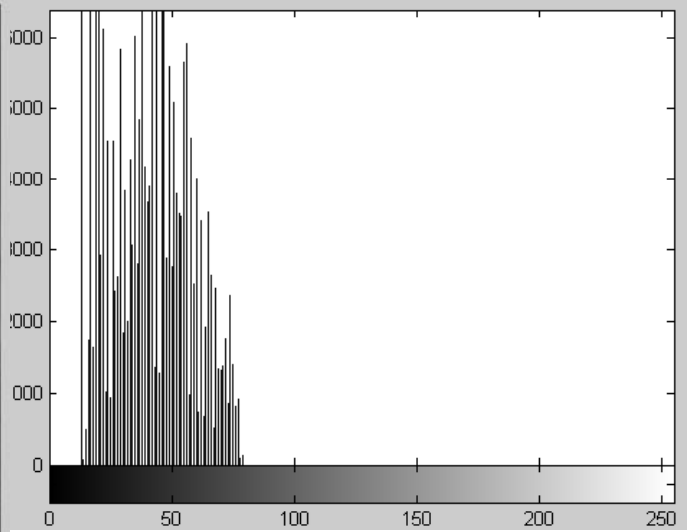
end







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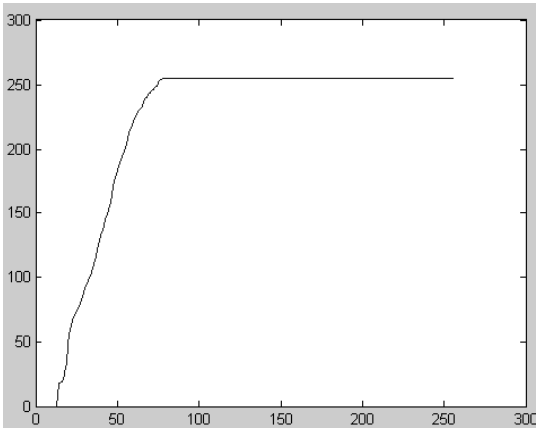
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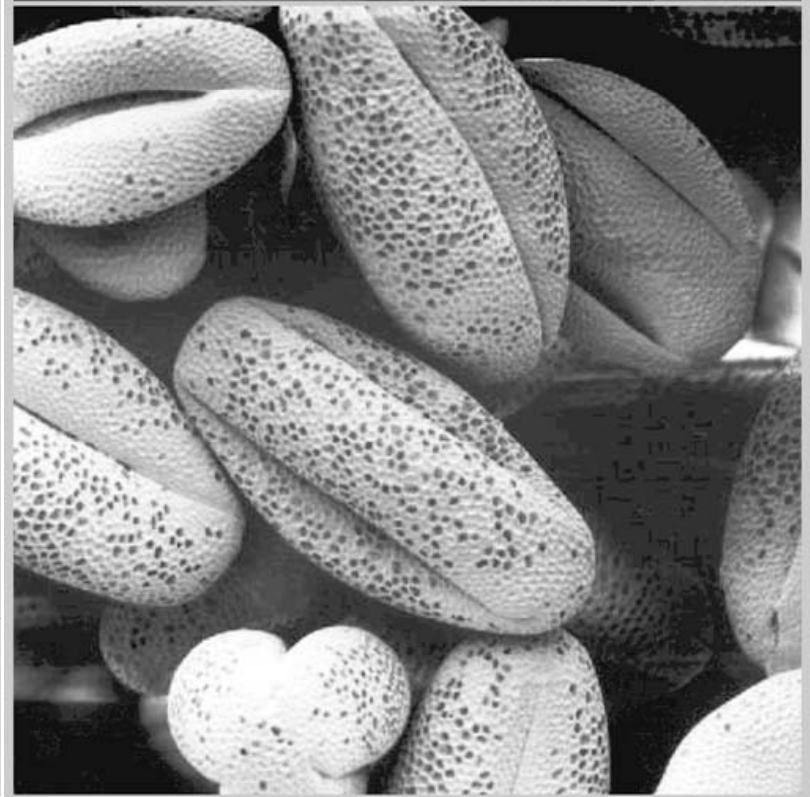

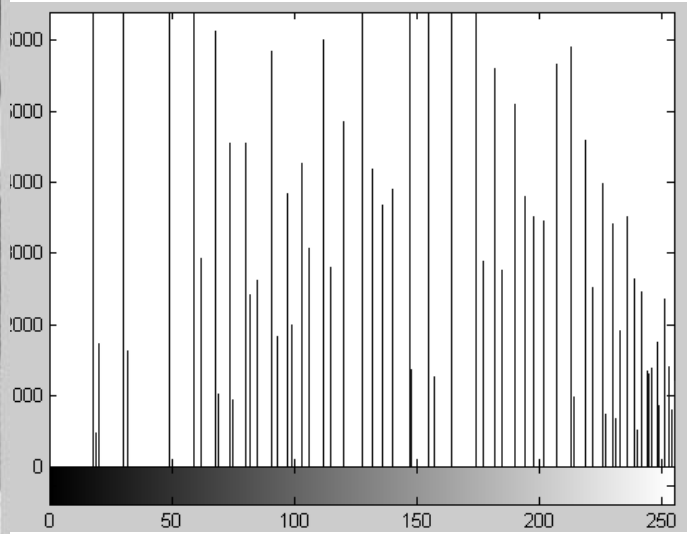
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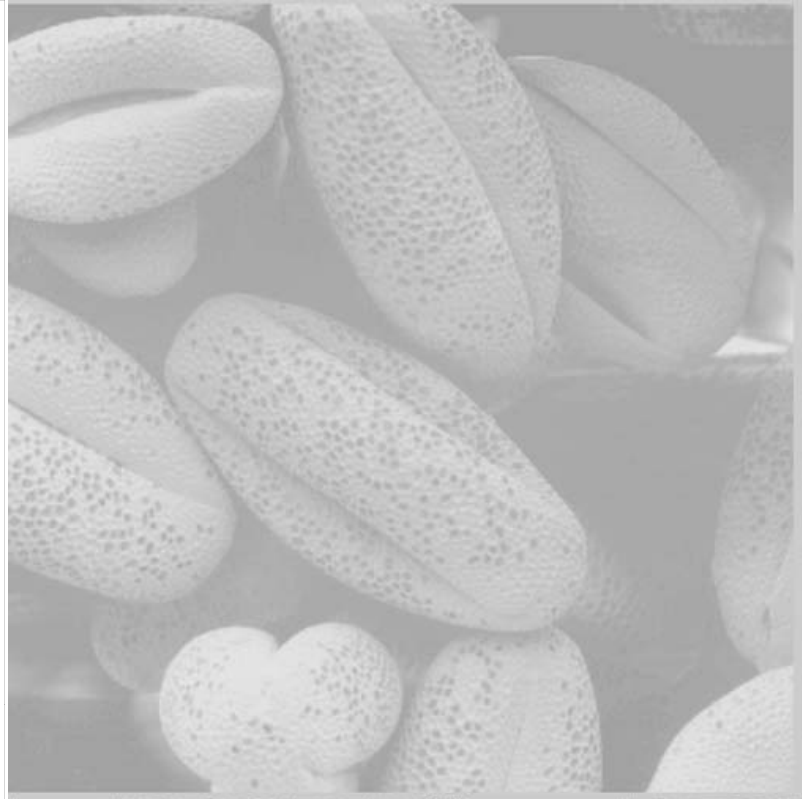

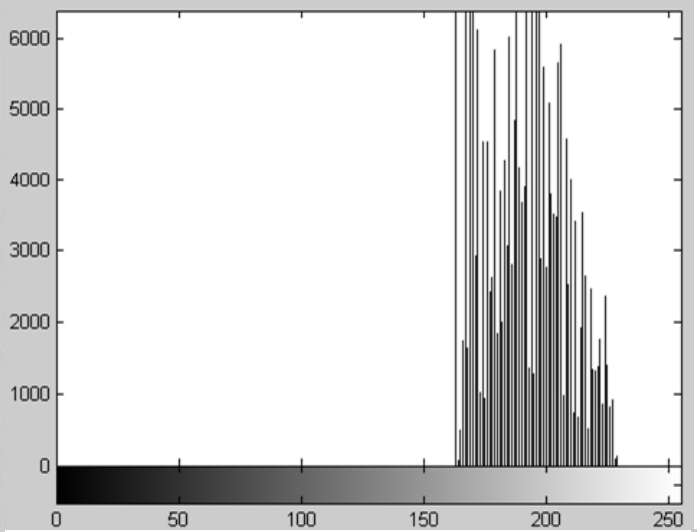
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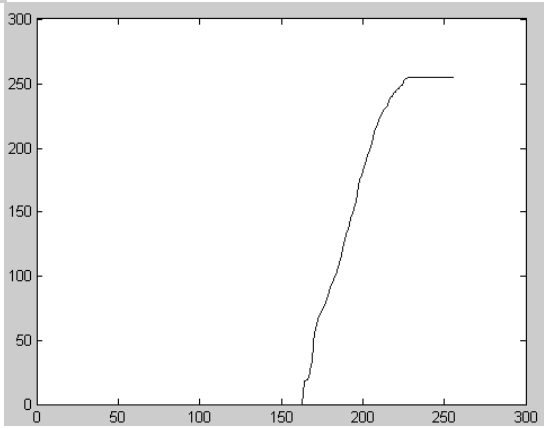
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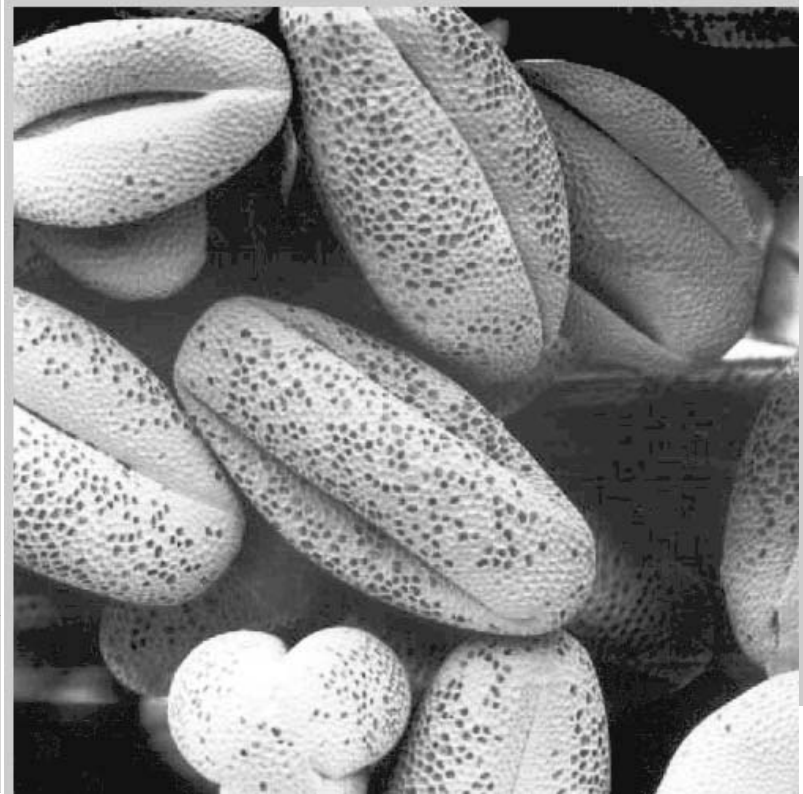

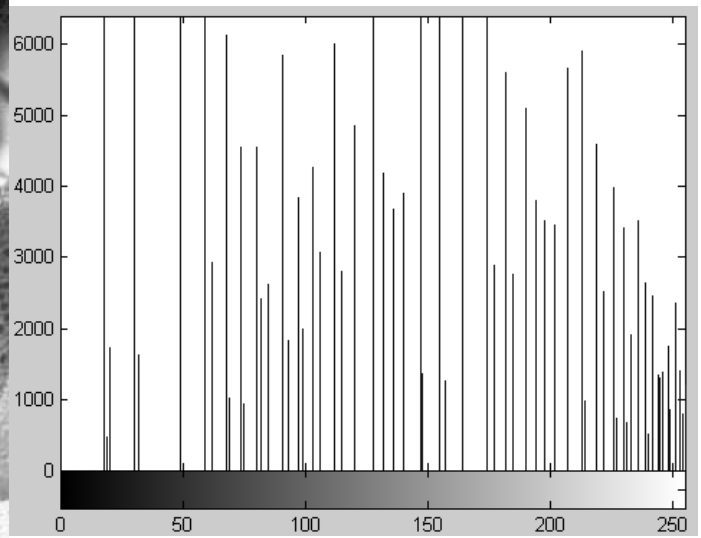
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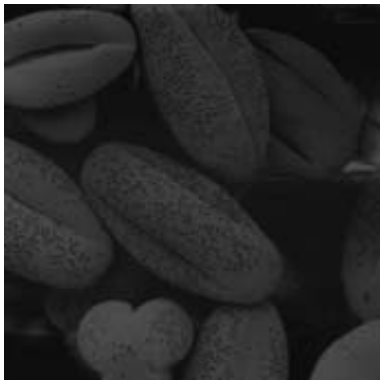
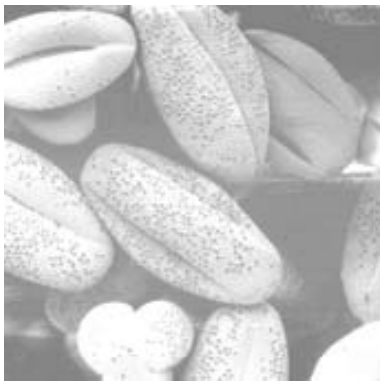
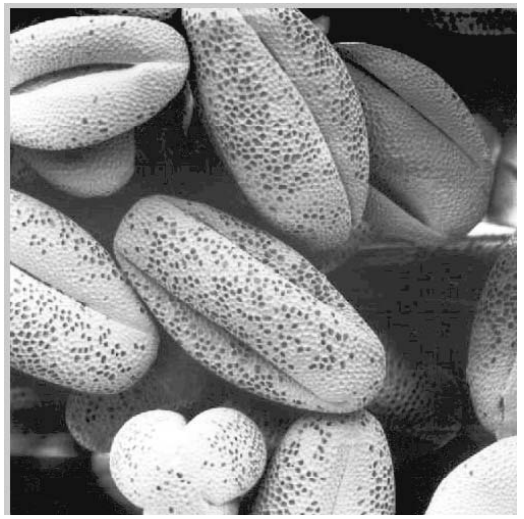
Histogram Equalization



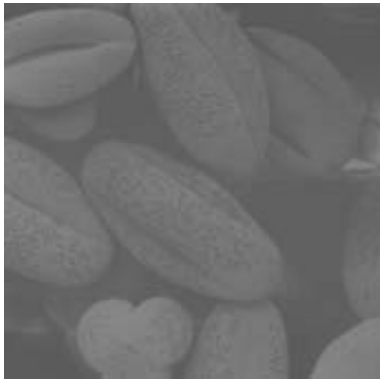


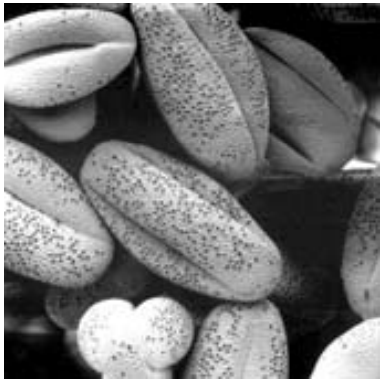
f(x)

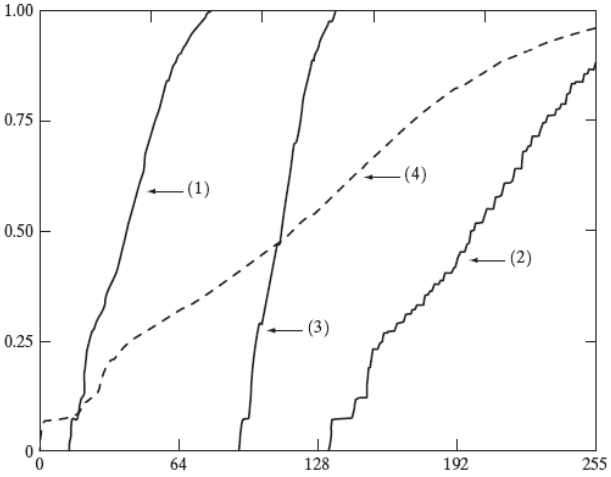
Histogram Equalization

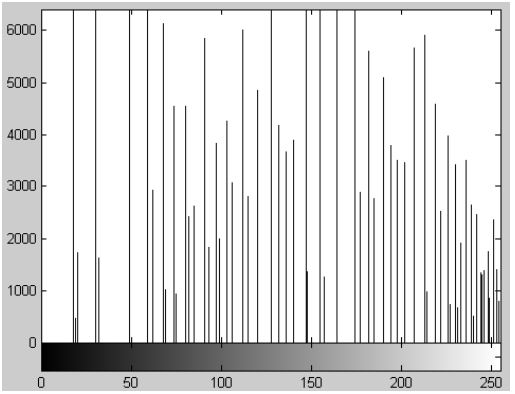


g(x)





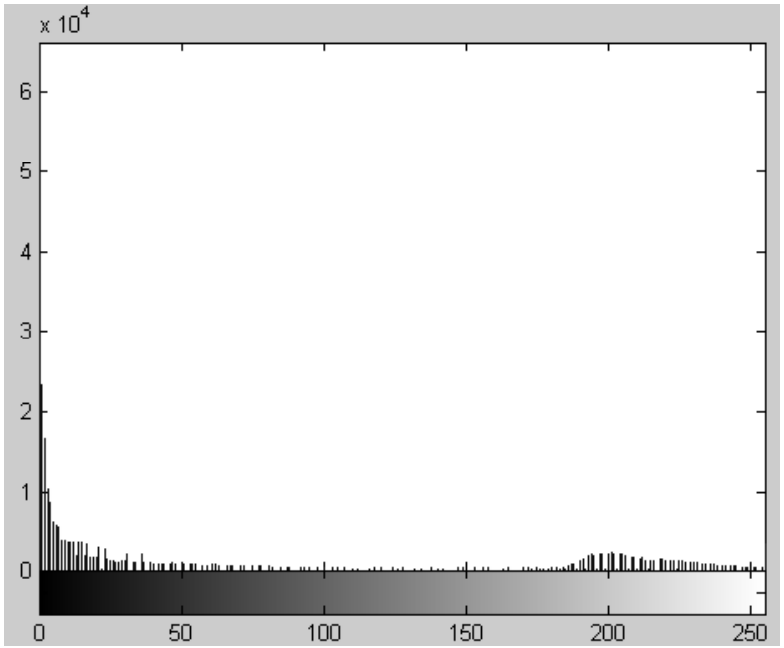


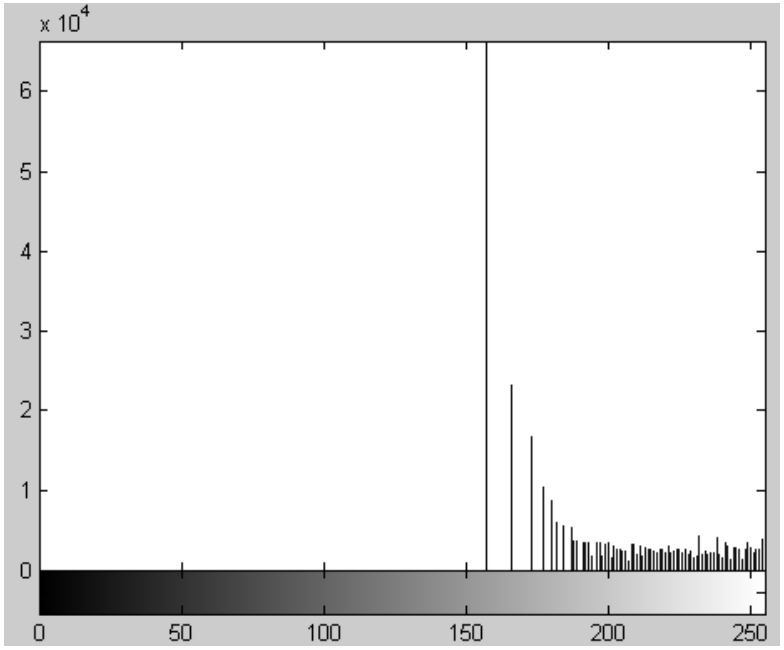


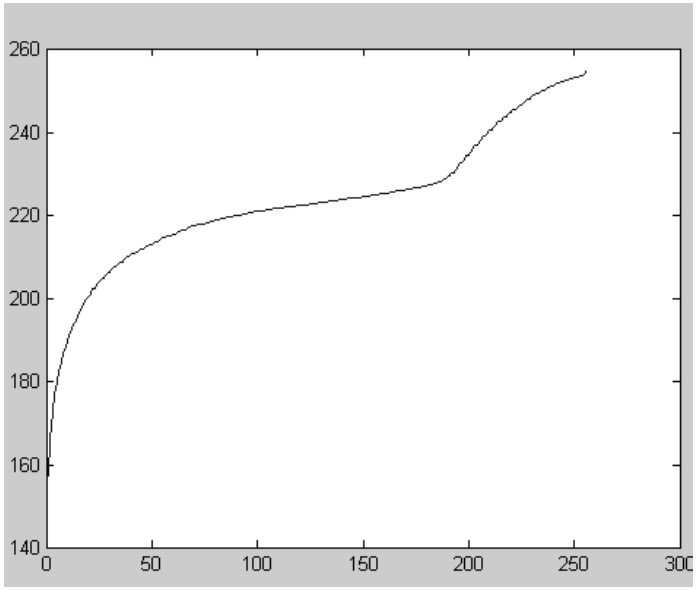
Histogram Equalization

- ช่วยเพิ่ม Contrast ให้กับภาพโดยเฉพาะบริเวณช่วงที่มีความหนาแน่นสูงใน Histogram
- ทำให้ Histogram กระจายตัวอย่างสม่ำเสมอ (uniform)
- ข้อดี
  - ทำงานได้อย่างอัตโนมัติ : ไม่ต้องมีการกำหนด Parameter
  - Function Transformations ได้จากภาพโดยตรง ปรับ Adaptive ตามลักษณะภาพ
- ข้อเสีย
  - ผลที่ได้อาจมีคุณภาพไม่ดีขึ้นในลักษณะการมองเห็น

Histogram Equalization







Histogram Matching (Specification)

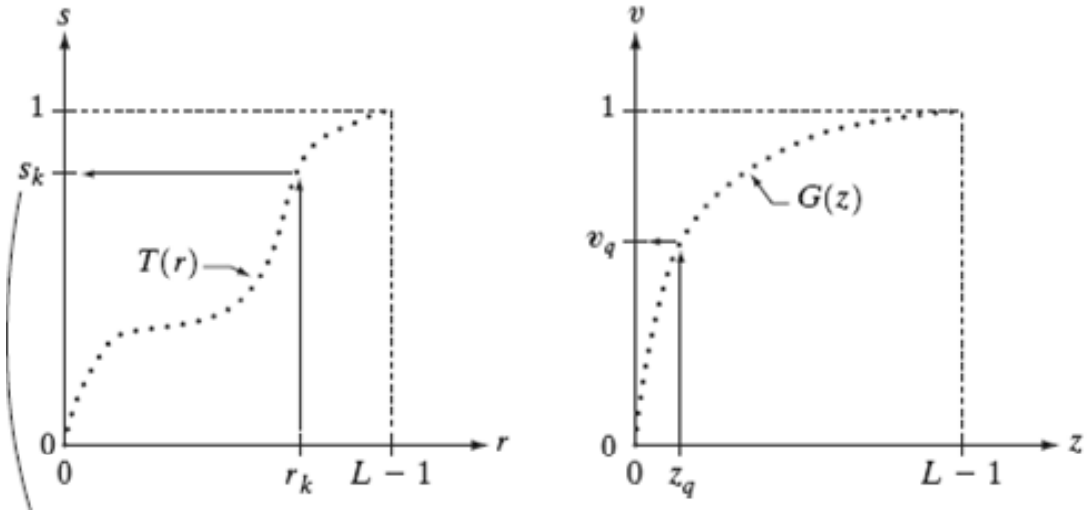
- เป็นการประมวลผลภาพเพื่อให้ Histogram ของภาพ Output มีรูปแบบตามที่ต้องการ โดยที่
  - $p_r(r)$ เป็นฟังก์ชันการกระจายความน่าจะเป็นของภาพต้นแบบ
  - $p_z(z)$ เป็นฟังก์ชันการกระจายความน่าจะเป็น specified ของภาพผลลัพธ์
    - Histogram Equalization ใช้ $s = T(r) = (L-1)\int_0^r p_r(w)\,dw$
    - ซึ่งผลลัพธ์ที่ได้อาจไม่ดีพอ
  - จึงทำการกำหนด $G(z) = (L-1)\int_0^z p_z(t)\,dt = s$

$$z = G^{-1}(s) = G^{-1}\left[T(r)\right]$$

Histogram Matching (Specification) Process

- คำนวณ Normalized Histogram ของภาพต้นแบบ $p_r(r)$
- คำนวนหา s_k จาก $s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j)$
- หาฟังก์ชัน $G(z_k)$ จาก $p_z(r)$ ที่กำหนดขึ้น

$$v_k = G(z_k) = \sum_{j=0}^{k} p_z(z_j)$$



Histogram Matching (Specification) Process

- หา s_k ที่เท่ากับ v_k เพื่อคำนวณหา z_k

$$s_k = v_k = G(z_k)$$

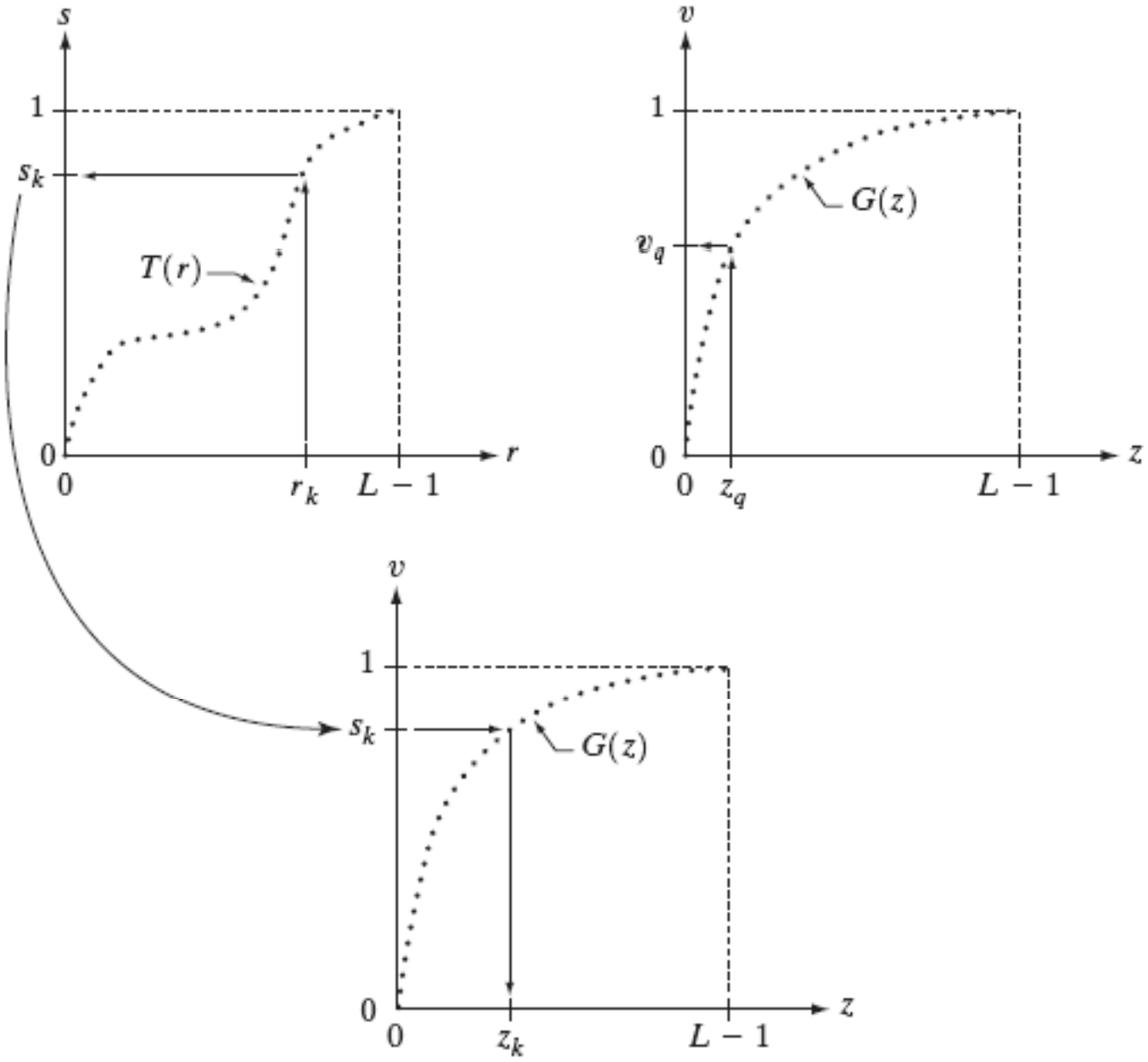
$$\left[G(z_k) - s_k\right] = 0$$

  - ซึ่งทำได้ยากมาก
  - จึงใช้วิธีการวนซ้ำ ซึ่งเป็นการประมาณค่าของ z_k

$$\left|s_k - v_l\right| = \min_{0 \le k \le L-1} \left|s_k - v_k\right| \approx 0$$

- ทำการแปลงจาก r_k เป็น s_k
  และทำการแปลงจาก s_k เป็น z_k

Histogram Matching (Specification)



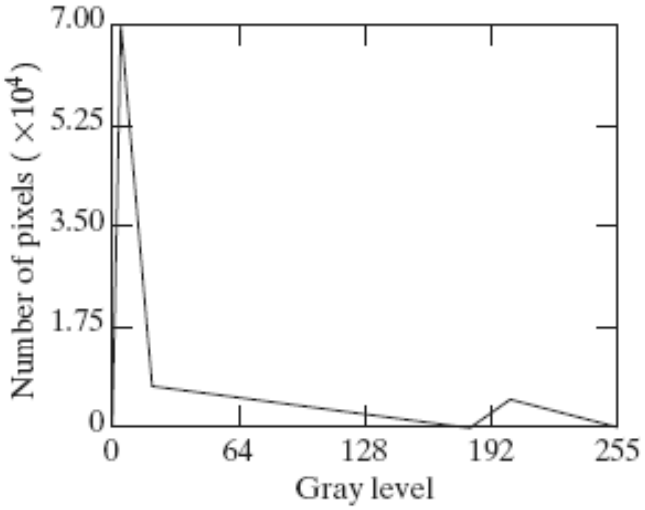
$$s_k = T(r_k)$$

$$v_k = G(z_k)$$

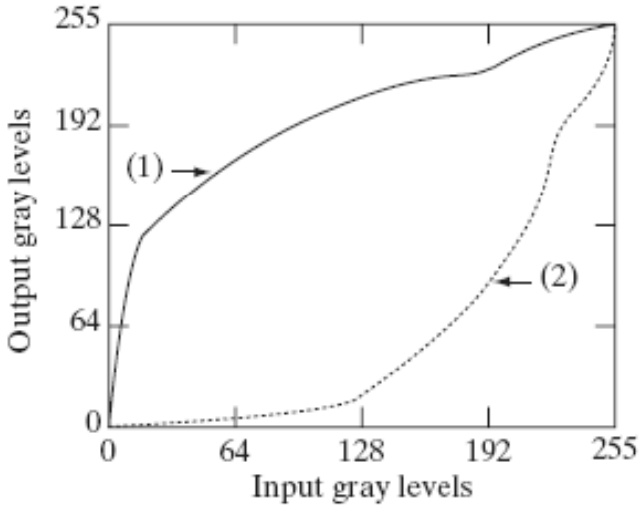
$$= s_k$$

Histogram Matching (Specification)

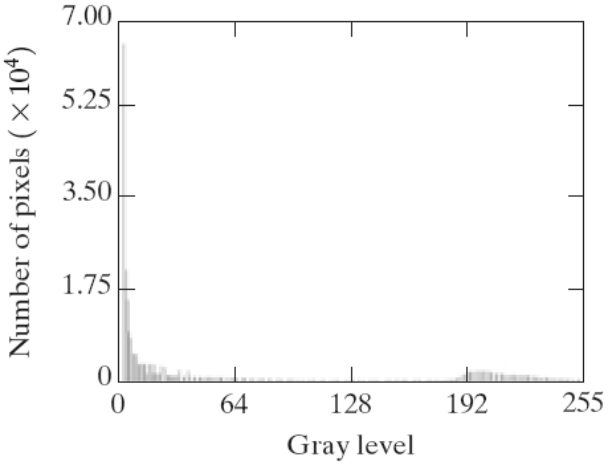
Specified histogram

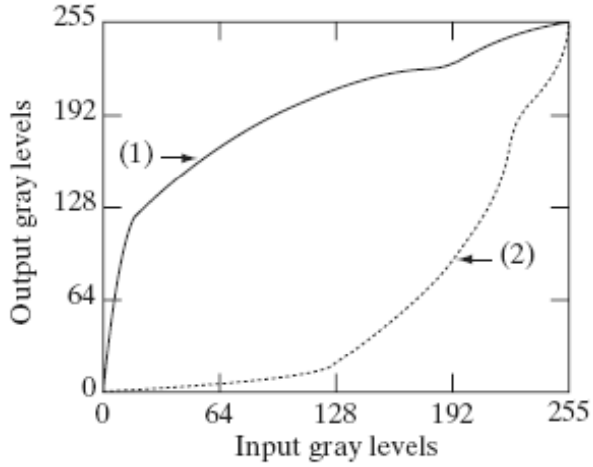


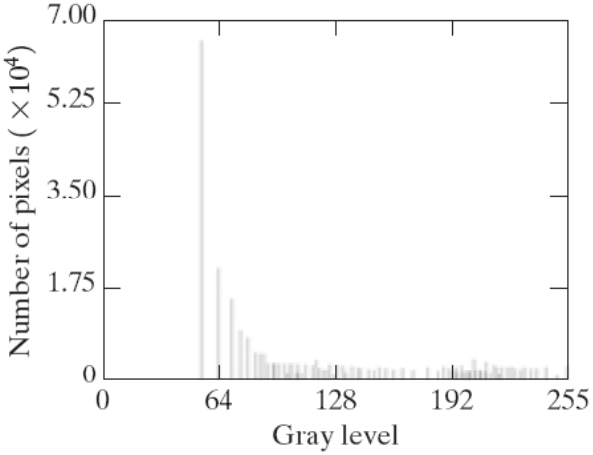
$$G(z_k) = \sum_{i=0}^{k} p_z(z_i) = s_k$$



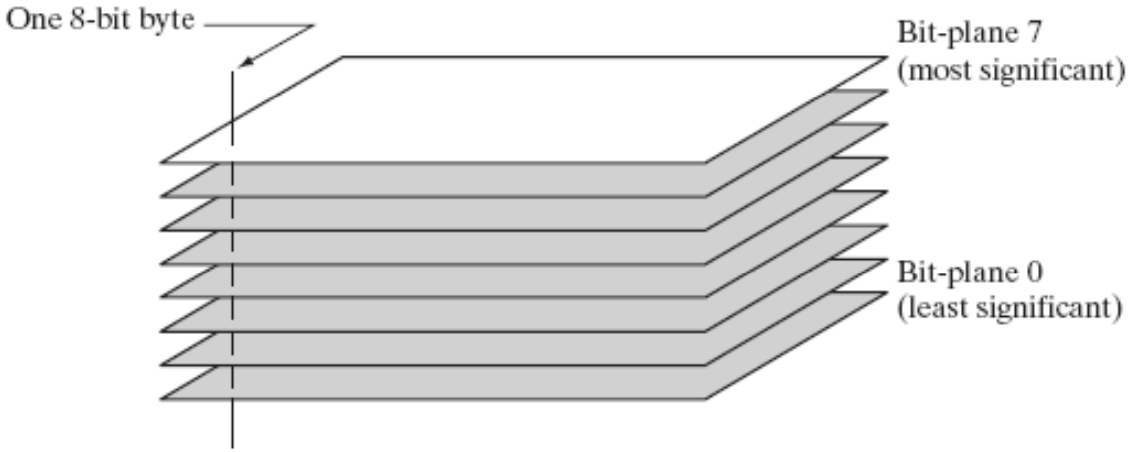
$$\left[G(\hat{z}) - s_k\right] \geq 0 \quad k = 0,1,2,\ldots,L-1$$

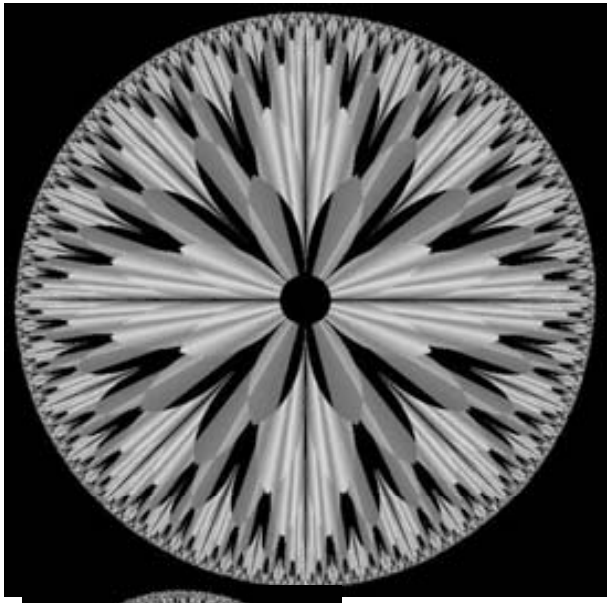

7.00
5.25
3.50
1.75
0
Number of pixels (× 10^4)
0
64
128
192
255
Gray level

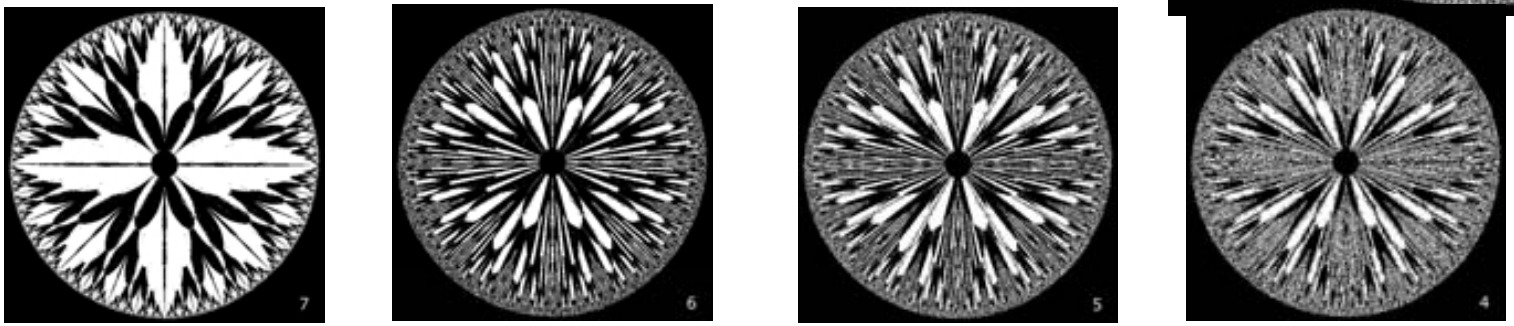

255
192
128
64
0
Output gray levels
(1)
(2)
0
64
128
192
255
Input gray levels

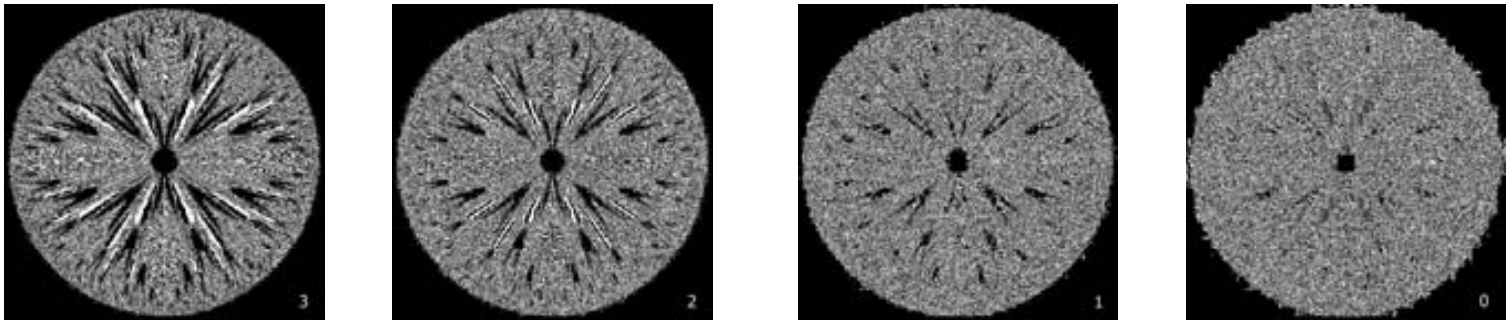

7.00
5.25
3.50
1.75
0
Number of pixels (× 10^4)
0
64
128
192
255
Gray level

Bit-plane slicing

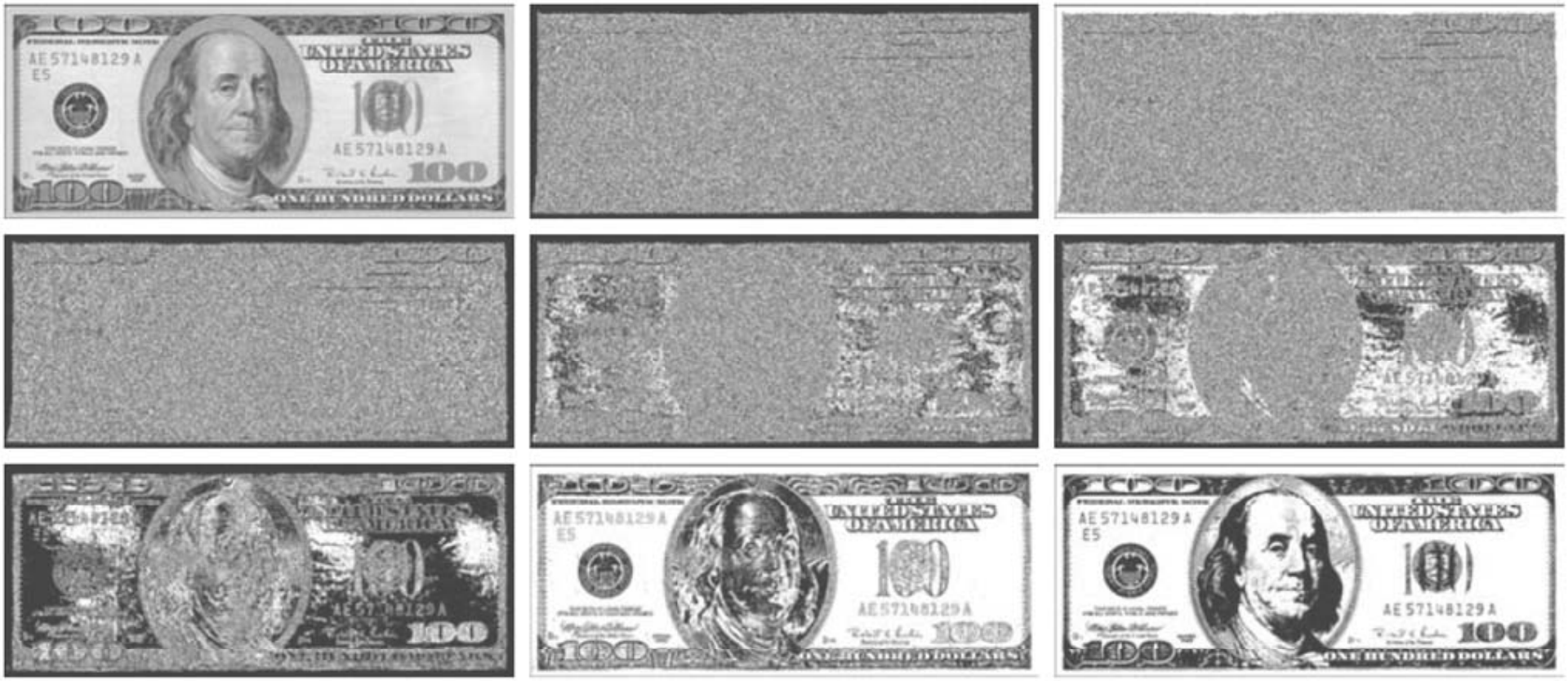








Bit-plane slicing



a b c
d e f
g h i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

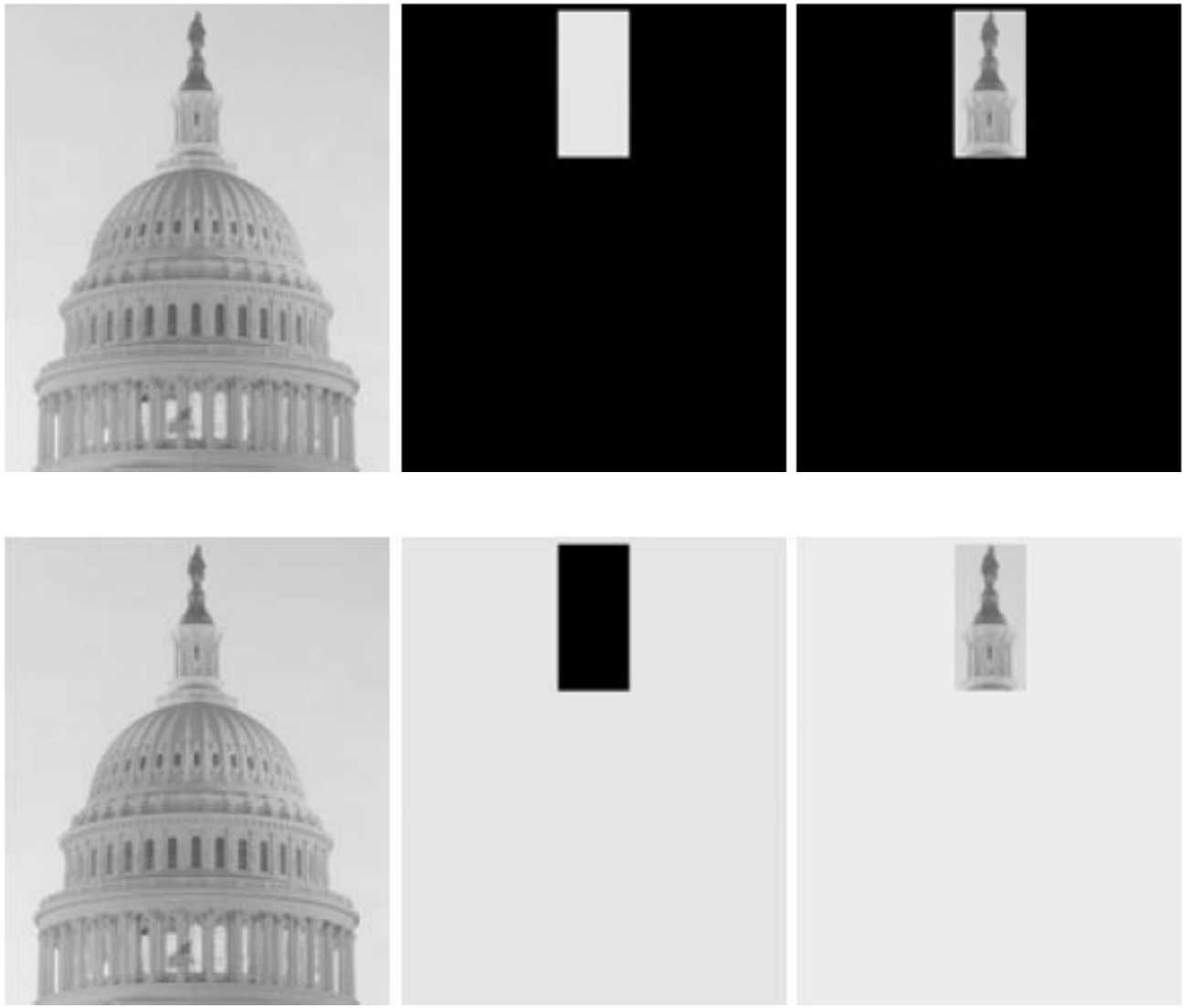
Bit-plane slicing

a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

Enhancement Using Arithmetic/Logic Operations

- เป็นการกระทำระหว่างภาพ 2 ภาพ ระหว่างพิกเซลต่อพิกเซลโดยตรง เช่น
  - การนำภาพ 2 ภาพมา AND หรือ OR กัน
  - หรือนำข้อมูลภาพมา +, -, * หรือ /



Enhancement Using Arithmetic/Logic Operations

- Image Subtraction

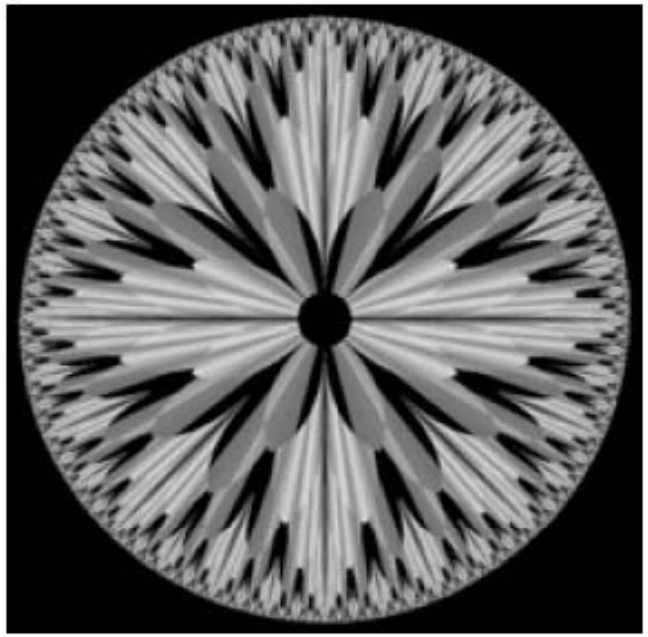
$$g(x, y) = f(x, y) - h(x, y)$$

- Image Averaging

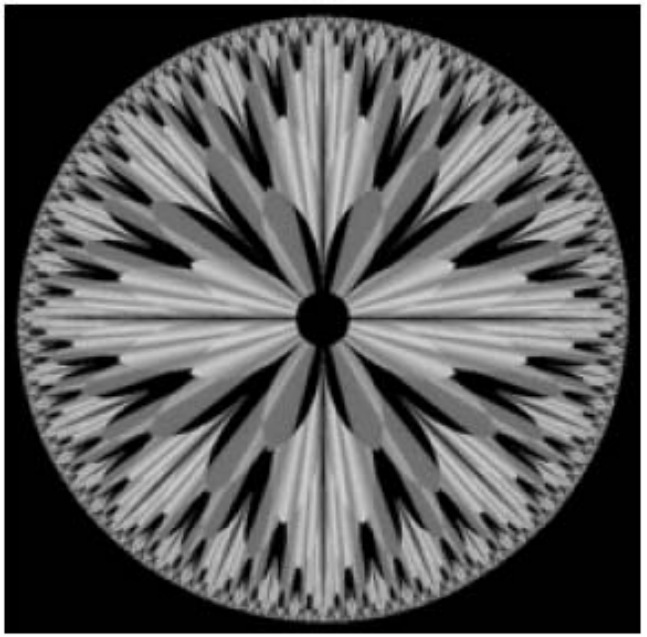
$$g(x, y) = f(x, y) + \eta(x, y)$$

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y)$$

Original fractal image



Result of setting the four lower-order bit planes to zero.



Difference between Original fractal image and the four lower-order bit planes to zero

Image of Galaxy Pair NGC 3314

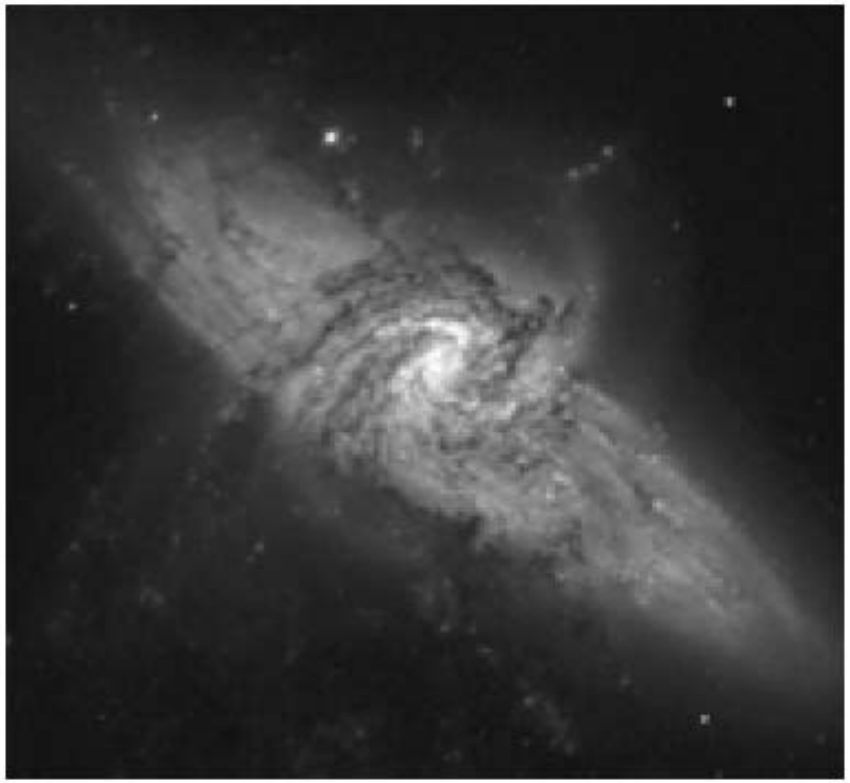
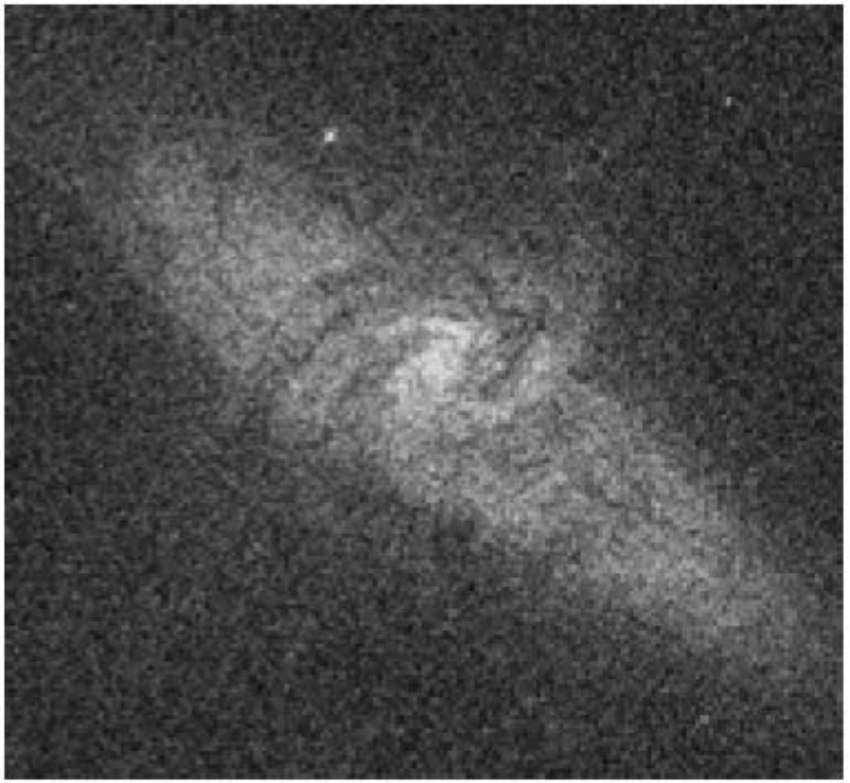
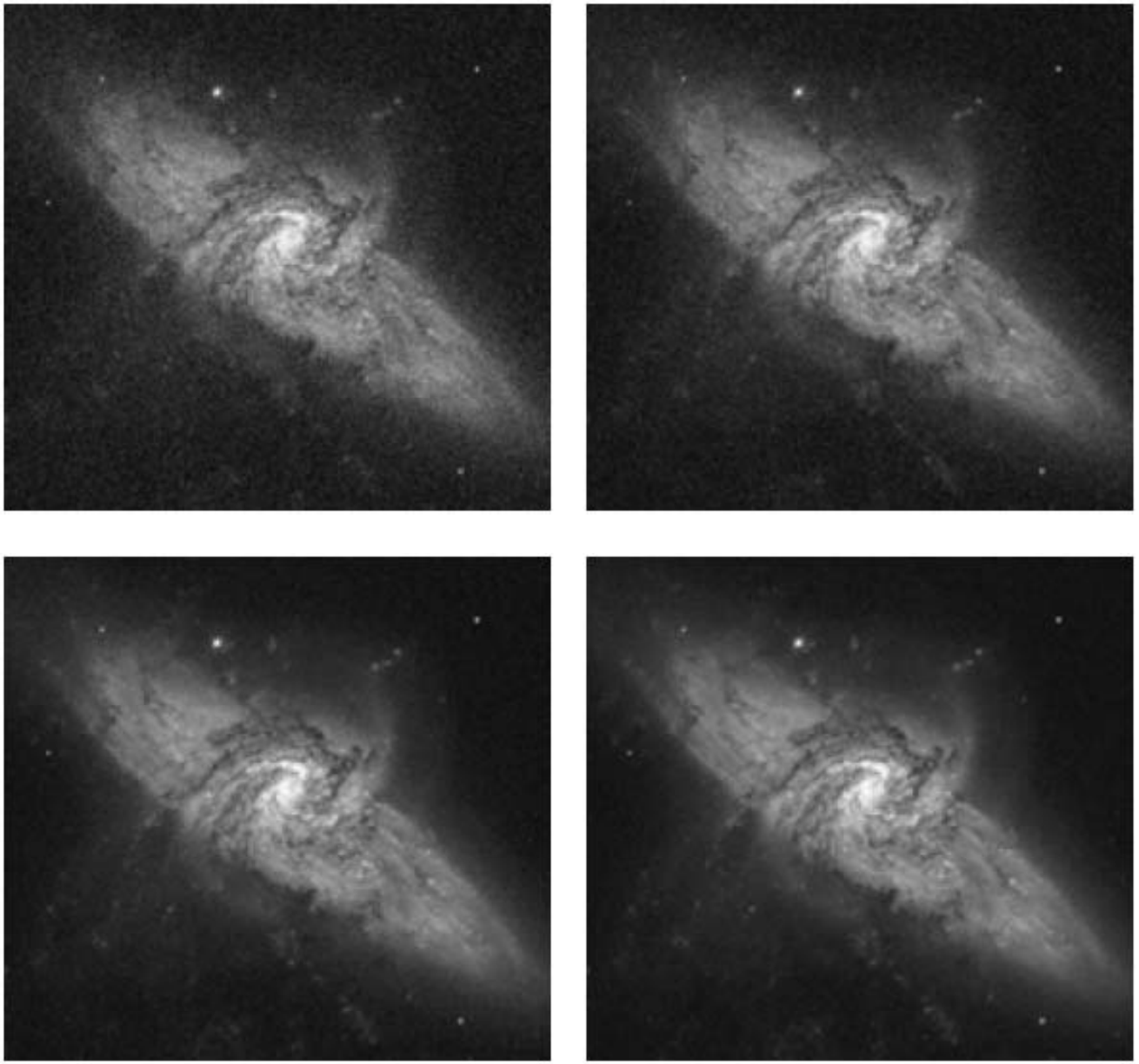


Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels.





c-f :Results of averaging K=8, 16, 64, and 128 noisy images.