HM1: Logistic Regression.

Name: Conor McGullam

Q1 (6%). $x = [5, 0, 1, -2]^T$ is a 4-dimensional vector. Calculate the following values:

1. the squared ℓ2-norm of x:

$\|x\|_2^2 = 5^2 + 0^2 + 1^2 + (-2)^2 = 30$ 2. the ℓ1-norm of x: $\|x\|_1 = 5 + 0 + 1 + (-2) = 4$ 3. the inner product of x and a, where $a = [4, -2, 6, -1]^T$: $a^T x = 5*4 + (-2)*0 + 1*6 + (-1)*(-2) = 28$

Q2 (4%). The matrix $A \in R^{2\times 3}$ and vector $b \in R^3$ are defined in the following:

$$A = \begin{bmatrix} 6 & 1 & 2 \\ -5 & 0 & -3 \end{bmatrix}$$

and

$$b = \begin{bmatrix} -4 \\ 5 \\ 0 \end{bmatrix}$$

Caculate the following values:

1. the matrix-vector product:

$Ab = \begin{bmatrix} (6*-4)+(5*1)+(0*2) \\ (-4*-5)+(5*0)+(-3*0) \end{bmatrix} = \begin{bmatrix} -19 \\ 20 \end{bmatrix}$ 2. the matrix-matrix product: $AA^T = \begin{bmatrix} 6 & 1 & 2 \\ -5 & 0 & -3 \end{bmatrix} \begin{bmatrix} 6 & -5 \\ 1 & 0 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 41 & -36 \\ -36 & 34 \end{bmatrix}$

Q3 (6%). Let $x = [x_1, x_2, x_3]$ and $y = \frac{x_1^2}{2} + log_e(x_2) - \frac{x_1}{x_3}$. Calculate the following:
$\frac{dy}{dx}$ at $x = [9, \frac{1}{2}, \frac{1}{3}]$:
$\frac{dy}{dx_1} = x_1 - \frac{1}{x_3}$
$\frac{dy}{dx_2} = \frac{1}{x_2}$
$\frac{dy}{dx_3} = \frac{x_1}{x_3^2}$
$\frac{dy}{dx} = [6, 2, 81]$
Hint: the answer should be a three dimensional vector

Q4 (4%). X is an $n \times d$ matrix, y is an $n \times 1$ vector, and w is an $d \times 1$ vector. Let $f(w) = \| Xw - y \|_2^2 + \lambda \| w\|_2^2$. Calculate the following:

$$\frac{\partial f(w)}{\partial w} = -\frac{1}{n}\sum_{i=1}^{n} \frac{y_i x_i}{1+exp(y_i x_i^T w)} + 2\lambda w$$