Edge Detection

CSE 576
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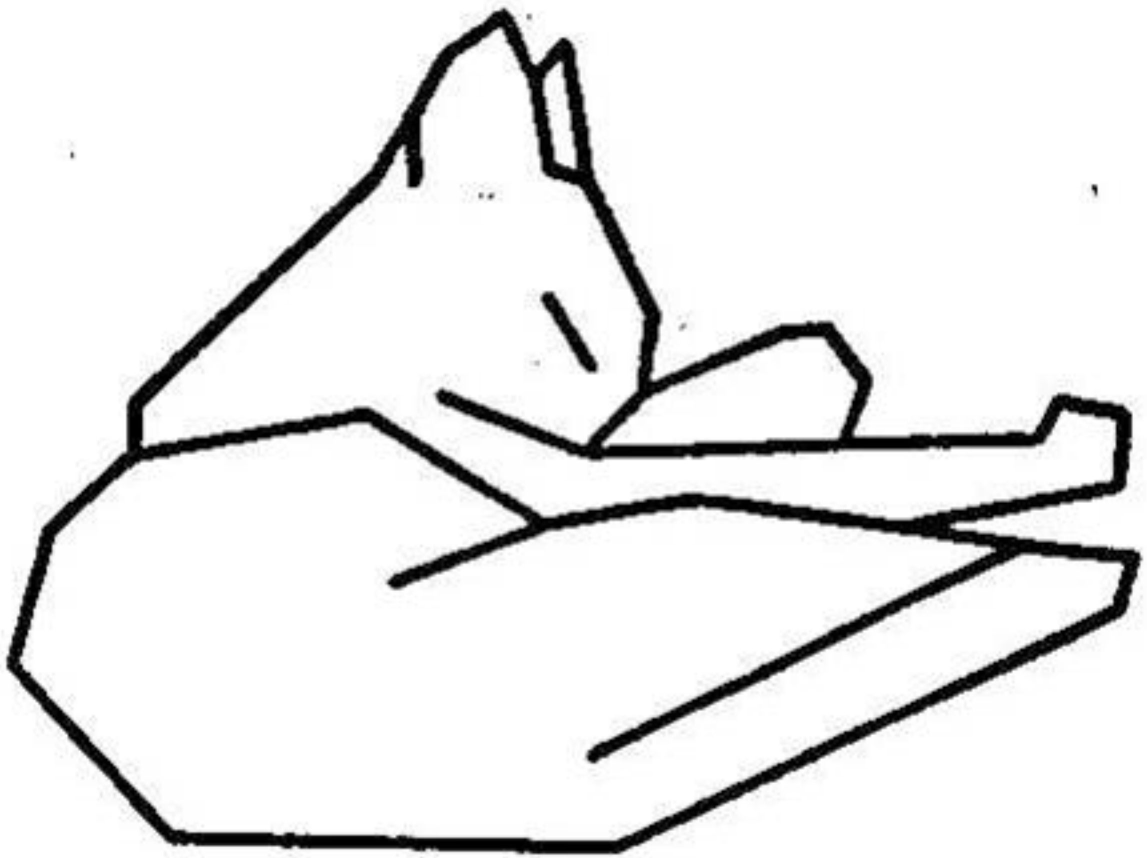
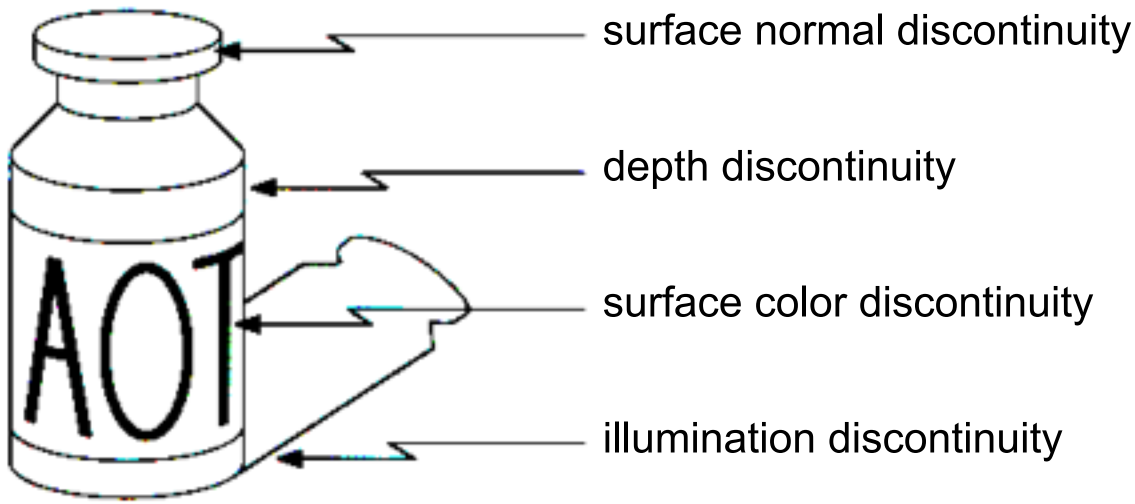
Many slides from Steve Seitz and Larry Zitnick

Edge



Attneave's Cat (1954)

Origin of edges



Edges are caused by a variety of factors

Characterizing edges

- An edge is a place of rapid change in the image intensity function

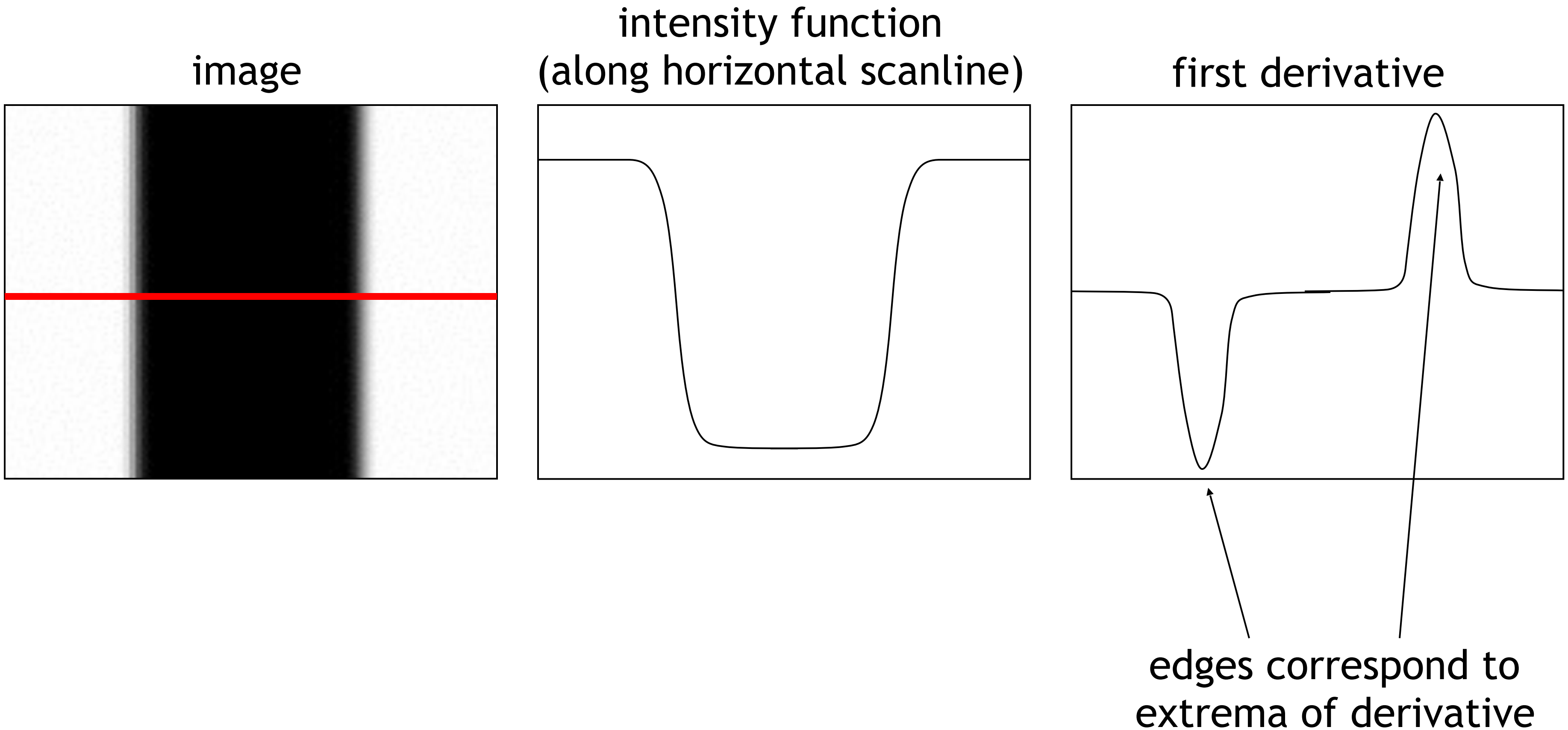
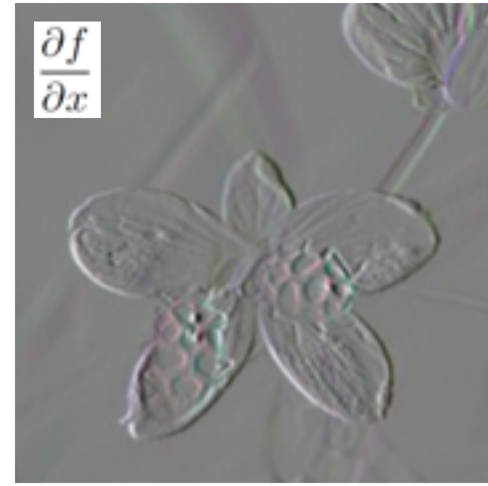


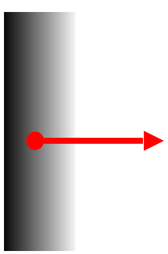
Image gradient



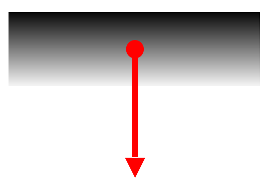
- The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

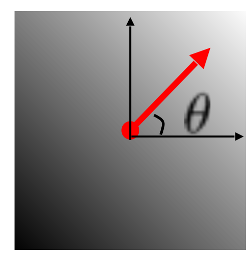
- The gradient points in the direction of most rapid change in intensity



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



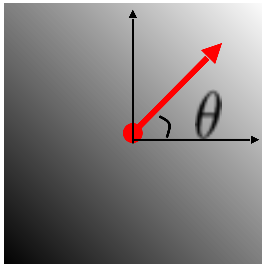
$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The discrete gradient

- How can we differentiate a *digital* image F[x,y]?
  - Option 1: reconstruct a continuous image, then take gradient
  - Option 2: take discrete derivative ("finite difference")

$$\frac{\partial f}{\partial x}[x, y] \approx F[x+1, y] - F[x, y]$$

Image gradient



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\frac{\partial f}{\partial x} = f(x+1, y) - f(x, y)$$

How would you implement this as a filter?

The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

How does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Sobel operator

In practice, it is common to use:

$$g_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad g_y = \begin{array}{|c|c|c|}\hline -1 & -2 & -1 \\ \hline 0 & 0 & 0 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

Magnitude:

$$g = \sqrt{g_x^2 + g_y^2}$$

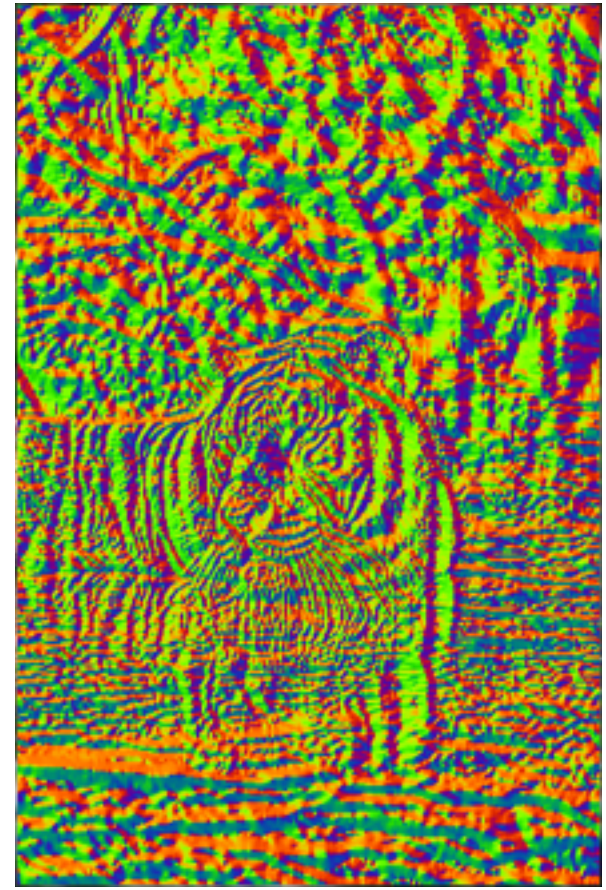
Orientation:

$$\Theta = \tan^{-1}\left(\frac{g_y}{g_x}\right)$$

Sobel operator

Original

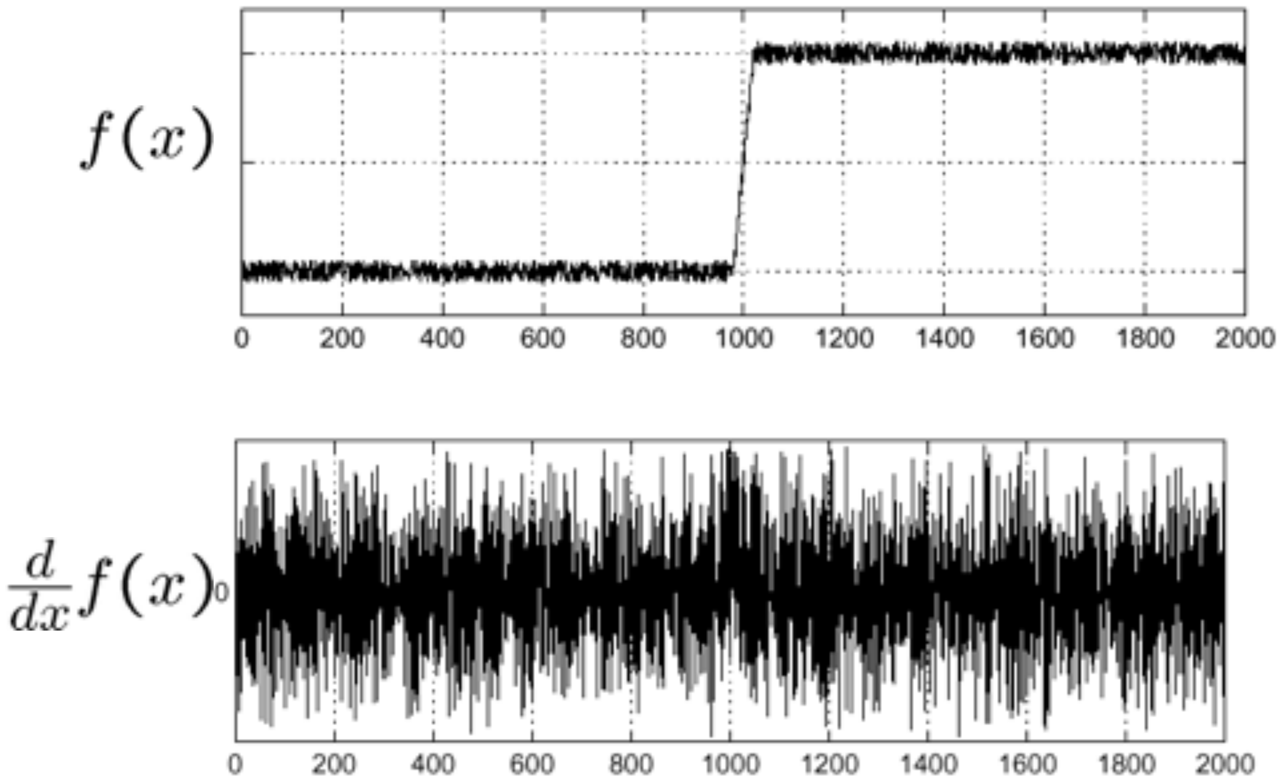
Magnitude



Orientation

Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal



Where is the edge?

Effects of noise

- Difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response
- What can we do about it?

Source: D. Forsyth

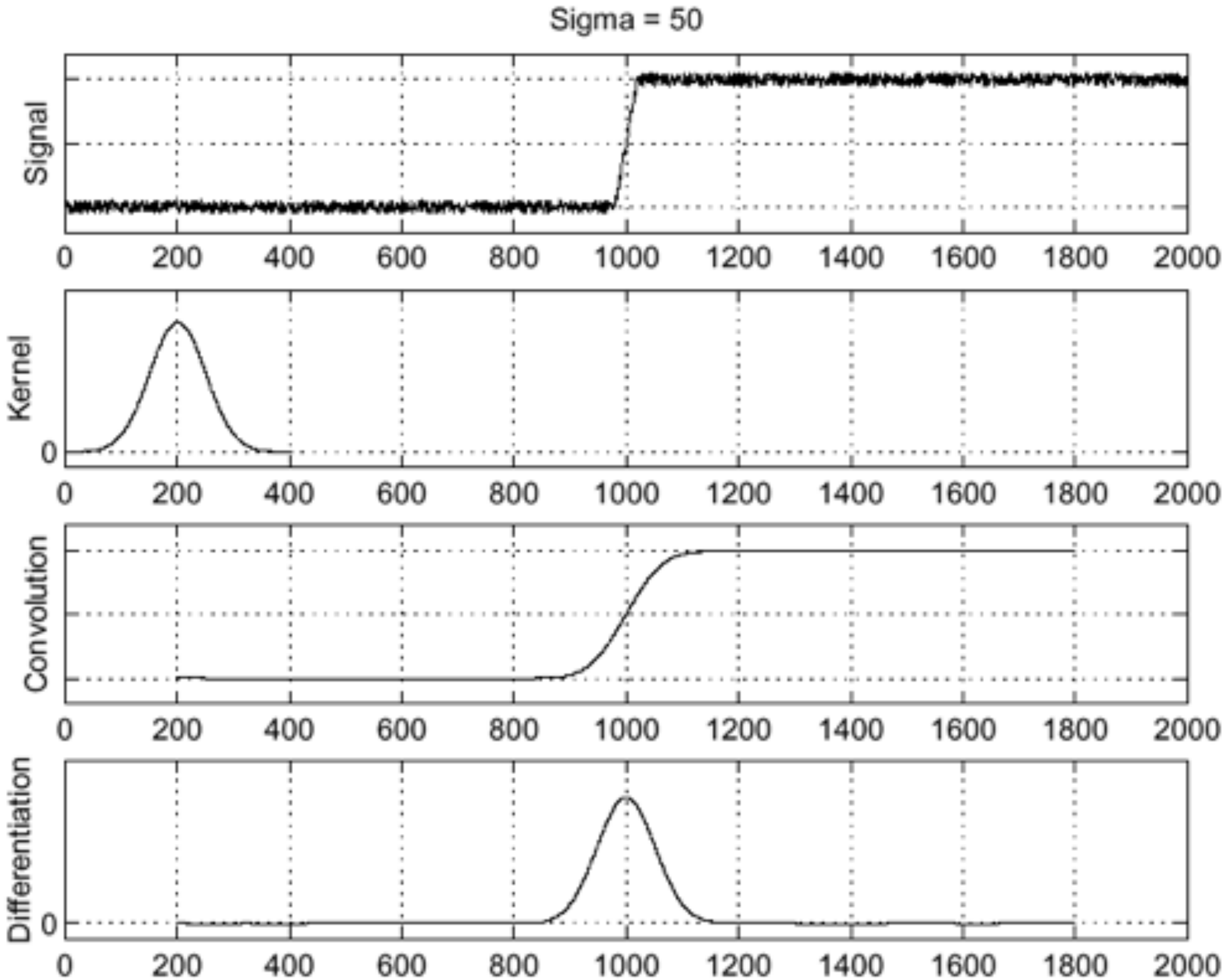
Solution: smooth first

f

h

$h \star f$

$\frac{\partial}{\partial x}(h \star f)$



Where is the edge?   Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

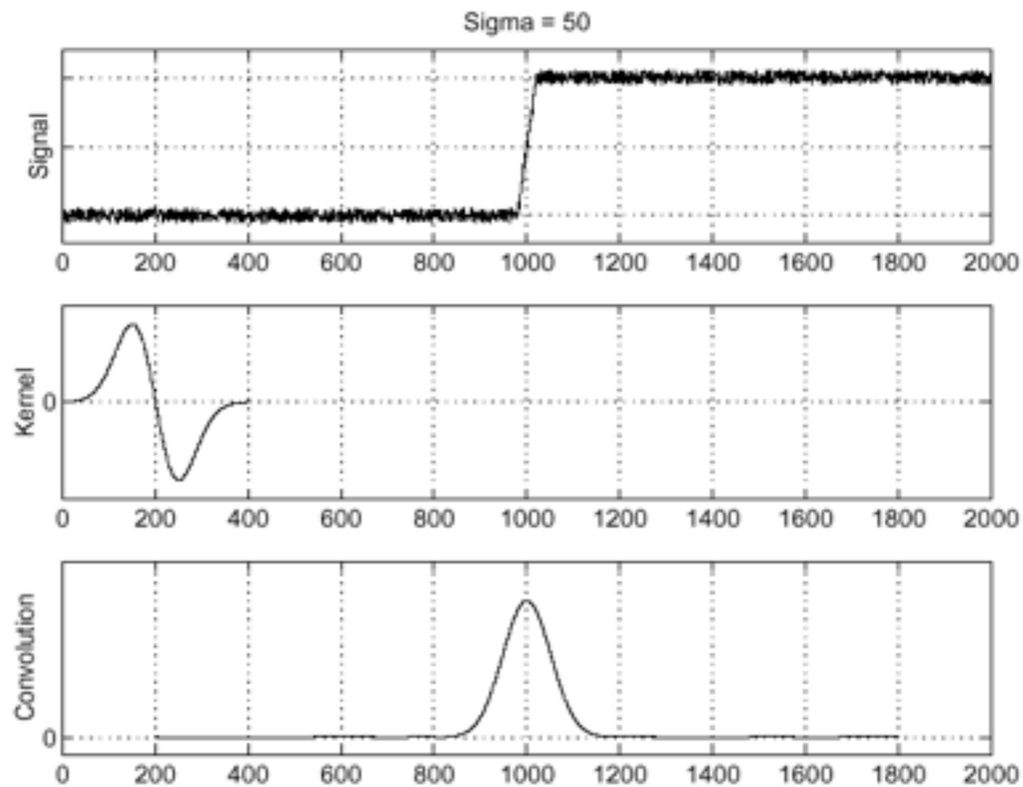
Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$
- This saves us one operation:

f

$\frac{d}{dx} g$

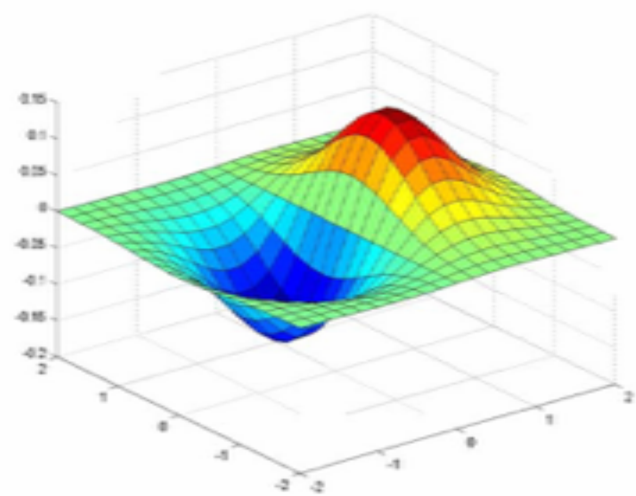
$f * \frac{d}{dx} g$

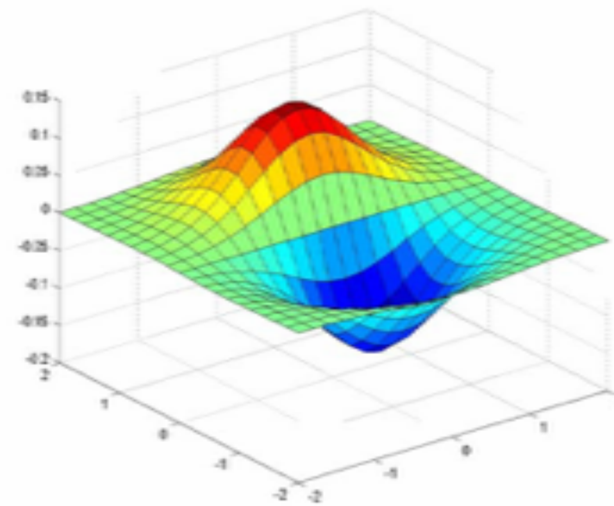


How can we find (local) maxima of a function?

Source: S. Seitz

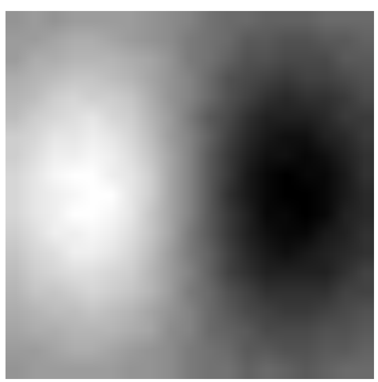
Remember:
Derivative of Gaussian filter

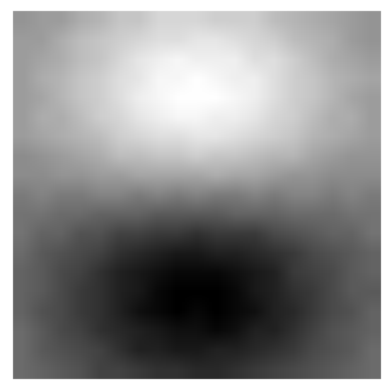




x-direction

y-direction





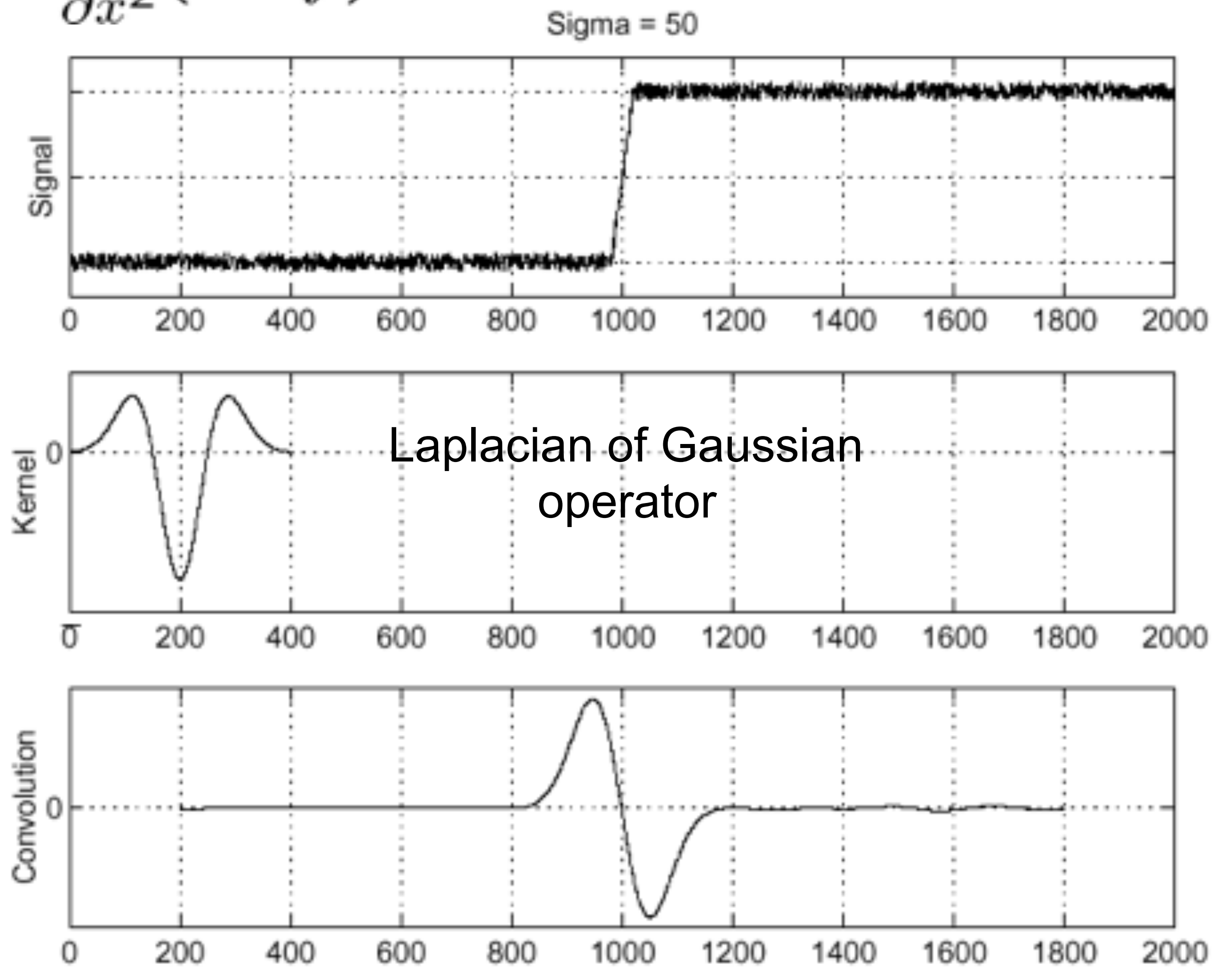
Laplacian of Gaussian

- Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

f

$\frac{\partial^2}{\partial x^2}h$

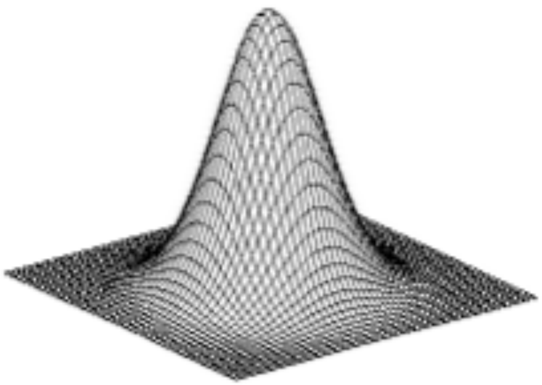
$(\frac{\partial^2}{\partial x^2}h) \star f$



Where is the edge?

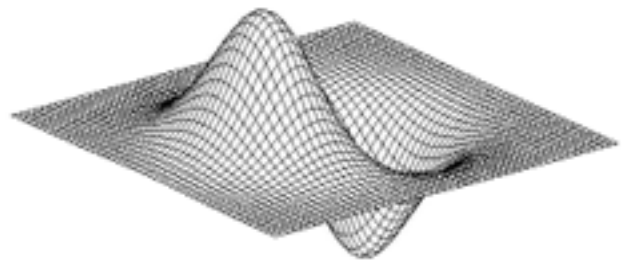
Zero-crossings of bottom graph

2D edge detection filters



Gaussian

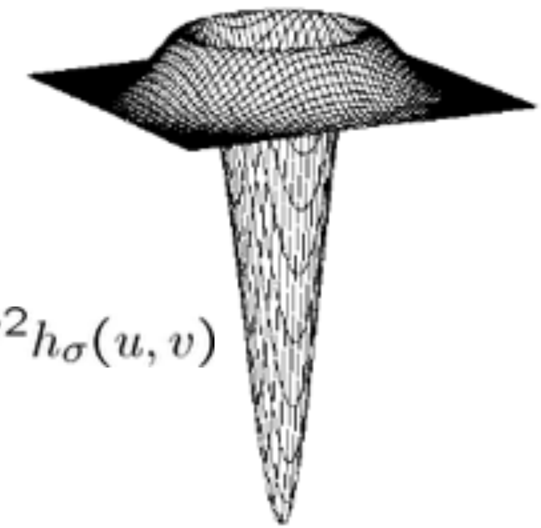
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Edge detection by subtraction

original

Edge detection by subtraction

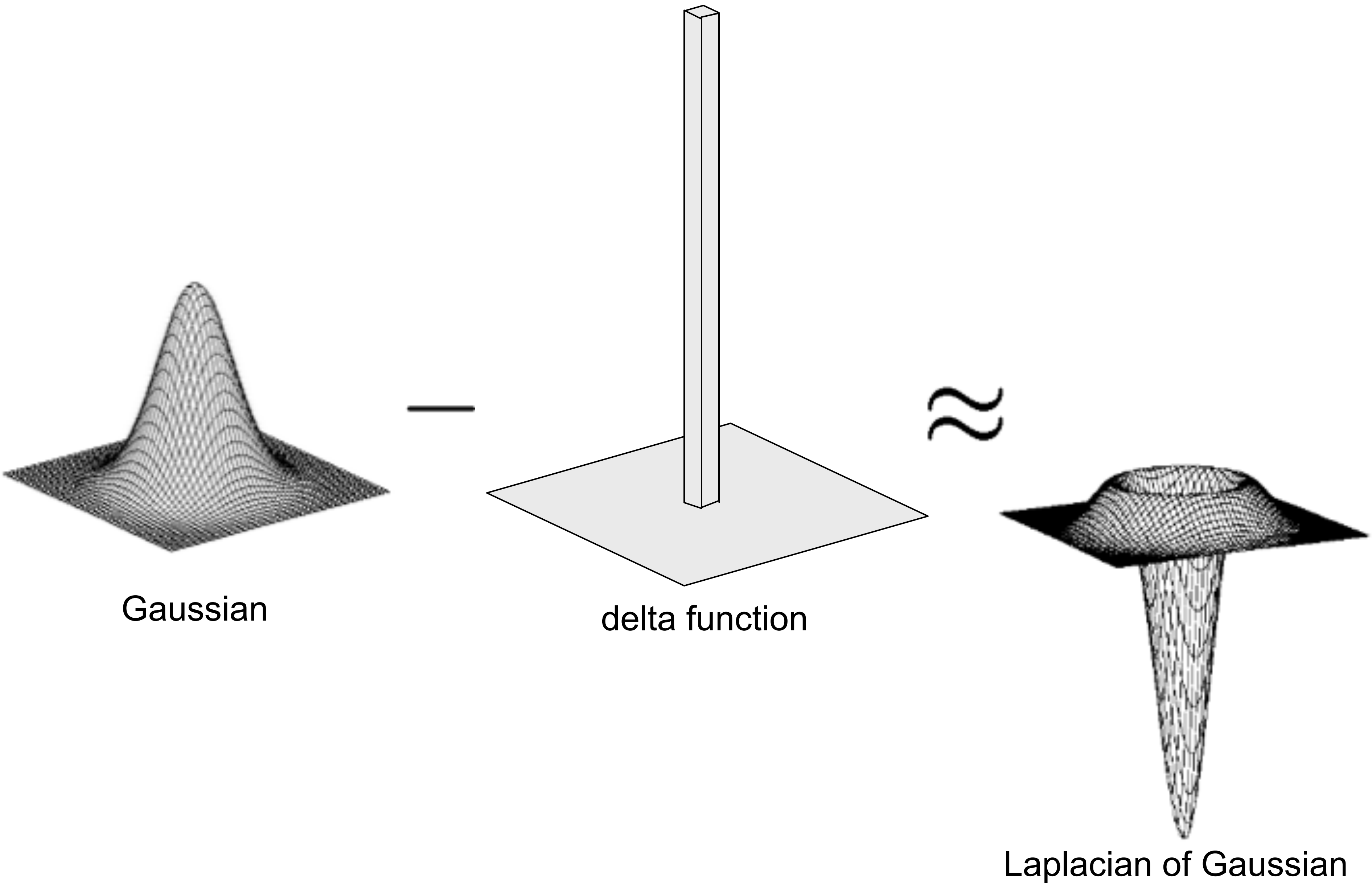
smoothed (5x5 Gaussian)

Edge detection by subtraction

smoothed – original
(scaled by 4, offset +128)

Why does this work?

Gaussian - image filter



Canny edge detector

- This is probably the most widely used edge detector in computer vision

J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Source: L. Fei-Fei

The Canny edge detector

- original image (Lena)

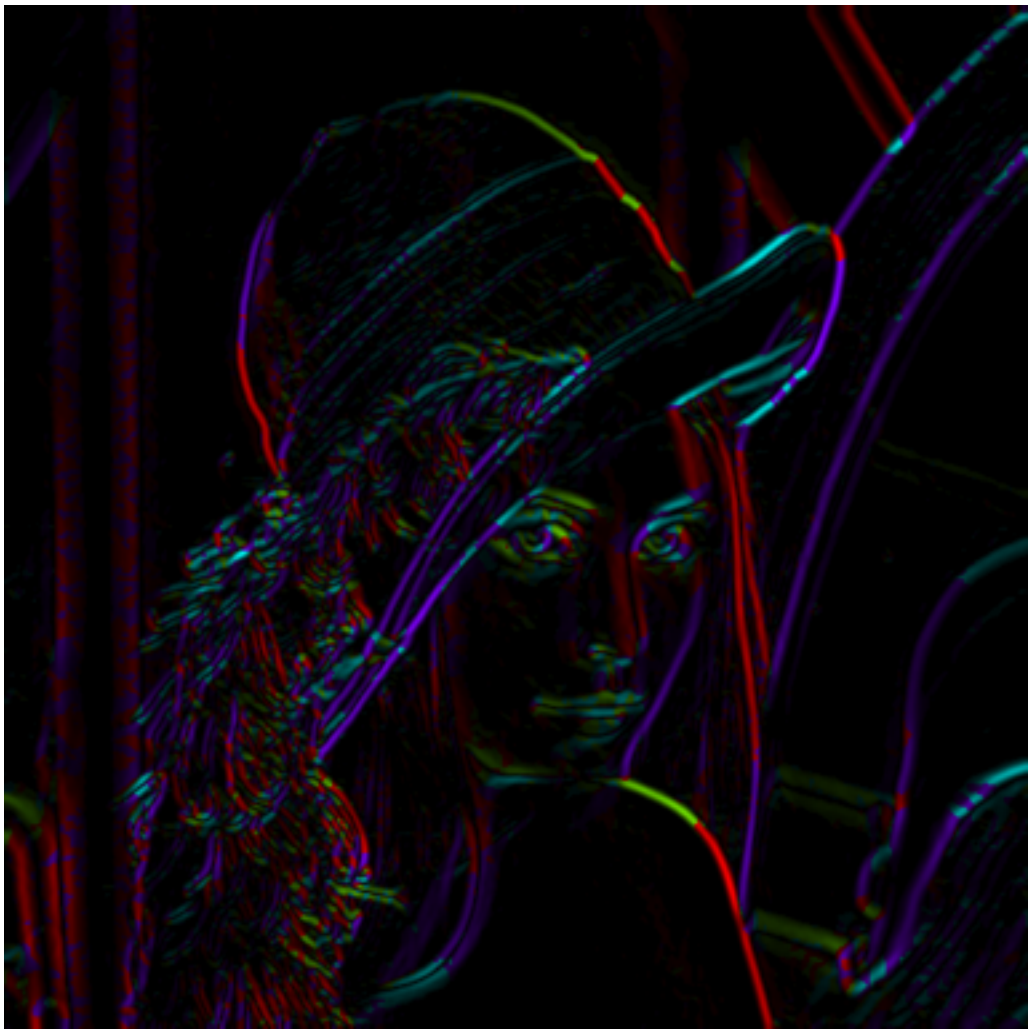
The Canny edge detector

norm of the gradient

The Canny edge detector

thresholding

Get Orientation at Each Pixel



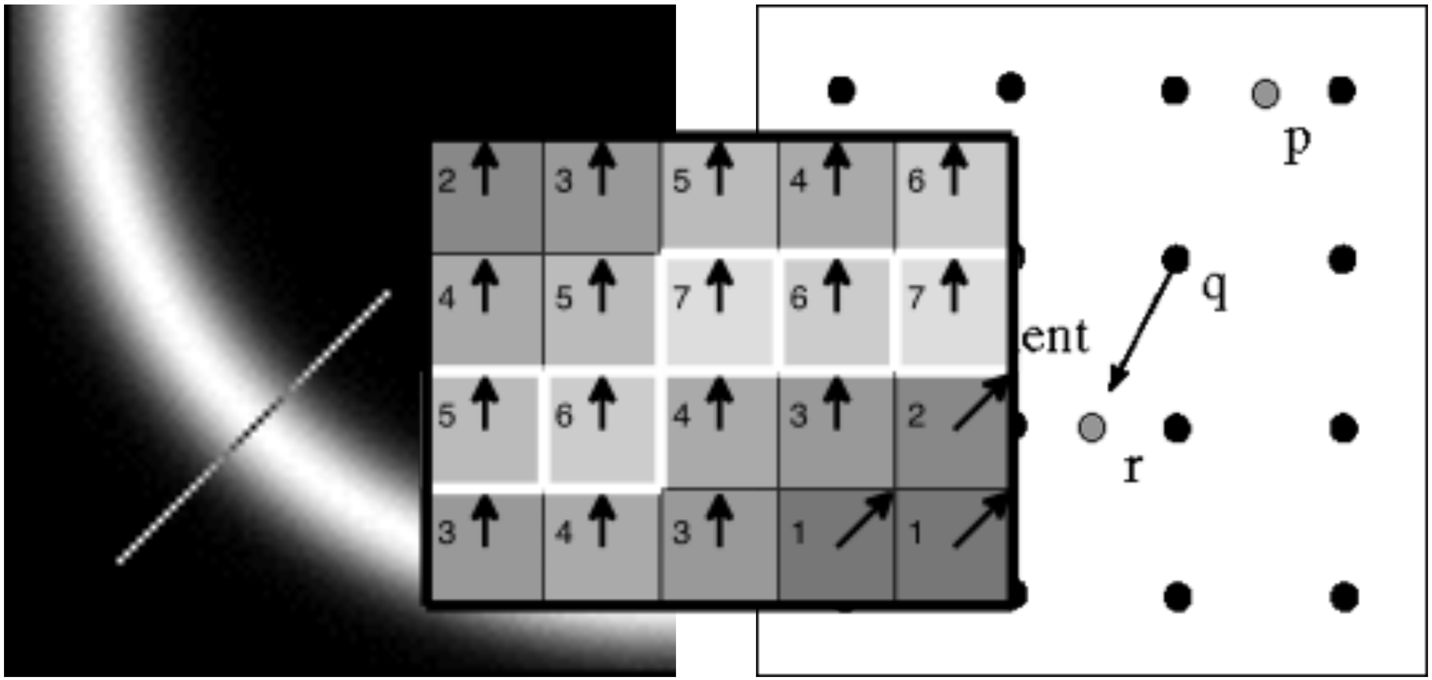
theta = atan2(-gy, gx)

The Canny edge detector

The Canny edge detector

thinning
(non-maximum suppression)

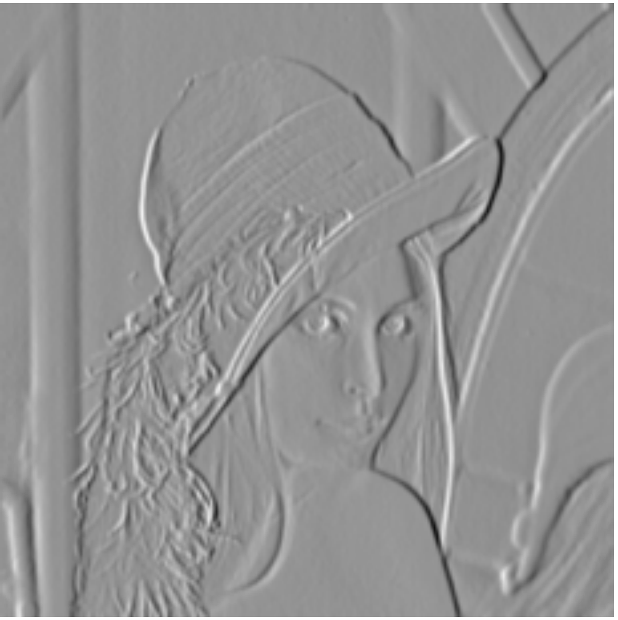
Non-maximum suppression



- Check if pixel is local maximum along gradient direction

Picture from Prem K Kalra

Compute Gradients (DoG)



X-Derivative of Gaussian

Y-Derivative of Gaussian

Gradient Magnitude

Canny Edges

Effect of σ (Gaussian kernel spread/size)

original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

An edge is not a line...

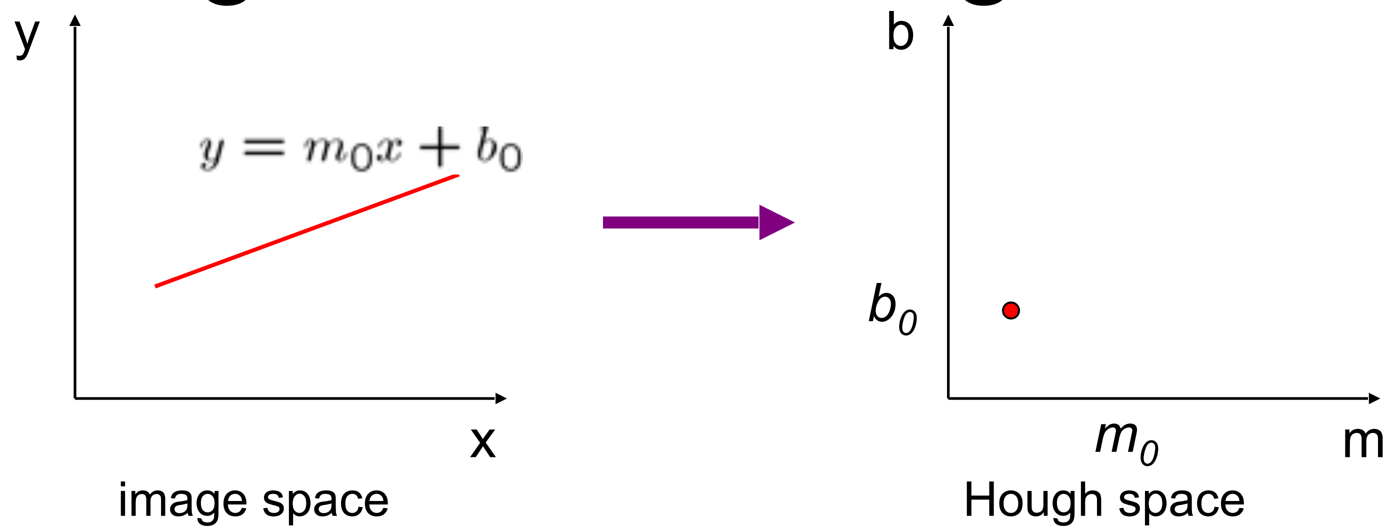
How can we detect ***lines*** ?

Finding lines in an image

- Option 1:
  - Search for the line at every possible position/ orientation
  - What is the cost of this operation?

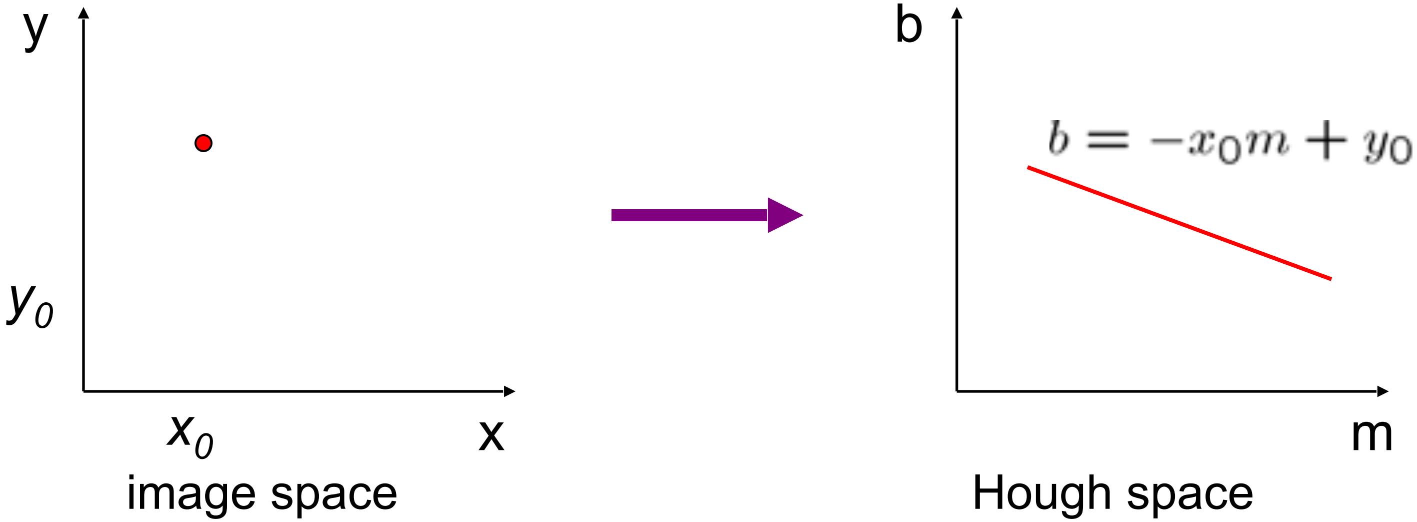
- Option 2:
  - Use a voting scheme: Hough transform

Finding lines in an image



- Connection between image (x,y) and Hough (m,b) spaces
  - A line in the image corresponds to a point in Hough space
  - To go from image space to Hough space:
    - given a set of points (x,y), find all (m,b) such that y = mx + b

Finding lines in an image



- Connection between image (x,y) and Hough (m,b) spaces
  - A line in the image corresponds to a point in Hough space
  - To go from image space to Hough space:
    - given a set of points (x,y), find all (m,b) such that y = mx + b
  - What does a point (x_0, y_0) in the image space map to?

  - A: the solutions of $b = -x_0 m + y_0$
  - this is a line in Hough space

Hough transform algorithm

- Typically use a different parameterization

$$d = x cos\theta + y sin\theta$$

  - d is the perpendicular distance from the line to the origin
  - θ is the angle

Hough transform algorithm

- Basic Hough transform algorithm
  1. Initialize H[d, θ]=0
  2. for each edge point I[x,y] in the image

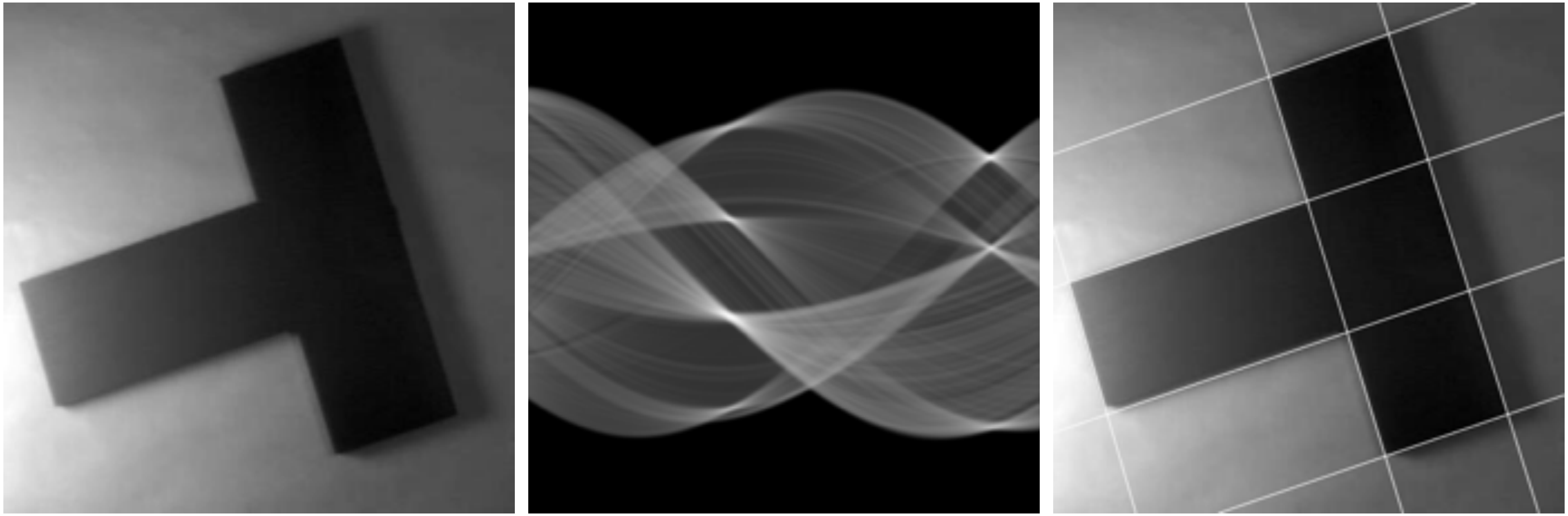
     for θ = 0 to 180

     $$d = x cos\theta + y sin\theta$$

     H[d, θ] += 1
  3. Find the value(s) of (d, θ) where H[d, θ] is maximum
  4. The detected line in the image is given by

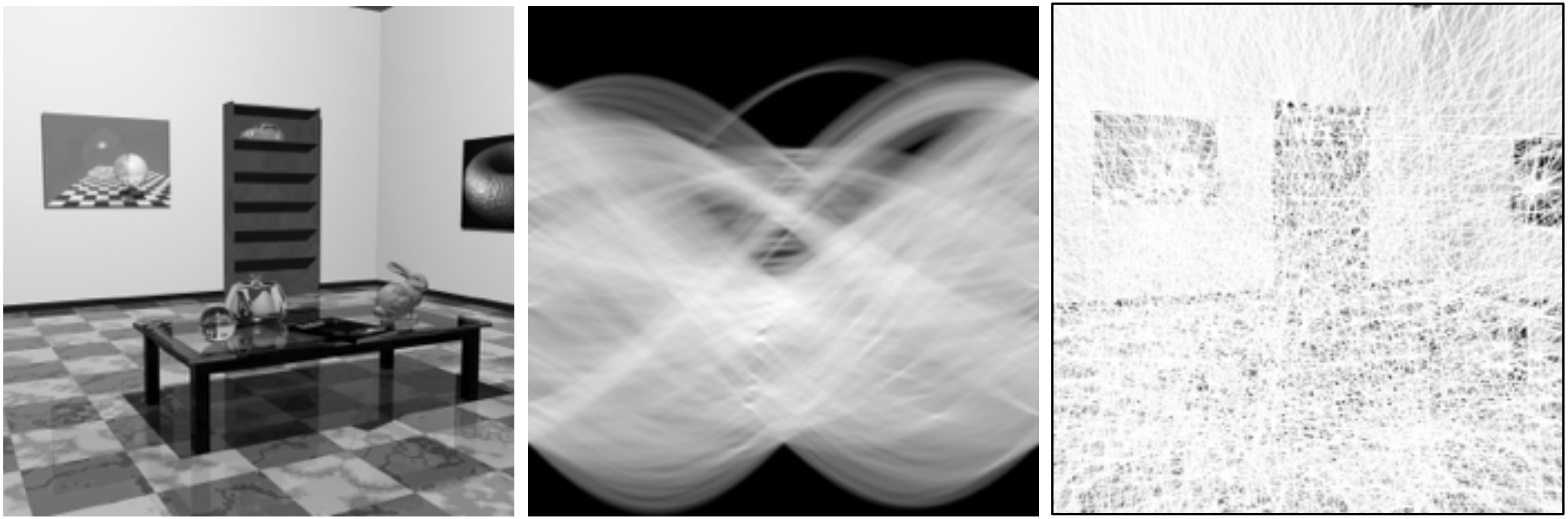
     $$d = x cos\theta + y sin\theta$$
- What's the running time (measured in # votes)?

Hough transform algorithm



http://www.cs.utah.edu/~vpegorar/courses/cs7966/

Hough transform algorithm



http://www.cs.utah.edu/~vpegorar/courses/cs7966/

Extensions

- Extension 1: Use the image gradient
  1. same
  2. for each edge point I[x,y] in the image

     compute unique (d, θ) based on image gradient at (x,y)

     $H[d, \theta]$ += 1
  3. same
  4. same
- What's the running time measured in votes?

- Extension 2
  – give more votes for stronger edges
- Extension 3
  – change the sampling of (d, θ) to give more/less resolution
- Extension 4
  – The same procedure can be used with circles, squares, or any other shape, How?