Edge Detection

CSE 576
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Edge
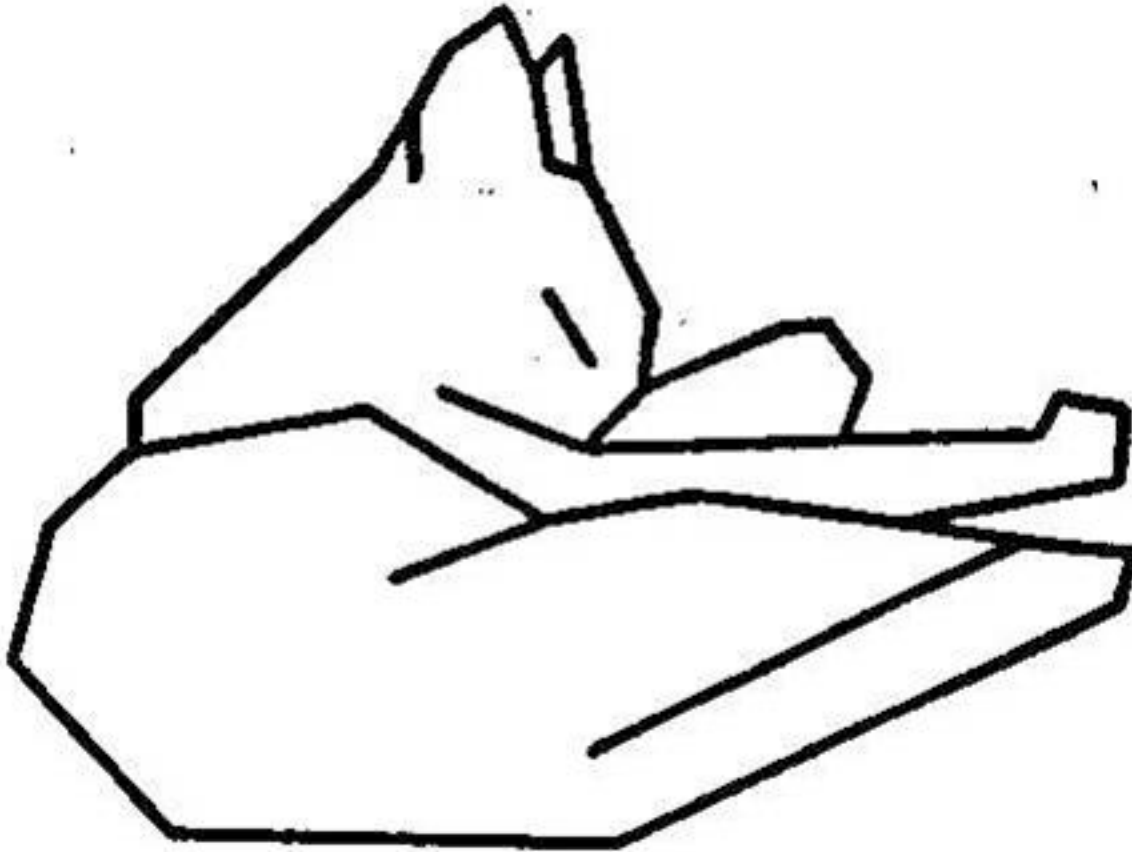
Attneave's Cat (1954)

Origin of edges

surface normal discontinuity

depth discontinuity

surface color discontinuity

illumination discontinuity
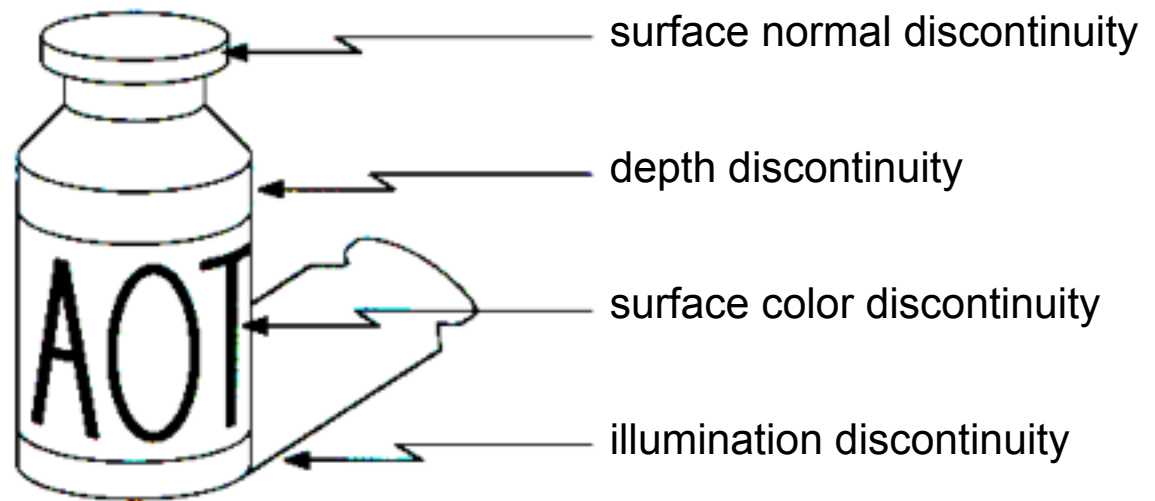
Edges are caused by a variety of factors

Characterizing edges

- An edge is a place of rapid change in the image intensity function

image

intensity function
(along horizontal scanline)

first derivative

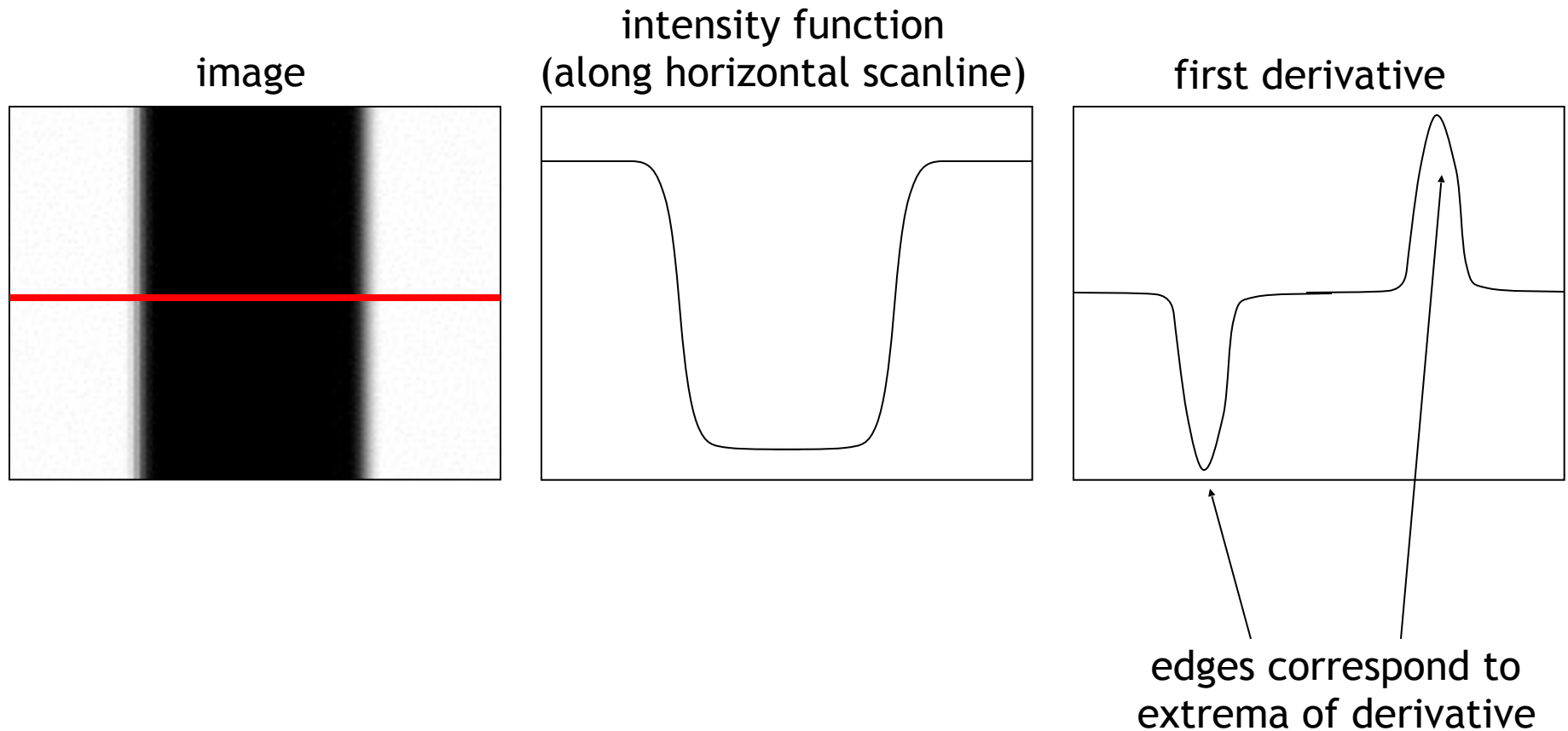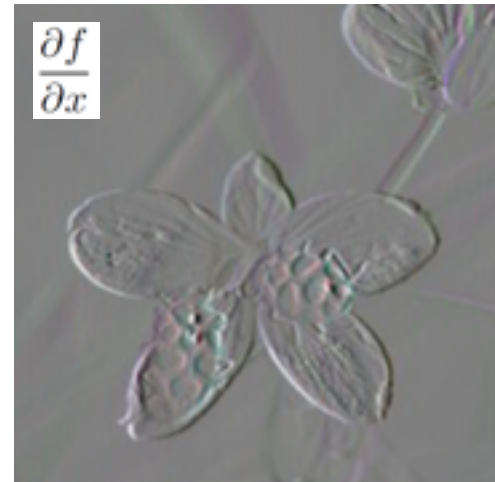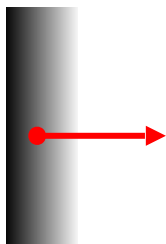edges correspond to
extrema of derivative

Image gradient

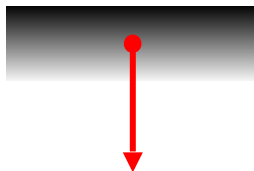- The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$
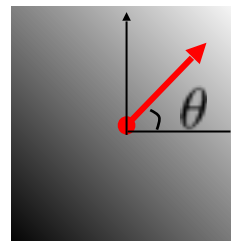
- The gradient points in the direction of most rapid change in intensity

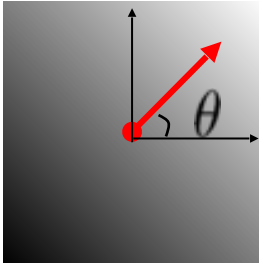$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right] \qquad \nabla f = \left[0, \frac{\partial f}{\partial y}\right] \qquad \nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The discrete gradient

- How can we differentiate a ==*digital*== image F[x,y]?
  - Option 1: reconstruct a continuous image, then take gradient
  - Option 2: ==take discrete derivative== ("finite difference")

$$\frac{\partial f}{\partial x}[x, y] \approx F[x + 1, y] - F[x, y]$$

Image gradient

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\frac{\partial f}{\partial x} = f(x+1, y) - f(x, y)$$

How would you implement this as a filter?

The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \middle/ \frac{\partial f}{\partial x}\right)$$

How does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Sobel operator

In practice, it is common to use:

$$g_x = \begin{array}{|c|c|c|} \hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad g_y = \begin{array}{|c|c|c|} \hline -1 & -2 & -1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

Magnitude:

$$g = \sqrt{g_x^2 + g_y^2}$$

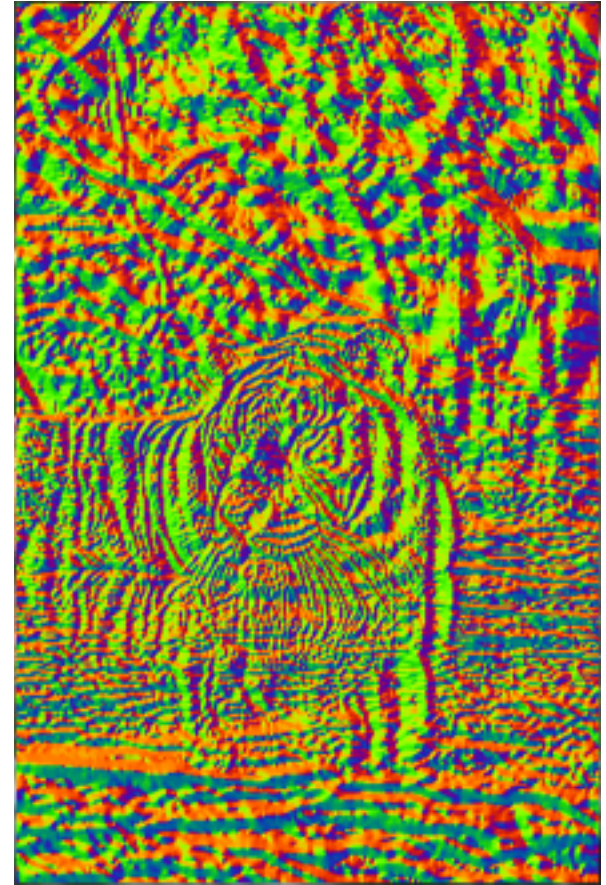Orientation:

$$\Theta = \tan^{-1}\left(\frac{g_y}{g_x}\right)$$
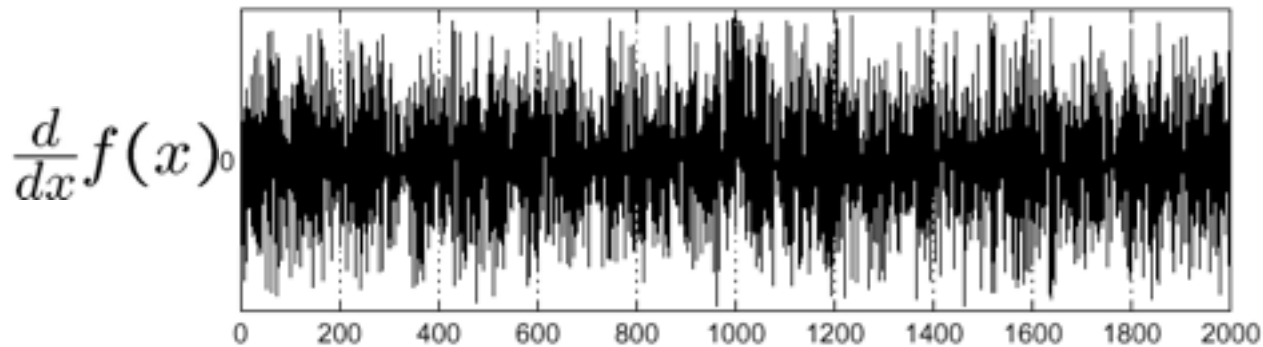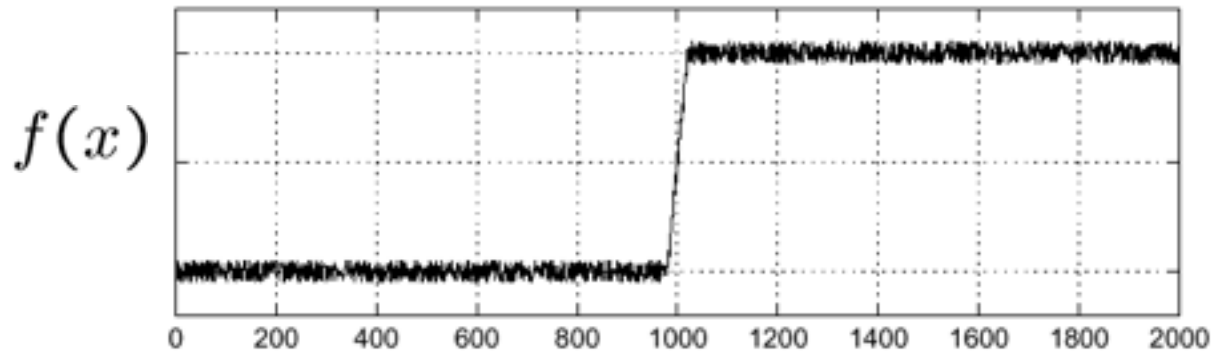
Sobel operator

| Original | Magnitude | Orientation |

Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal
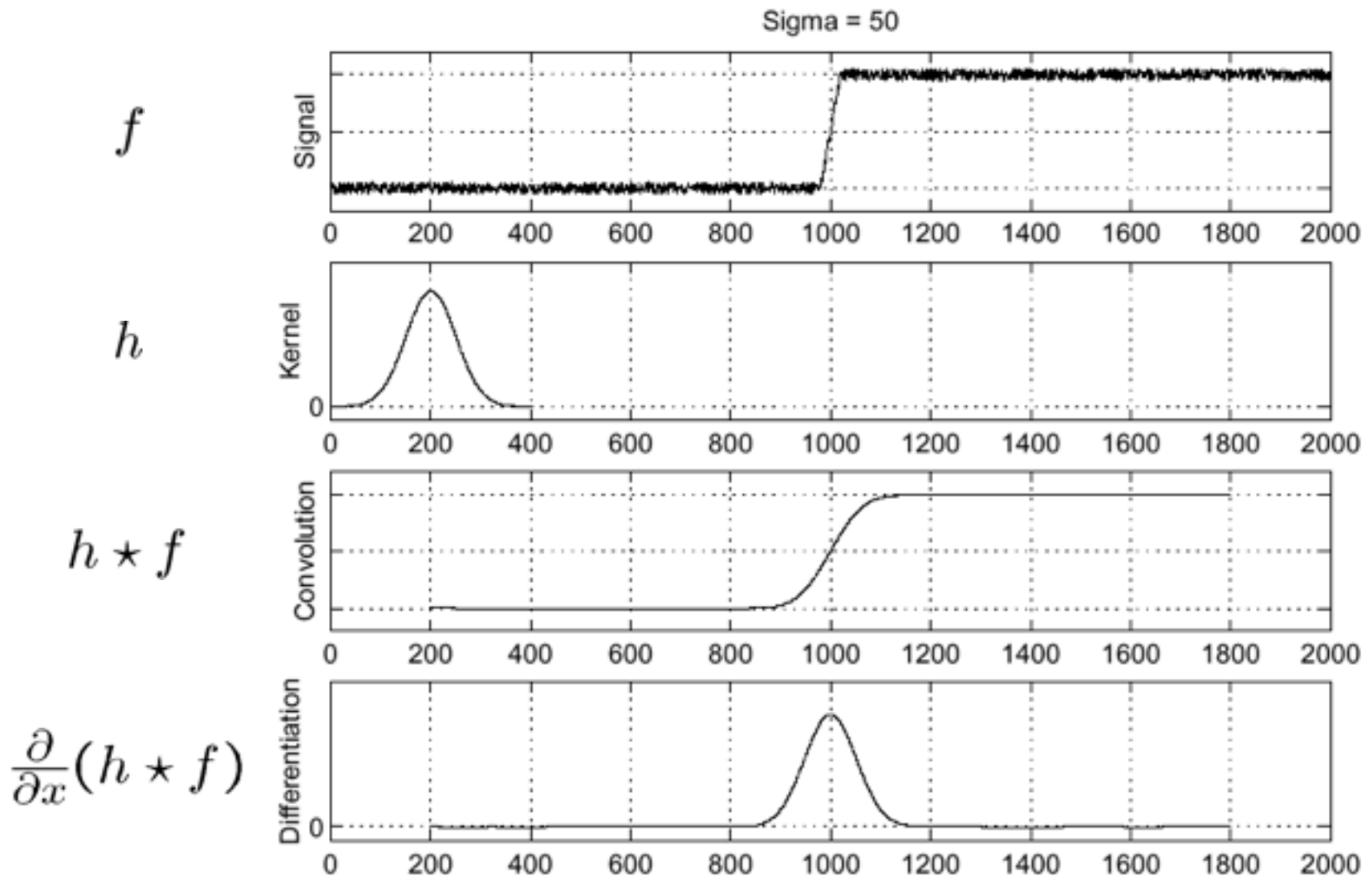
$f(x)$

$\frac{d}{dx}f(x)$

Where is the edge?

Effects of noise

- Difference filters respond strongly to noise

  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response

- What can we do about it?
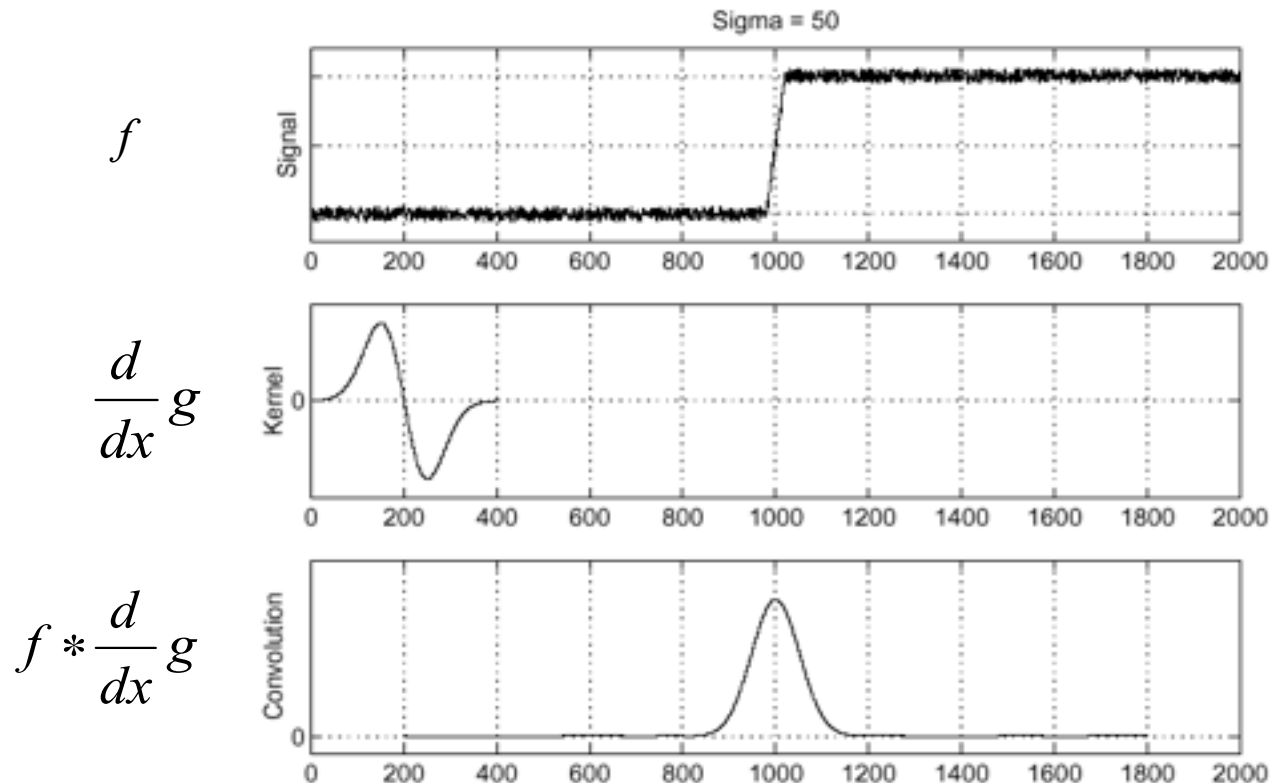
Solution: smooth first

Sigma = 50

f — Signal

h — Kernel

$h \star f$ — Convolution

$\frac{\partial}{\partial x}(h \star f)$ — Differentiation

Where is the edge?   Look for peaks in $\frac{\partial}{\partial x}(h \star f)$
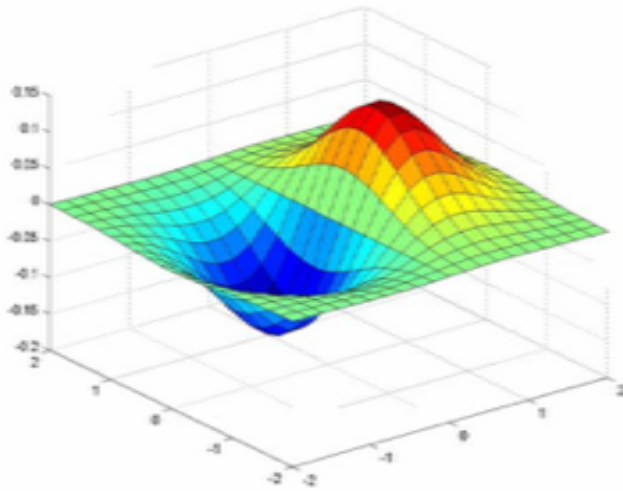
Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative: $\dfrac{d}{dx}(f * g) = f * \dfrac{d}{dx}g$
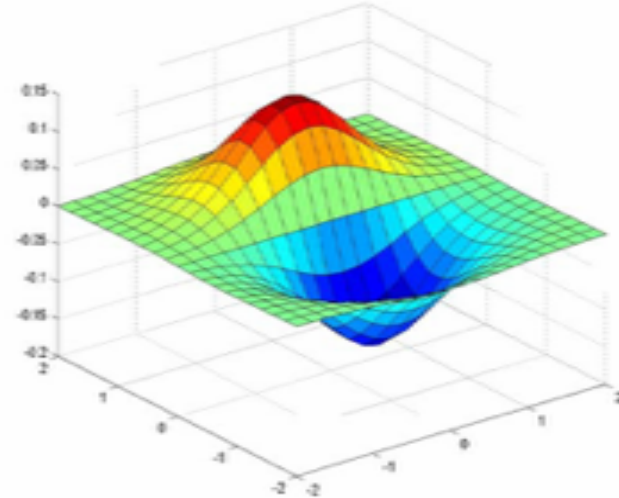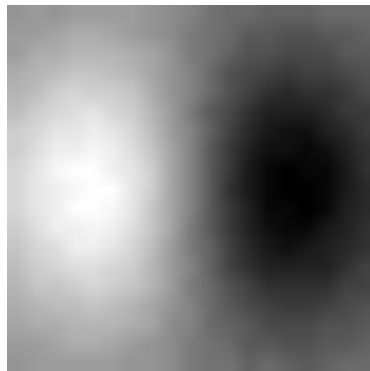
- This saves us one operation:

f

$\dfrac{d}{dx}g$

$f * \dfrac{d}{dx}g$

How can we find (local) maxima of a function?

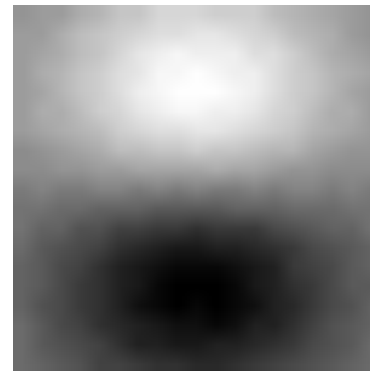Source: S. Seitz

Remember:
Derivative of Gaussian filter

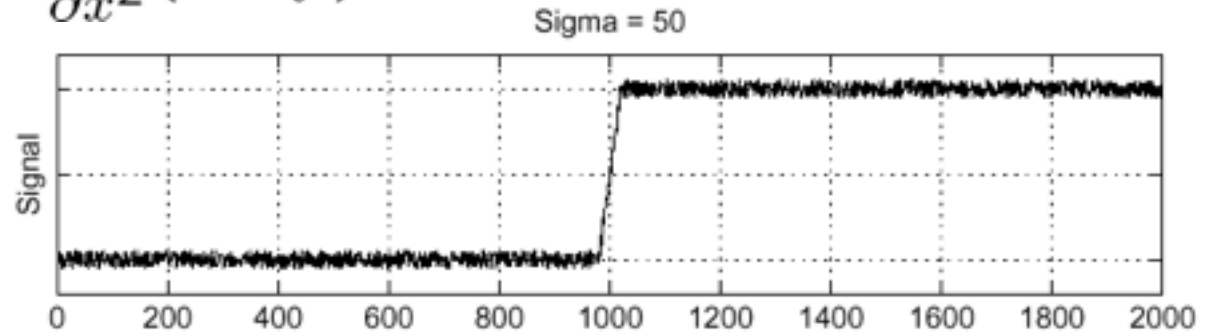*x*-direction   *y*-direction

Laplacian of Gaussian
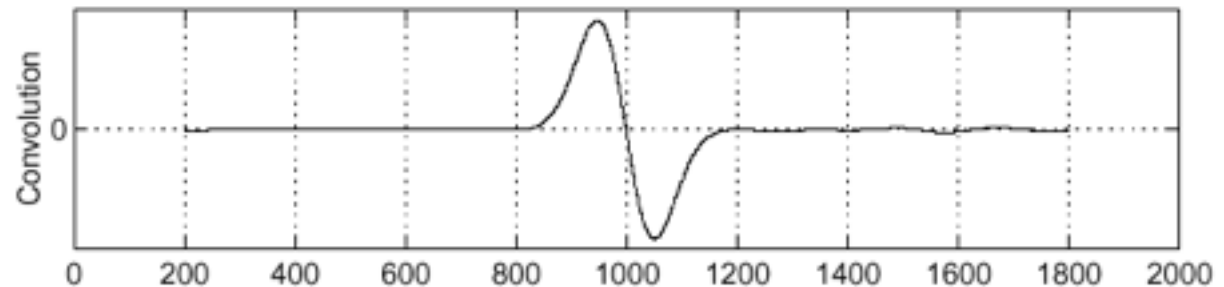
- Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

Sigma = 50

f

$\frac{\partial^2}{\partial x^2}h$
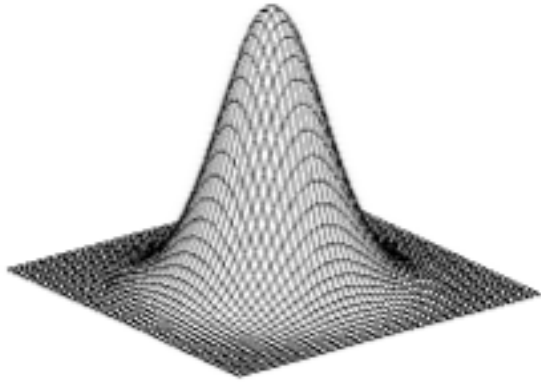
Laplacian of Gaussian operator
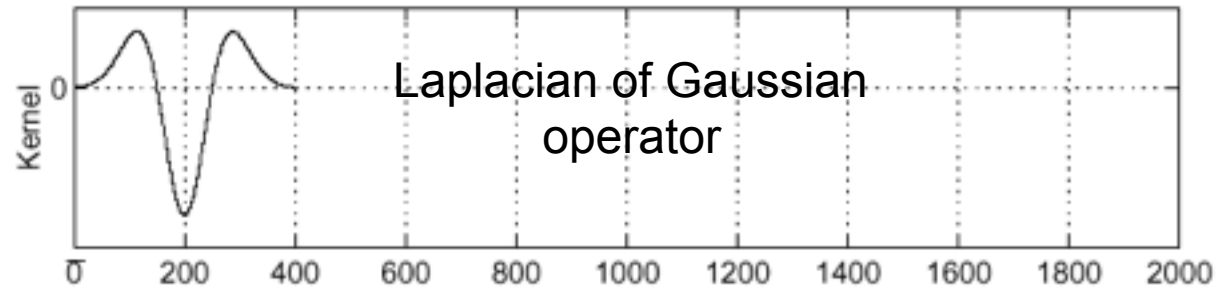
$(\frac{\partial^2}{\partial x^2}h) \star f$

Where is the edge?   Zero-crossings of bottom graph

2D edge detection filters

Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$
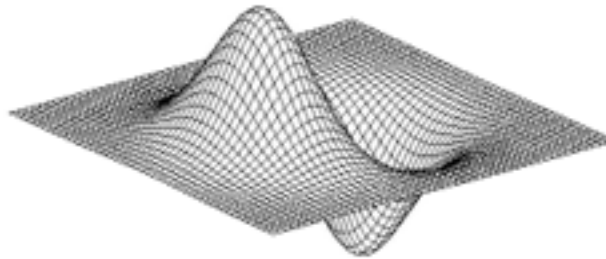
derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$
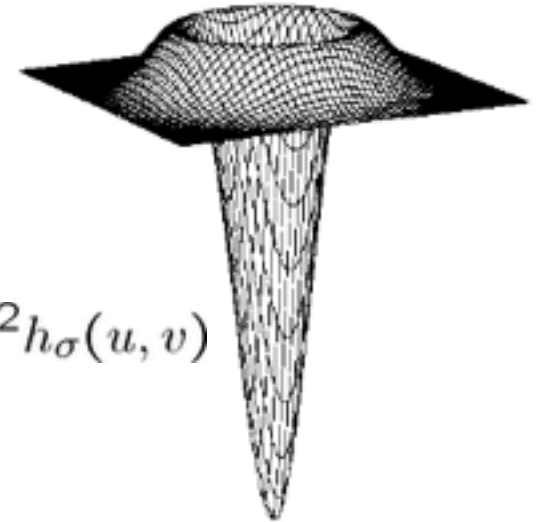
Laplacian of Gaussian

$$\nabla^2 h_\sigma(u,v)$$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Edge detection by subtraction

original

Edge detection by subtraction

smoothed (5x5 Gaussian)

Edge detection by subtraction

smoothed – original
(scaled by 4, offset +128)

Why does
this work?

Gaussian - image filter

Gaussian   delta function

Laplacian of Gaussian
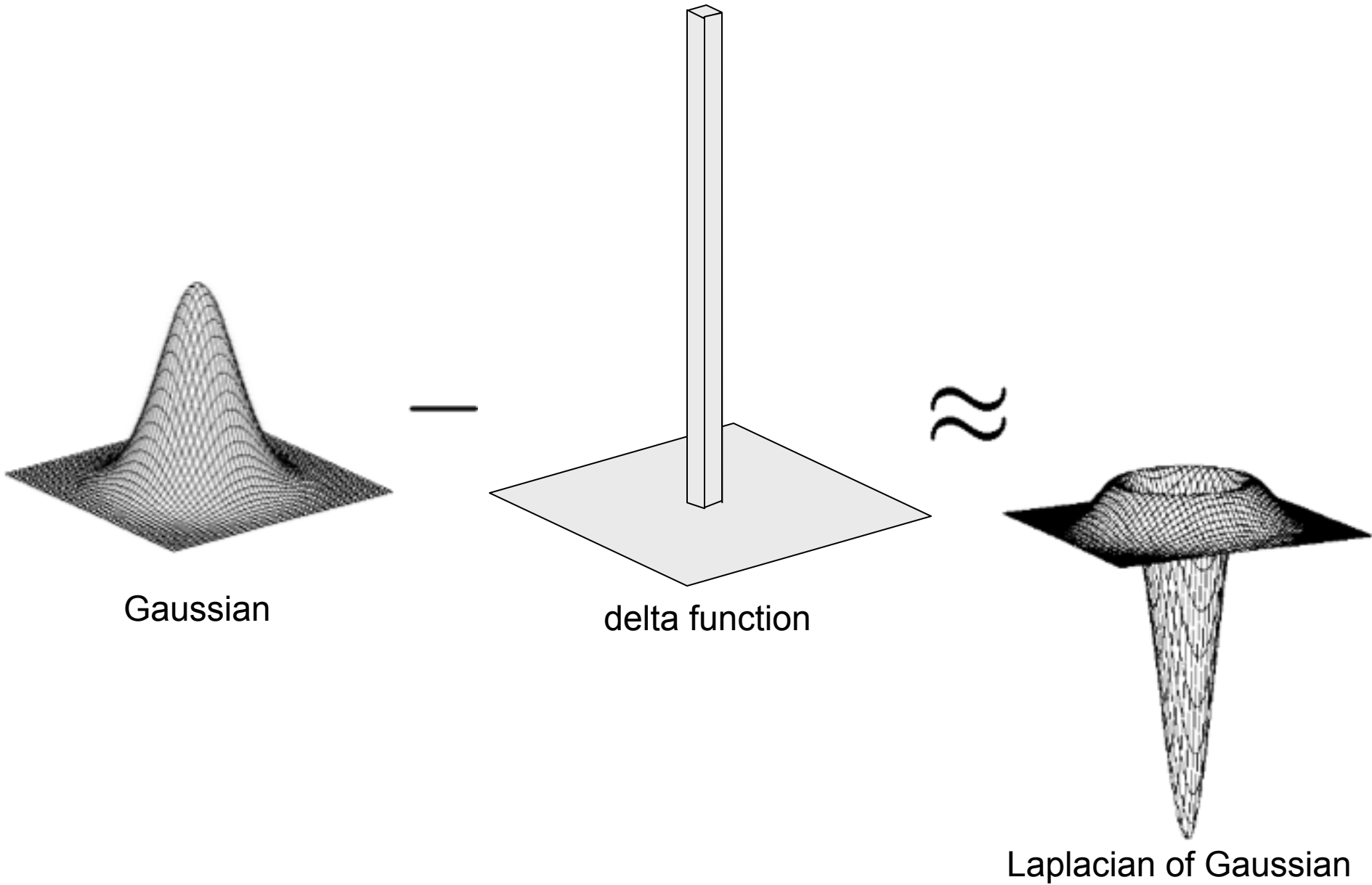
Canny edge detector

- This is probably the most widely used edge detector in computer vision

J. Canny, *A Computational Approach To Edge Detection*, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Source: L. Fei-Fei

The Canny edge detector

-          original image (Lena)

The Canny edge detector

norm of the gradient

The Canny edge detector

thresholding

Get Orientation at Each Pixel

theta = atan2(-gy, gx)
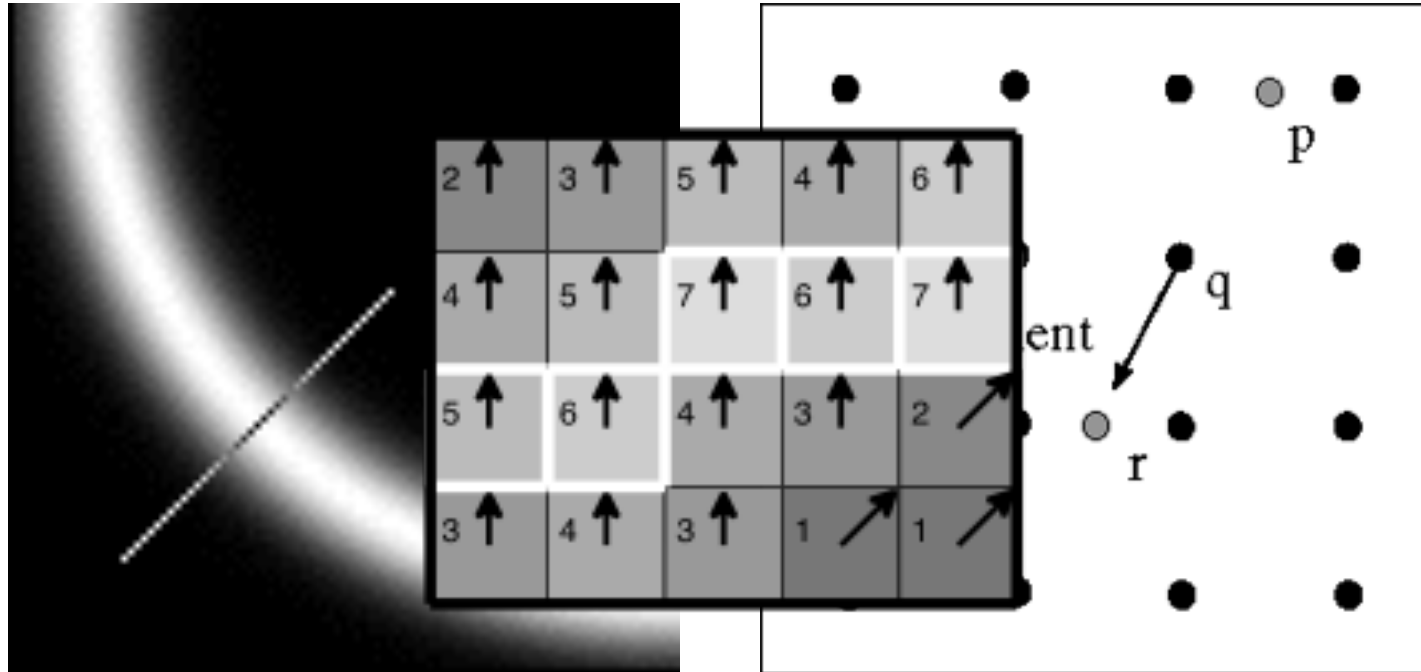
The Canny edge detector

The Canny edge detector

thinning
(non-maximum suppression)

Non-maximum suppression

- Check if pixel is local maximum along gradient direction
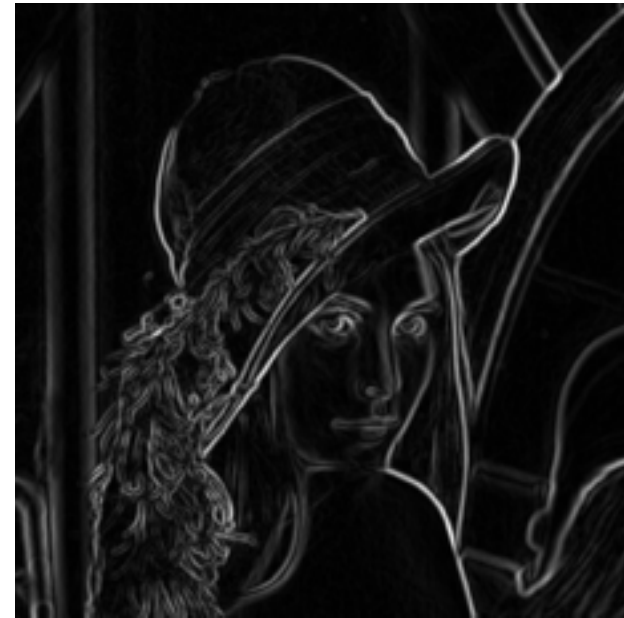
Compute Gradients (DoG)

X-Derivative of Gaussian   Y-Derivative of Gaussian   Gradient Magnitude

Canny Edges

Effect of σ (Gaussian kernel spread/size)

original   Canny with $\sigma = 1$ Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
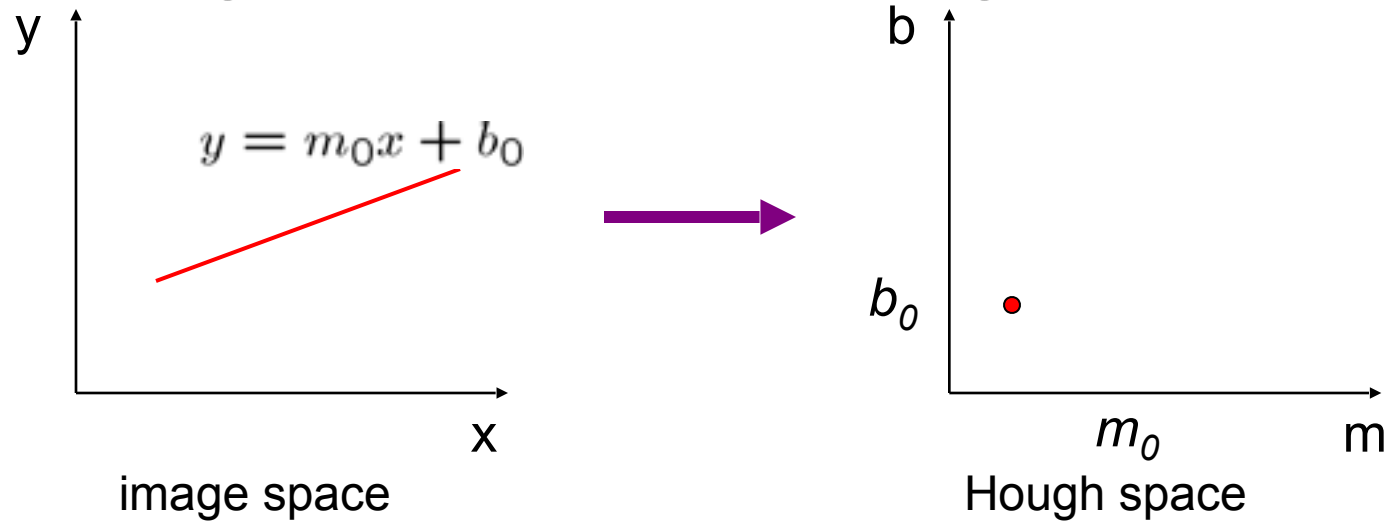- small σ detects fine features

An edge is not a line…

How can we detect *lines* ?

Finding lines in an image

- Option 1:
  - Search for the line at every possible position/orientation
  - What is the cost of this operation?

- Option 2:
  - Use a voting scheme: Hough transform

Finding lines in an image

y

$$y = m_0 x + b_0$$

b_0

x

image space

b

m_0 m

Hough space

- Connection between image (x,y) and Hough (m,b) spaces
  - A line in the image corresponds to a point in Hough space
  - To go from image space to Hough space:
    - given a set of points (x,y), find all (m,b) such that y = mx + b

Finding lines in an image
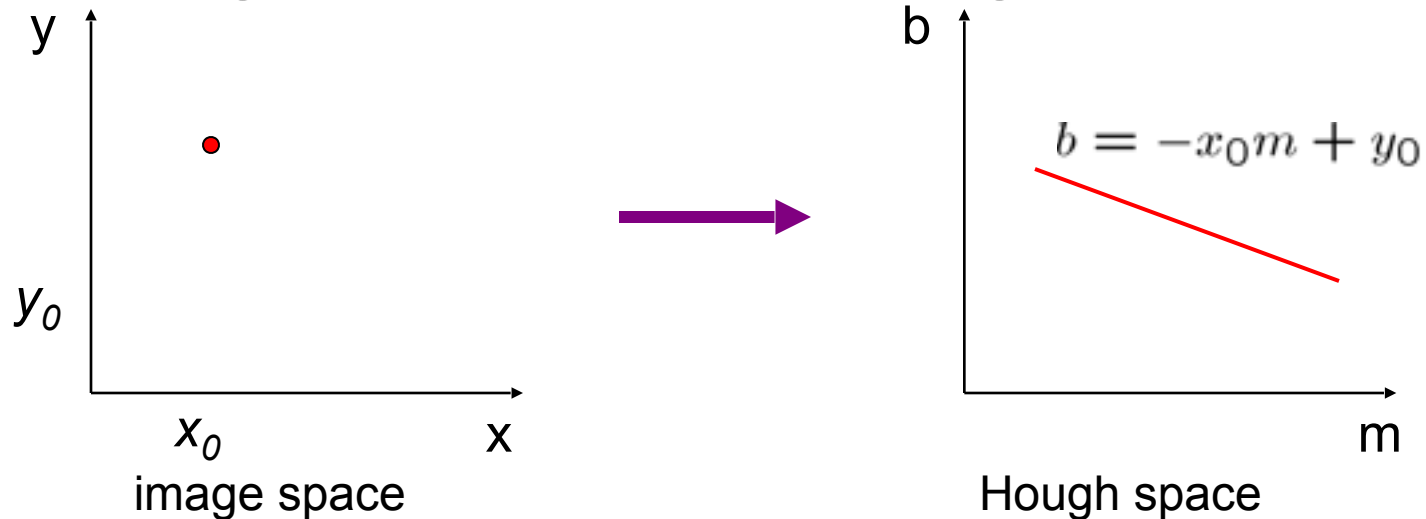
y

y_0

x_0 x

image space

b

$$b = -x_0 m + y_0$$

m

Hough space

- Connection between image (x,y) and Hough (m,b) spaces
  - A line in the image corresponds to a point in Hough space
  - To go from image space to Hough space:
    - given a set of points (x,y), find all (m,b) such that y = mx + b
  - What does a point (x_0, y_0) in the image space map to?

    - A: the solutions of b = $-x_0$m + y_0
    - this is a line in Hough space

Hough transform algorithm

- Typically use a different
  param $d = x cos\theta + y sin\theta$ . . . .

  - d is the perpendicular distance from the line to the origin
  - θ is the angle

Hough transform algorithm

- Basic Hough transform algorithm
  1. Initialize H[d, θ]=0
  2. for each edge point I[x,y] in the image
         for θ = 0 to 180
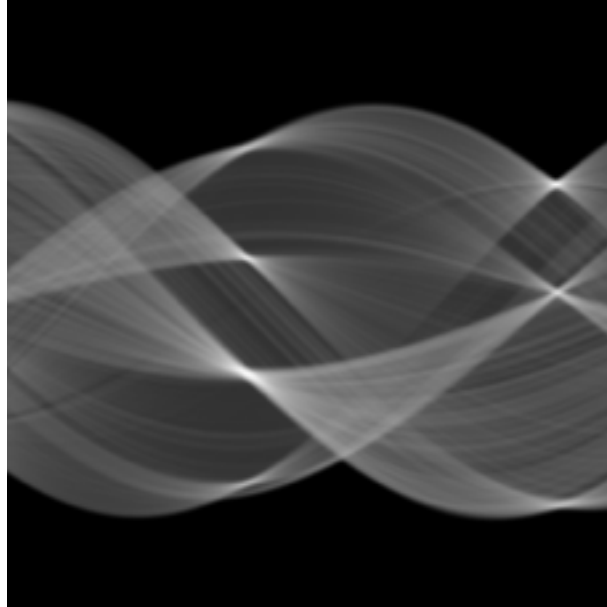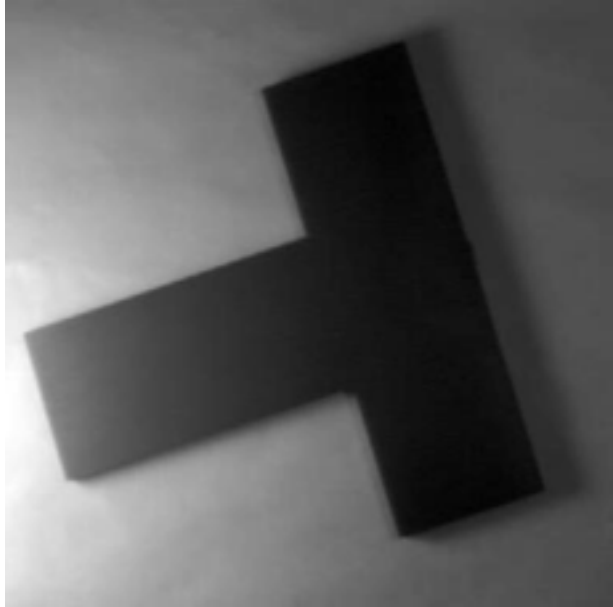 $$d = xcos\theta + ysin\theta$$
         H[d, θ] += 1
  3. Find the value(s) of (d, θ) where H[d, θ] is maximum

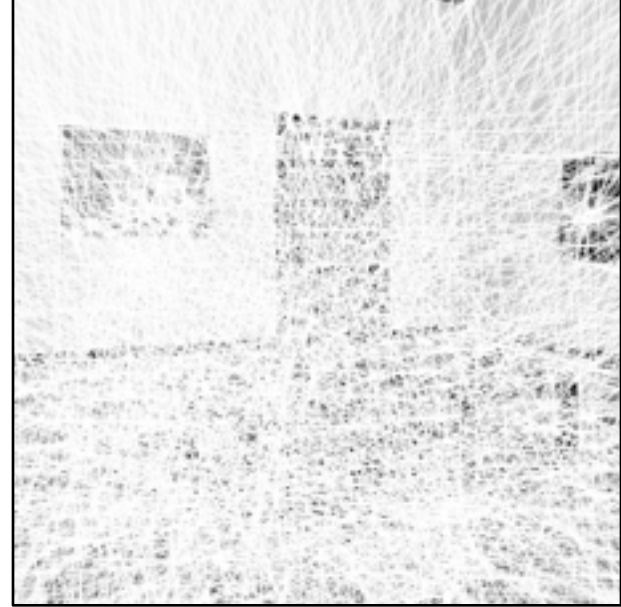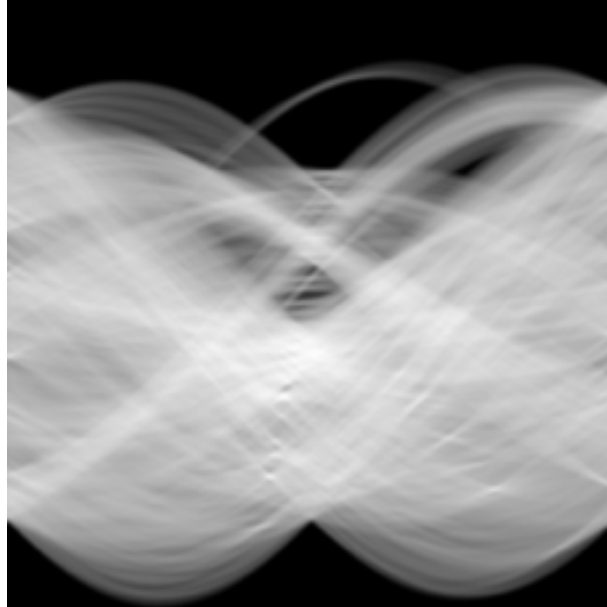 $$d = xcos\theta + ysin\theta$$
  4. The detected line in the image is given by
- What's the running time (measured in # votes)?

Hough transform algorithm

Hough transform algorithm

Extensions

- Extension 1: Use the image gradient
  1. same
  2. for each edge point I[x,y] in the image

     compute unique (d, θ) based on image gradient at (x,y)

     H[d, θ] += 1
  3. same
  4. same
- What's the running time measured in votes?

- Extension 2
  – give more votes for stronger edges
- Extension 3
  – change the sampling of (d, θ) to give more/less resolution
- Extension 4
  – The same procedure can be used with circles, squares, or any other shape, How?