

3η Σειρά Ασκήσεων - Λύση 4ης άσκησης

(α)

```
[0] sum(X,s(s(0)),s(s(s(s(0)))))
    {X = X}
    1 --> 4
    sum(X1,s(Y1),s(Z1)) :- sum(X1,Y1,Z1).
    {X1 = X, Y1 = s(0), Z1 = s(s(s(s(0))))}
```

```
[1] sum(X,s(0),s(s(s(s(0)))))
    {X = X}
    1 --> 4
    sum(X2,s(Y2),s(Z2)) :- sum(X2,Y2,Z2).
    {X2 = X, Y2 = 0, Z2 = s(s(s(s(0))))}
```

```
[2] sum(X,0,s(s(s(0))))
    {X = X}
    1 --> 3
    sum(X3,0,X3) :- nat(X3).
    {X = s(s(s(s(0)))), X3 = s(s(s(s(0))))}
```

```
[4] nat(s(s(s(0))))
    {X = s(s(s(s(0))))}
    1 --> 2
    nat(s(X4)) :- nat(X4).
    {X4 = s(s(s(0)))}
```

```
[4] nat(s(s(0)))
    {X = s(s(s(s(0))))}
    1 --> 2
    nat(s(X5)) :- nat(X5).
    {X5 = s(0)}
```

```
[5] nat(s(0))
    {X = s(s(s(s(0))))}
    1 --> 2
    nat(s(X6)) :- nat(X6).
    {X6 = 0}
```

```
[6] nat(0)
    {X = s(s(s(s(0))))}
    1 --> 1
    nat(0).
    {}
```

[7] #  
    {X = s(s(s(0)))}

-----  
X = s(s(s(0)))

( $\beta$ )

[0] gcdEuc(72,120,X)  
    {X = X}  
    1 --> 2  
    gcdEuc(M1,N1,D1) :- M1 < N1, K1 is N1-M1, gcdEuc(M1,K1,D1).  
    {M1 = 72, N1 = 120, D1 = X}

[1] 72 < 120, K1 is 120-72, gcdEuc(72,K1,X)  
    {X = X}  
    < : build-in  
    {}

[2] K1 is 120-72, gcdEuc(72,K1,X)  
    {X = X}  
    is : build-in  
    {K1 = 48}

[3] gcdEuc(72,48,X)  
    {X = X}  
    1 --> 2  
    gcdEuc(M2,N2,D2) :- M2 < N2, K2 is N2-M2, gcdEuc(M2,K2,D2).  
    {M2 = 72, N2 = 48, D2 = X}

[4] 72 < 48, K2 is 48-72, gcdEuc(72,K2,D2)  
    {X = X}  
    < : build-in  
    failure - backtracking

[3] gcdEuc(72,48,X)  
    {X = X}  
    3 --> 3  
    gcdEuc(M3,N3,D3) :- M3 > N3, K3 is M3-N3, gcdEuc(N3,K3,D3).  
    {M3 = 72, N3 = 48, D3 = X}

[4] 72 > 48, K3 is 72-48, gcdEuc(48,K3,X)  
    {X = X}  
    < : build-in  
    {}

```
[5] K3 is 72-48, gcdEuc(48,K3,X)
    {X = X}
    is : build-in
    {K3 = 24}
```

```
[6] gcdEuc(48,24,X)
    {X = X}
    1 --> 2
    gcdEuc(M4,N4,D4) :- M4 < N4, K4 is N4-M4, gcdEuc(M4,K4,D4).
    {M4 = 48, N4 = 24, D4 = X}
```

```
[7] 48 < 24, K4 is 24-48, gcdEuc(48,K4,X).
    {X = X}
    < : build-in
    failure - backtracking
```

```
[6] gcdEuc(48,24,X)
    {X = X}
    3 --> 3
    gcdEuc(M5,N5,D5) :- M5 > N5, K5 is M5-N5, gcdEuc(N5,K5,D5).
    {M5 = 48, N5 = 24, D5 = X}
```

```
[7] 48 > 24, K5 is 48-24, gcdEuc(24,K5,X)
    {X = X}
    < : build-in
    {}
```

```
[8] K5 is 48-24, gcdEuc(24,K5,X).
    {X = X}
    is : build-in
    {K5 = 24}
```

```
[9] gcdEuc(24,24,X)
    {X = X}
    3 --> 3
    gcdEuc(N6,N6,N6).
    {X = 24, N6 = 24}
```

```
[10] #
     {X = 24}
```

---

X = 24

( $\gamma$ )

```
[0] gcdFast(455,196,X)
    {X = X}
    1 --> 2
    gcdFast(M1,N1,D1) :- M1 < N1, K1 is N1 mod M1, gcdFast(K1,M1,D1).
    {M1 = 455, N1 = 196, D1 = X}

[1] 455 < 196, K1 is 196 mod 455, gcdFast(K1,455,X)
    {X = X}
    < : build-in
    failure - backtracking

[0] gcdFast(455,196,X)
    {X = X}
    3 --> 3
    gcdFast(M2,N2,D2) :- M2 >= N2, K2 is M2 mod N2, gcdFast(K2,N2,D2).
    {M2 = 455, N2 = 196, D2 = X}

[1] 455 >= 196, K2 is 455 mod 196, gcdFast(K2,196,X)
    {X = X}
    >= : build-in
    {}

[2] K2 is 455 mod 196, gcdFast(K2,196,X)
    {X = X}
    is : build-in
    {K2 = 63}

[3] gcdFast(63,196,X)
    {X = X}
    1 --> 2
    gcdFast(M3,N3,D3) :- M3 < N3, K3 is N3 mod M3, gcdFast(K3,M3,D3).
    {M3 = 63, N3 = 196, D3 = X}

[4] 63 < 196, K3 is 196 mod 63, gcdFast(K3,63,X)
    {X = X}
    < : build-in
    {}

[5] K3 is 196 mod 63, gcdFast(K3,63,X)
    {X = X}
    is : build-in
    {K3 = 7}
```

```
[6] gcdFast(7,63,X)
    {X = X}
    1 --> 2
    gcdFast(M4,N4,D4) :- M4 < N4, K4 is N4 mod M4, gcdFast(K4,M4,D4).
    {M4 = 7, N4 = 63, D4 = X}
```

```
[7] 7 < 63, K4 is 63 mod 7, gcdFast(K4,7,X)
    {X = X}
    < : build-in
    {}
```

```
[8] K4 is 63 mod 7, gcdFast(K4,7,X)
    {X = X}
    is : build-in
    {K4 = 0}
```

```
[9] gcdFast(0,7,X)
    {X = X}
    1 --> 1
    gcdFast(0,M5,M5).
    {X = 7, M5 = 7}
```

```
[10] #
     {X = 7}
```

---

X = 7

(δ)

```
[0] conc([1,a,[3,4],[ ]], [ ], L)
    {L = L}
    1 --> 2
    conc([H1|T1],L1,[H1|Q1]) :- conc(T1,L1,Q1).
    {H1 = 1, T1 = [a,[3,4],[ ]], L1 = [ ], L = [1|Q1]}
```

```
[1] conc([a,[3,4],[ ]], [ ], Q1)
    {L = [1|Q1]}
    1 --> 2
    conc([H2|T2],L2,[H2|Q2]) :- conc(T2,L2,Q2).
    {H2 = a, T2 = [[3,4],[ ]], L2 = [ ], Q1 = [a|Q2]}
```

```
[2] conc([[3,4],[ ]], [ ], Q2)
    {L = [1,a|Q2]}
    1 --> 2
    conc([H3|T3],L3,[H3|Q3]) :- conc(T3,L3,Q3).
    {H3 = [3,4], T3 = [[ ]], L3 = [ ], Q2 = [[3,4]|Q3]}
```

```
[3] conc([[ ]], [ ], Q3)
    {L = [1,a,[3,4]|Q3]}
    1 --> 2
    conc([H4|T4],L4,[H4|Q4]) :- conc(T4,L4,Q4).
    {H4 = [ ], T4 = [ ], L4 = [ ], Q3 = [[ ]|Q4]}
```

```
[4] conc([ ], [ ], Q4)
    {L = [1,a,[3,4],[ ]|Q4]}
    1 --> 1
    conc([ ],L5,L5).
    {L5 = [ ], Q4 = [ ]}
```

```
[5] #
    {L = [1,a,[3,4],[ ]]}
```

---

L = [1,a,[3,4],[ ]]

( $\epsilon$ )

```
[0] delete(0,L,[1])
    {L = L}
    1 --> 1
    delete(X1,[X1|T1],T1).
    {L = [0,1], X1 = 0, T1 = [1]}
```

```
[1] #
    {L = [0,1]}
```

---

```
L = [0,1] ;
```

---

```
[0] delete(0,L,[1])
    {L = L}
    2 --> 2
    delete(X2,[H2|T2],[H2|S2]) :- delete(X2,T2,S2).
    {L = [1|T2], X2 = 0, H2 = 1, S2 = []}
```

```
[1] delete(0,T2,[])
    {L = [1|T2]}
    1 --> 1
    delete(X3,[X3|T3],T3).
    {T2 = [0], X3 = 0, T3 = []}
```

```
[2] #
    {L = [1,0]}
```

---

```
L = [1,0] ;
```

---

```
[1] delete(0,T2,[])
    {L = [1|T2]}
    2 --> EOP
    failure - backtraching
```

```
[0] delete(0,L,[1])
    {L = L}
    EOP
    failure
```

---

no