Images and Filters

CSE 576
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Many slides from Steve Seitz and Larry Zitnick

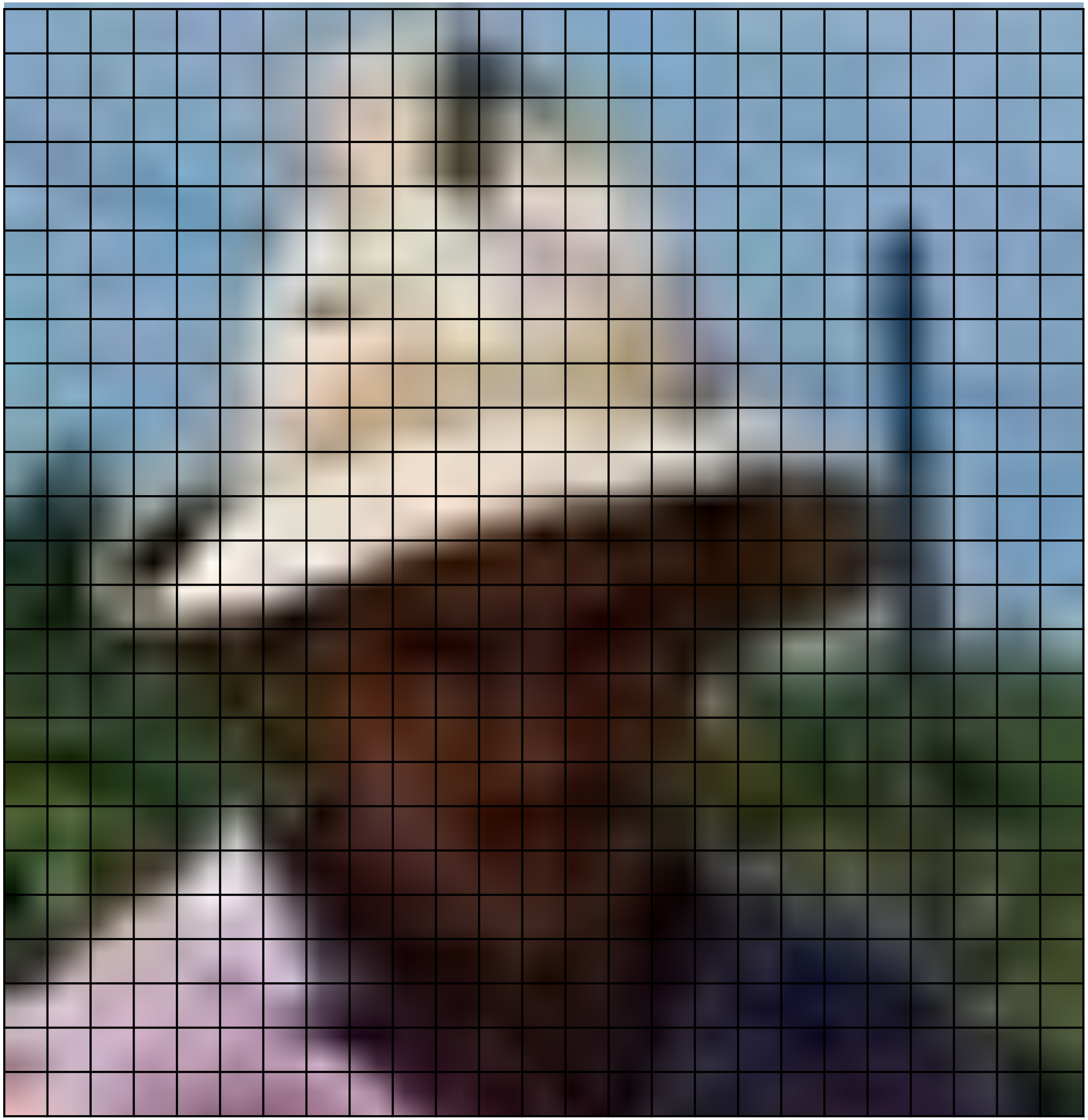
Administrative Stuff

- See the setup instructions on the course web page
- Setup your environment
- Project
  - Topic
  - Team up (discussion board)
  - The project proposal is due on 4/6
    - Use the dropbox link on the course webpage to upload
- HW1
  - Due on 4/8
  - Use the dropbox link on the course webpage to upload

What is an image?

$$P = f(x, y)$$

$$f : R^2 \Rightarrow R$$



$$P = f(x, y)$$

$$f : R^2 \Rightarrow R$$

Image Operations
(functions of functions)

F() = 

Image Operations

(functions of functions)

F() = 

Image Operations

(functions of functions)

F() =

0.1
0
0.8
0.9
0.9
0.9
0.2
0.4
0.3
0.6
0
0
0.1
0.5
0.9
0.9
0.2
0.4
0.3
0.6
0
0
0.1
0.9
0.9
0.2
0.4
0.3
0.6
0
0

Image Operations
(functions of functions)

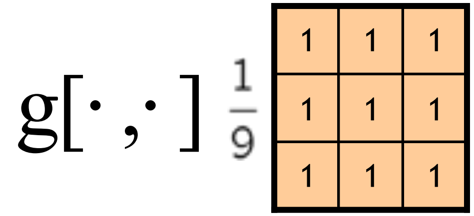
F( , ) = 0.23

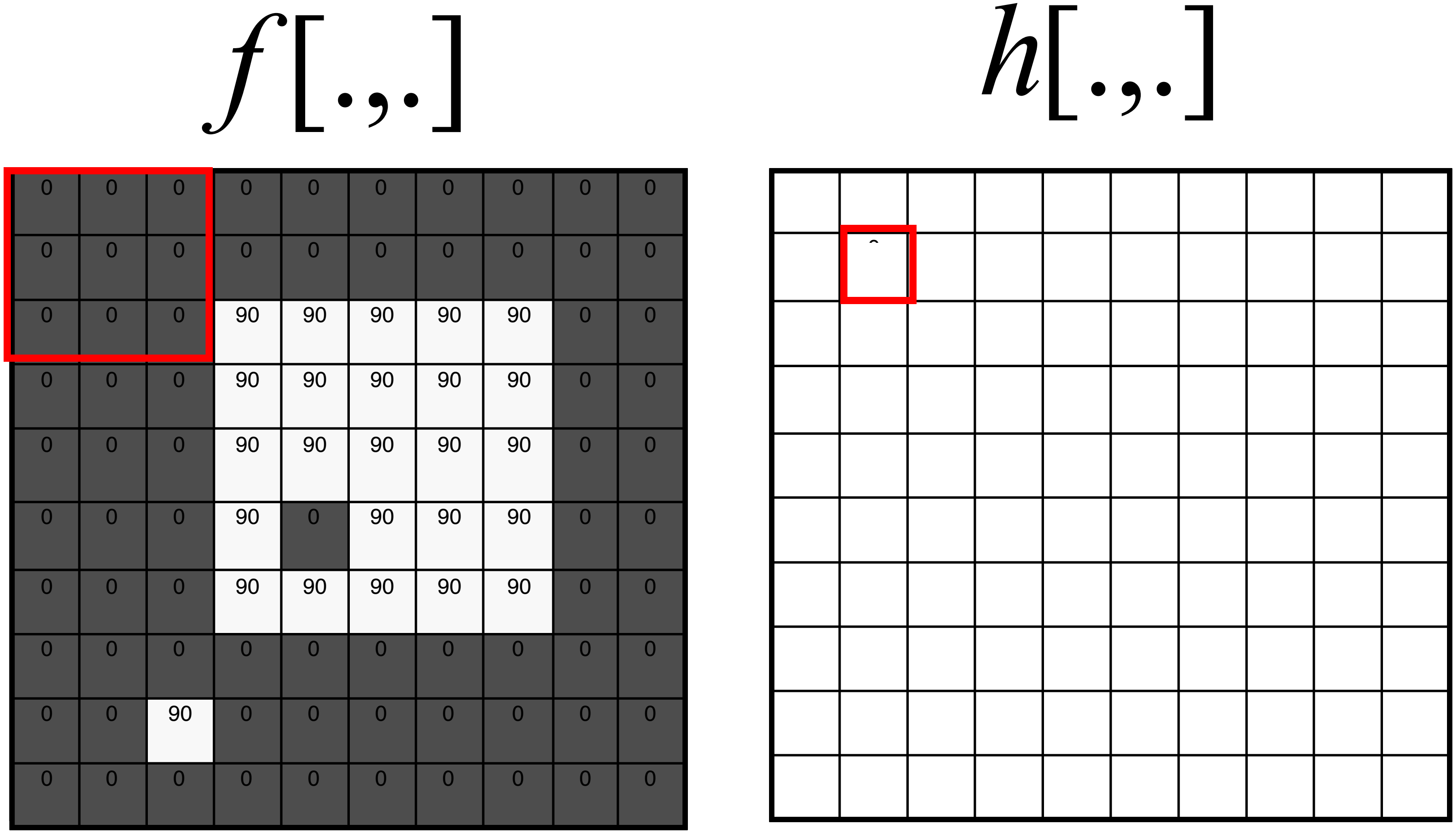
Local image functions

$F($  $) =$ 

How can I get rid of the noise in this image?

Image filtering

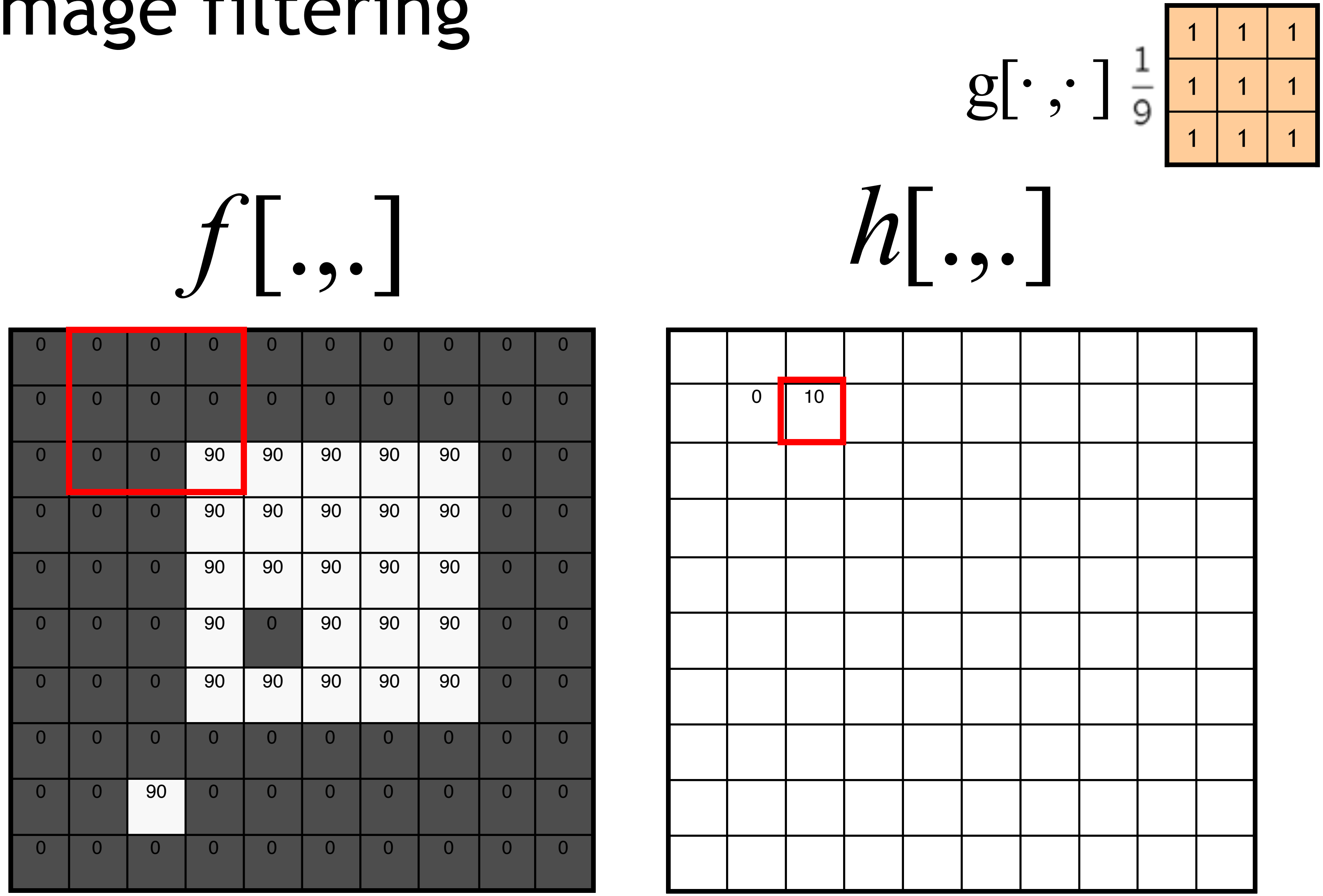




$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Image filtering



$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20						

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30					

$$h[m,n]=\sum_{k,l} g[k,l]\,f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$$g[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
				?					

$$h[m,n]=\sum_{k,l} g[k,l]\,f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
						?			
				50					

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$$g[\cdot,\cdot] \quad \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$$f[.,.]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[.,.]$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n]=\sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood

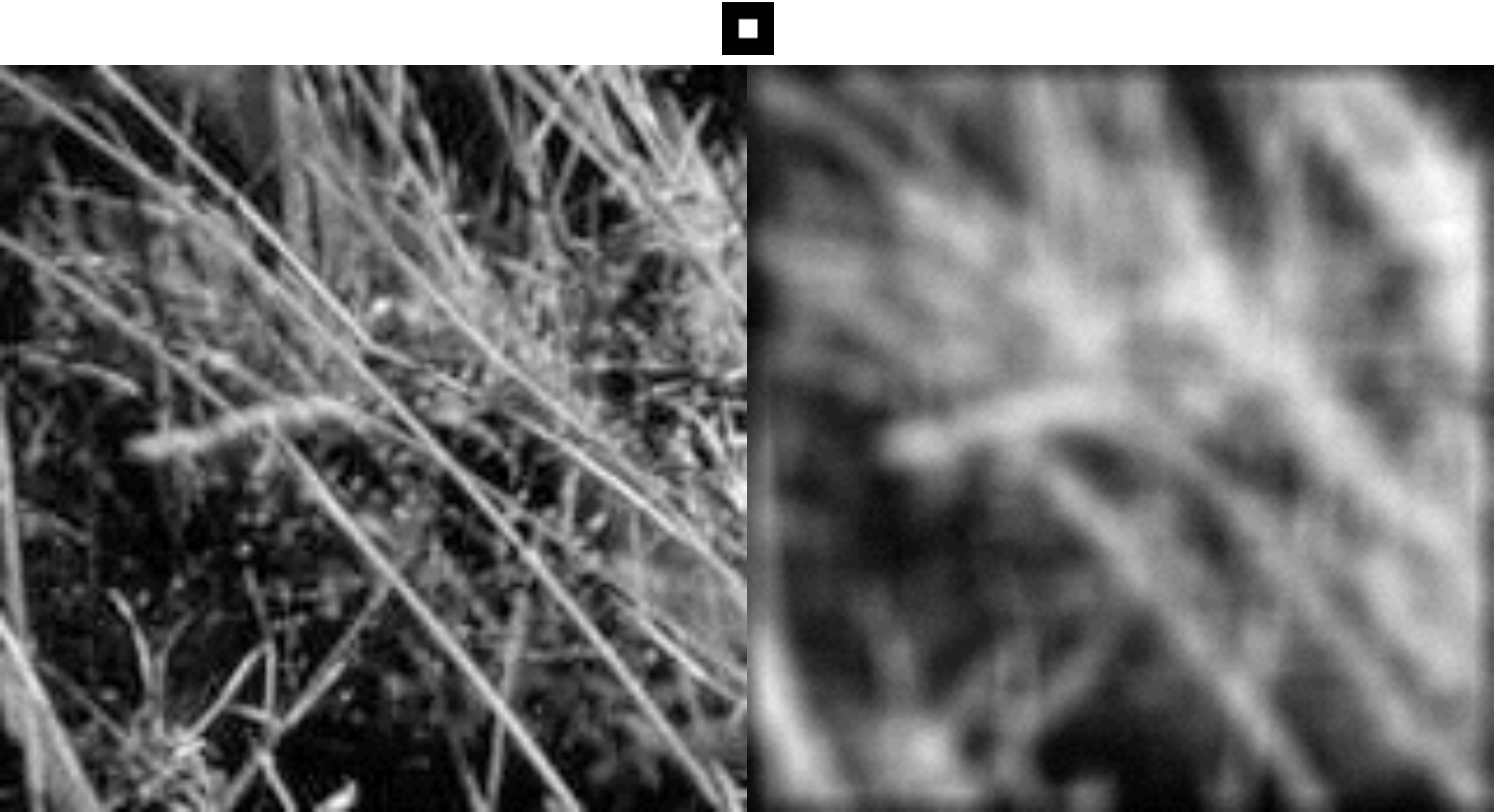
- Achieve smoothing effect (remove sharp features)

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Smoothing with box filter



Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

?

Source: D. Lowe

Practice with linear filters

0	0	0
0	0	1
0	0	0

Original

Shifted left
By 1 pixel

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

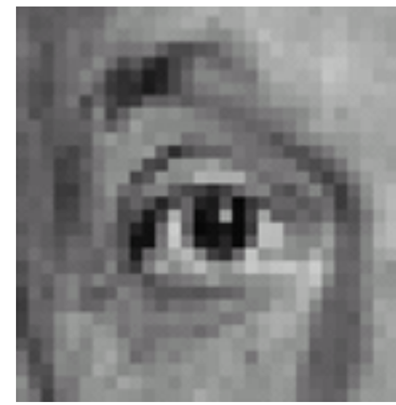
?

(Note that filter sums to 1)

Source: D. Lowe

Practice with linear filters

$$\begin{bmatrix}0 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



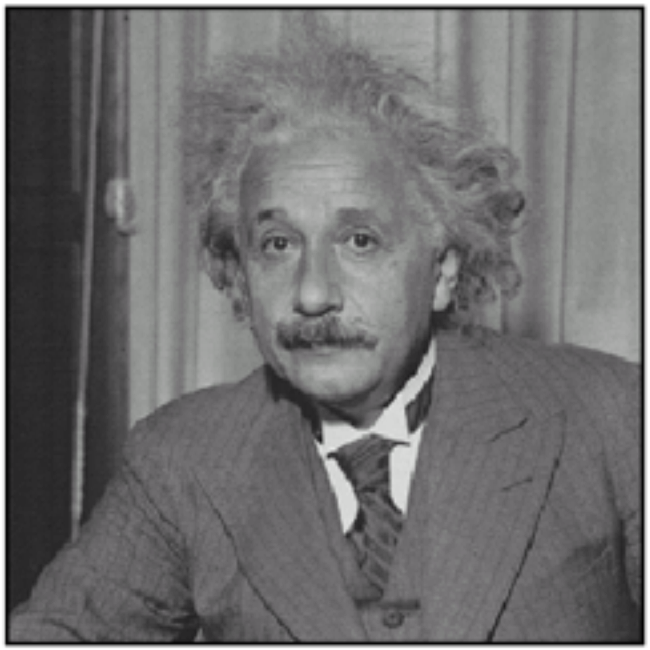
Original

Sharpening filter

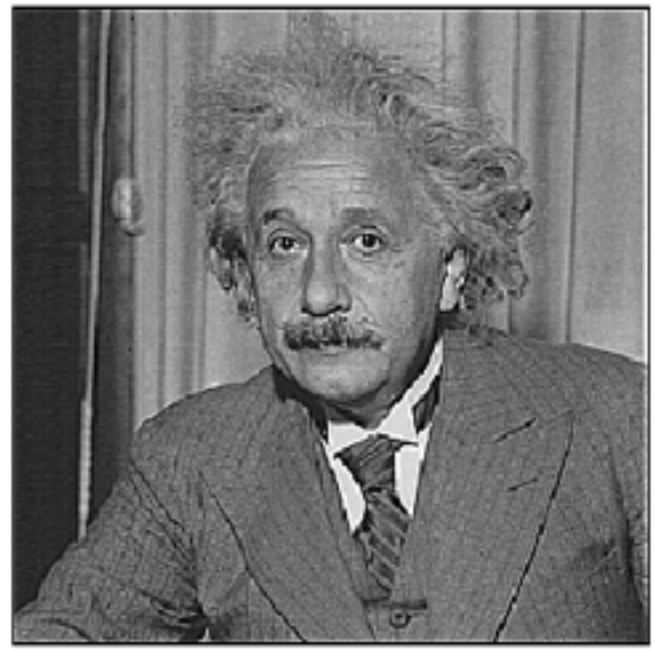
- Accentuates differences with local average

Source: D. Lowe

Sharpening



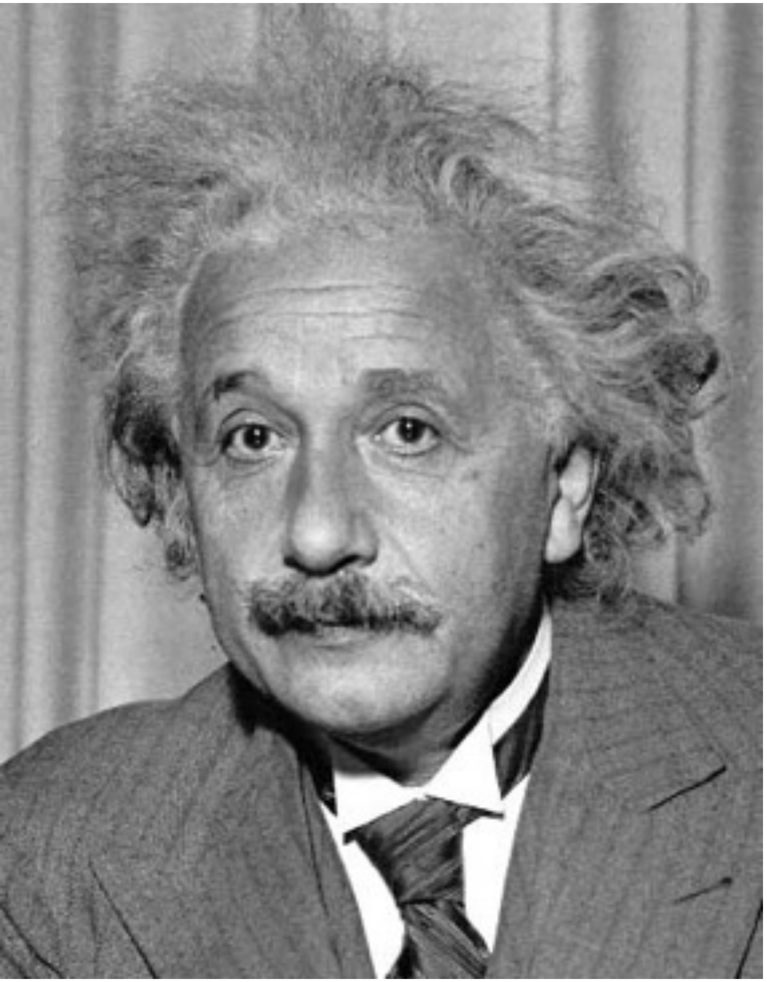
before



after

Source: D. Lowe

Other filters

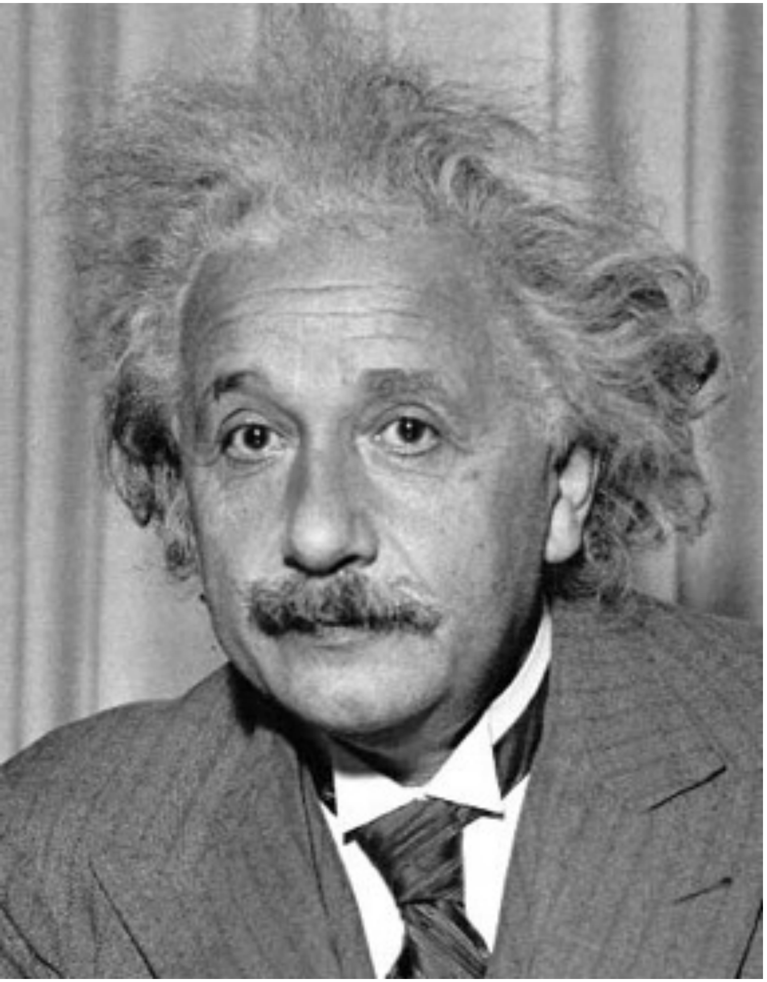


1	0	-1
2	0	-2
1	0	-1

Sobel

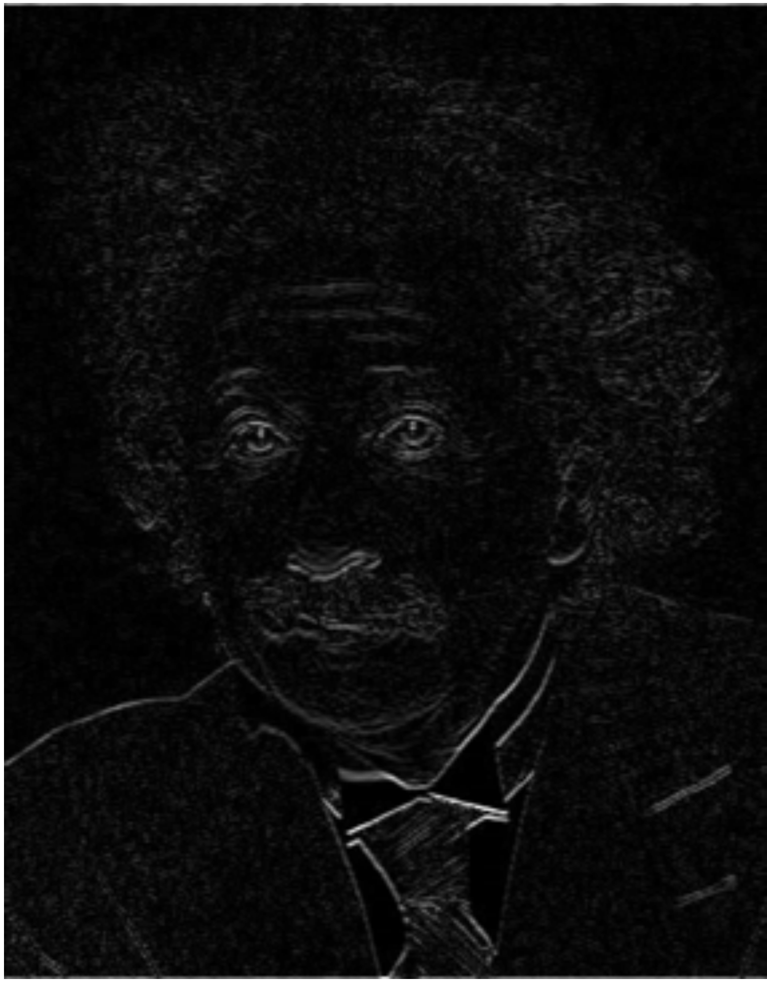
Vertical Edge
(absolute value)

Other filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Basic gradient filters

Horizontal Gradient

0	0	0
-1	0	1
0	0	0

or

-1	0	1

Vertical Gradient

0	1	0
0	0	0
0	-1	0

or

-1
0
1

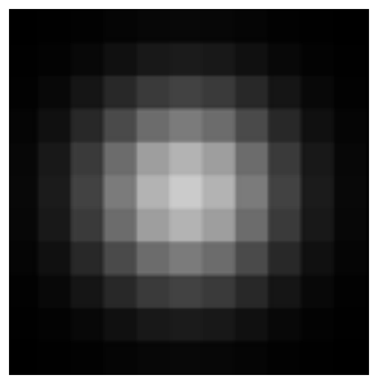
Gaussian filter

$$\mathcal{G}_\sigma(x,y) = \frac{1}{Z} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

Compute empirically

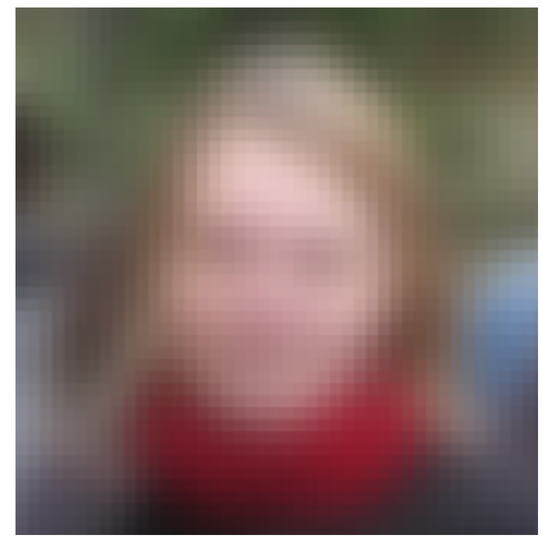
Input image f

*



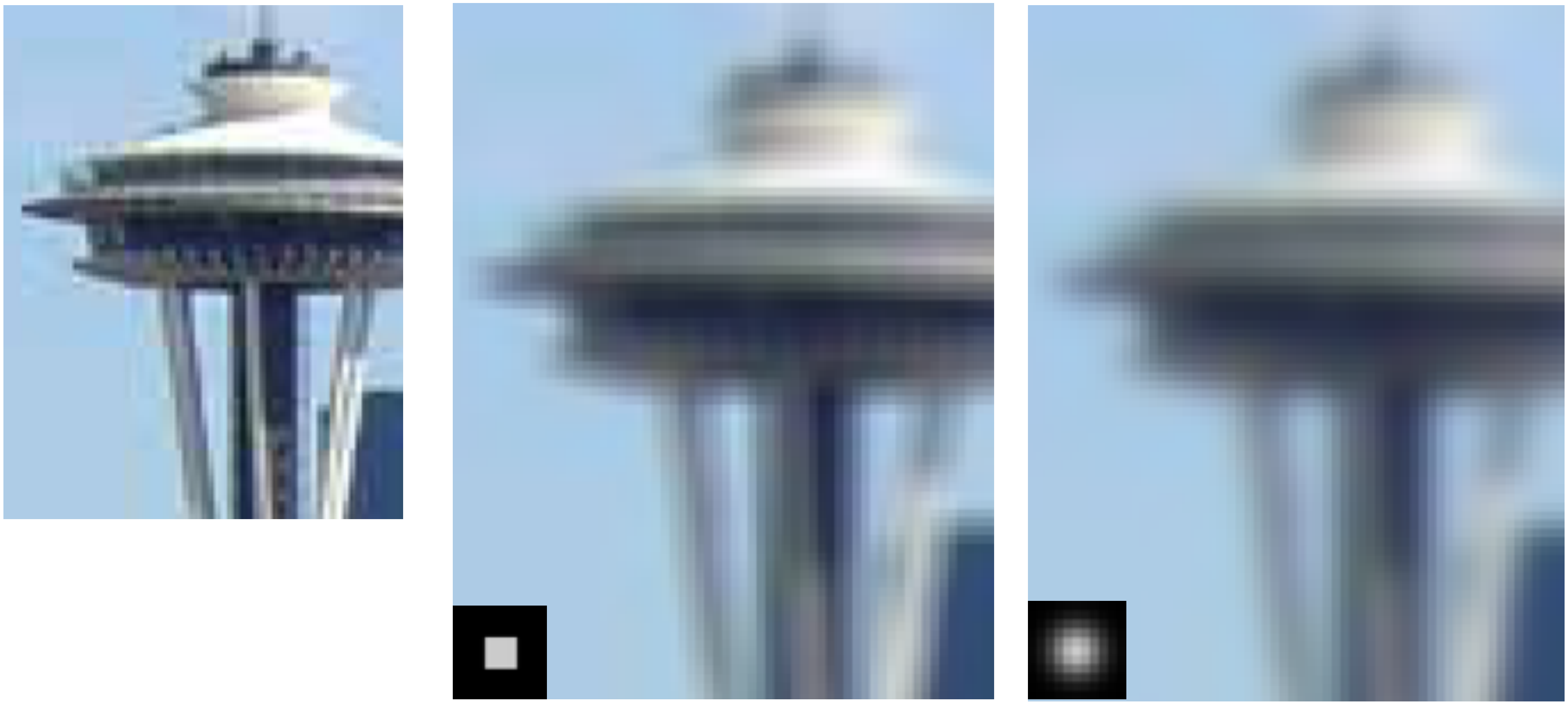
Filter h

=



Output image g

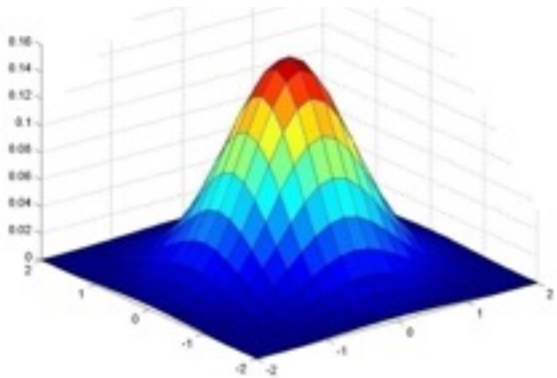
Gaussian vs. mean filters



What does real blur look like?

Important filter: Gaussian

- Spatially-weighted average



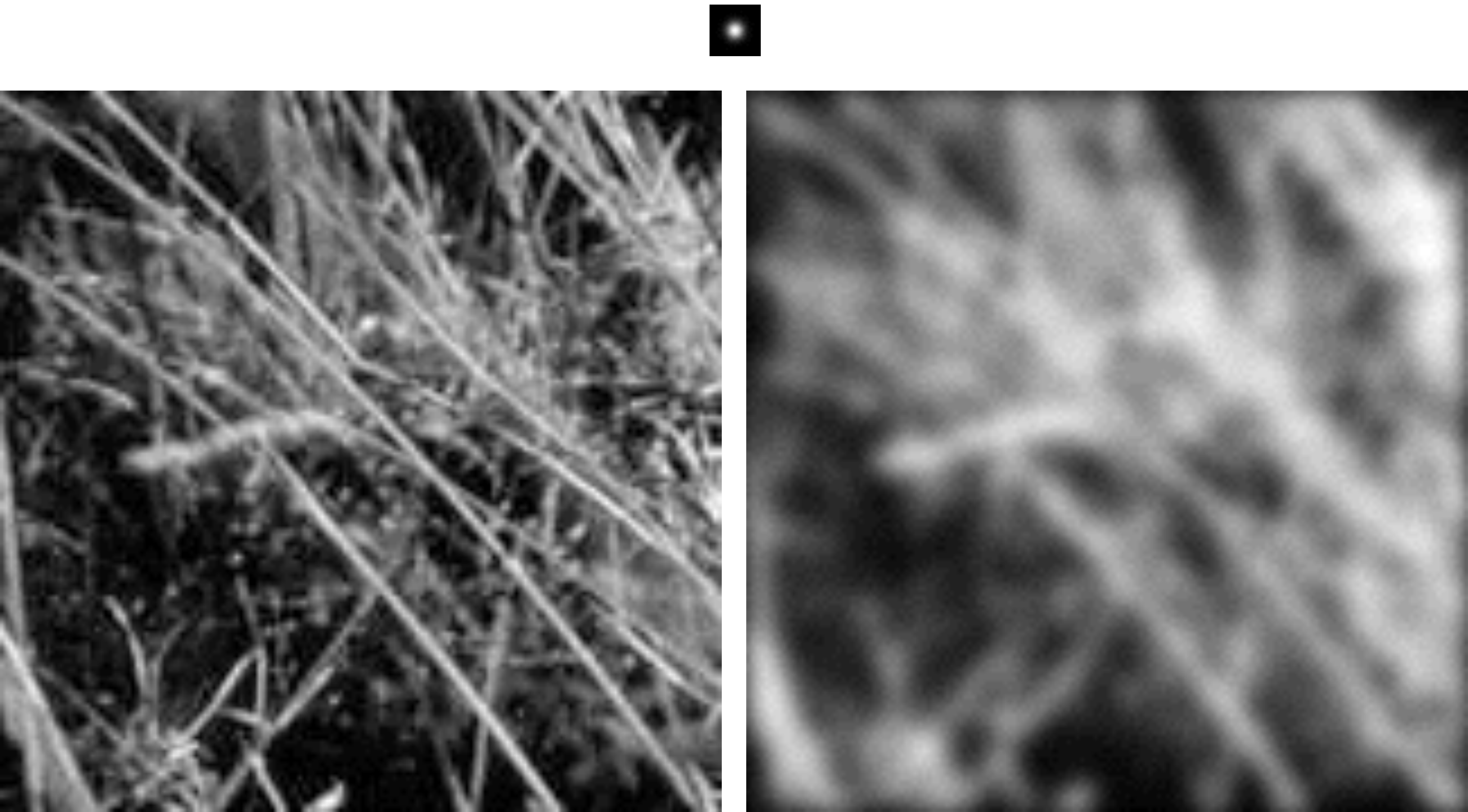
0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

5 x 5, $\sigma = 1$

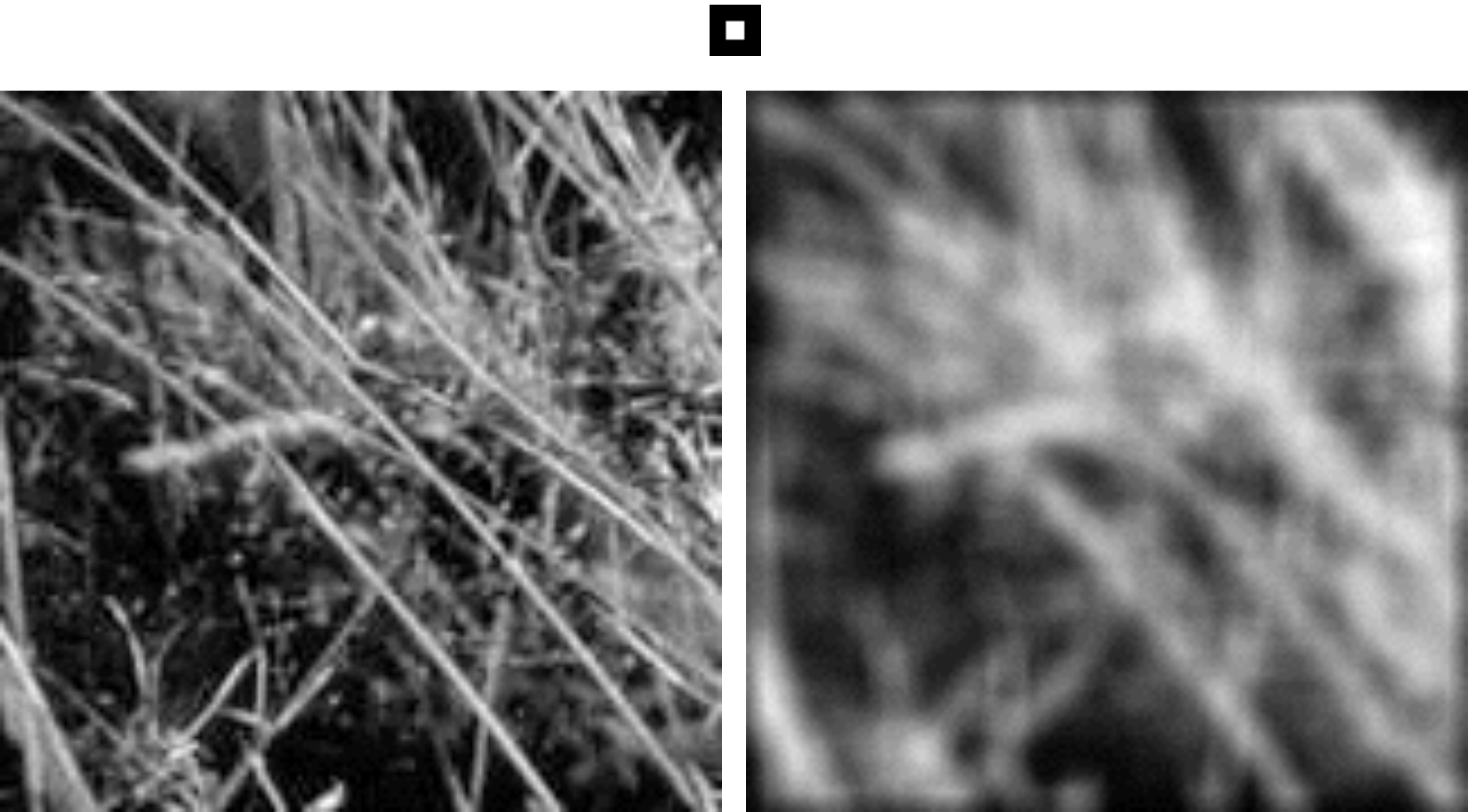
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Slide credit: Christopher Rasmussen

Smoothing with Gaussian filter

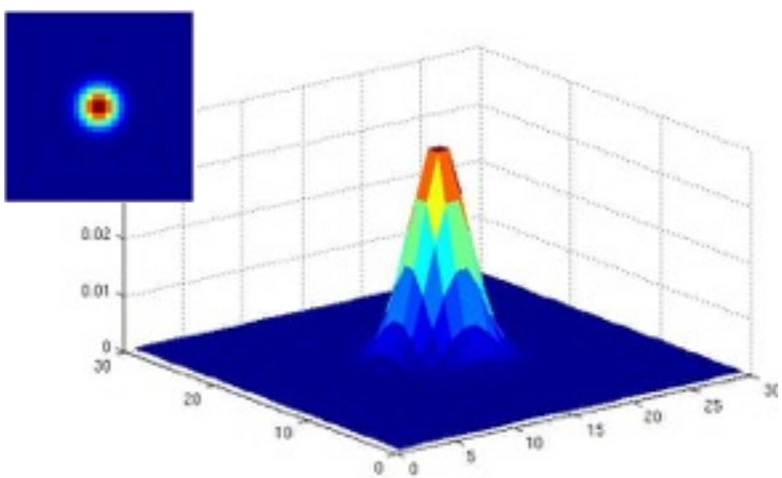


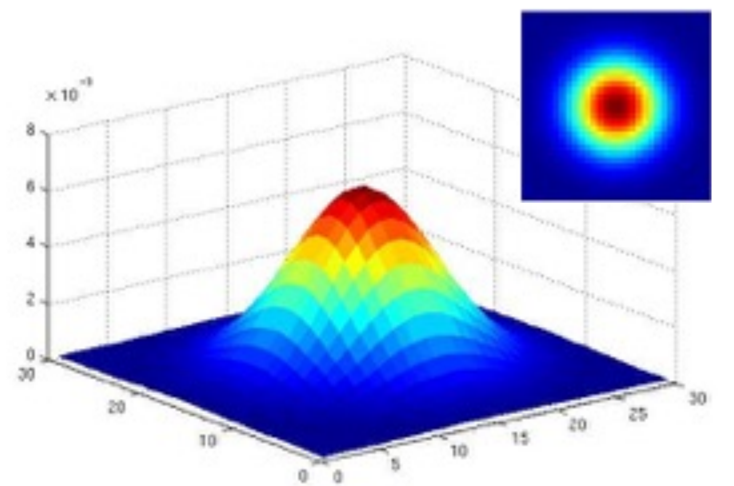
Smoothing with box filter



Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing

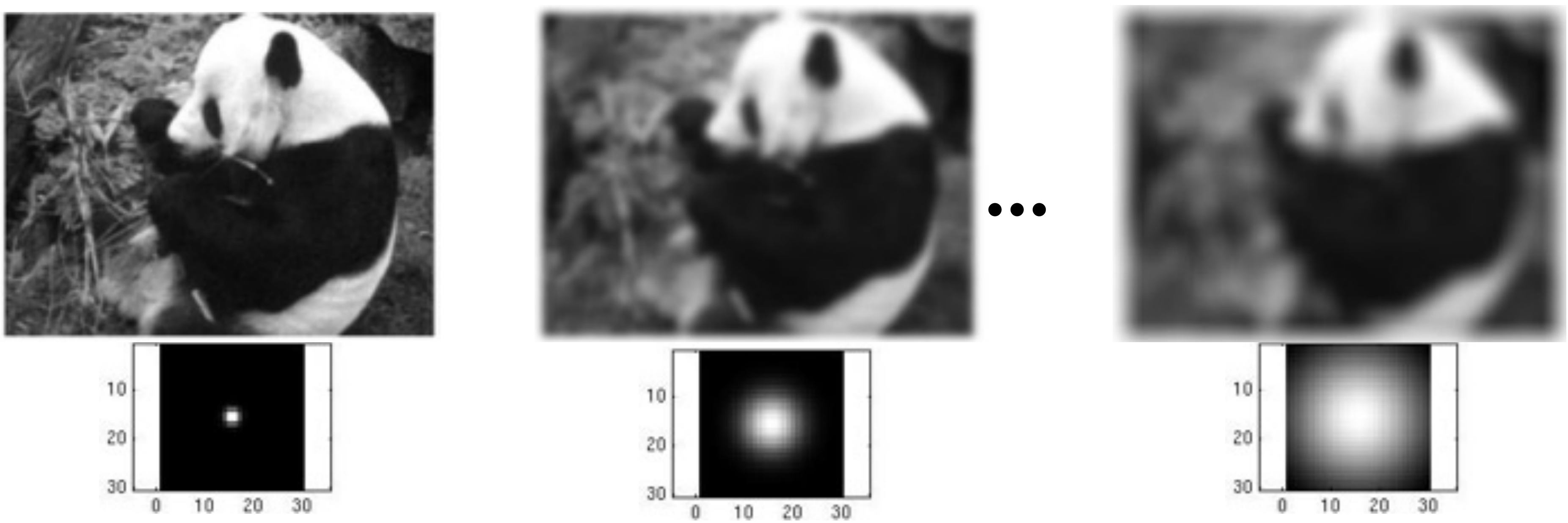




Source: K. Grauman

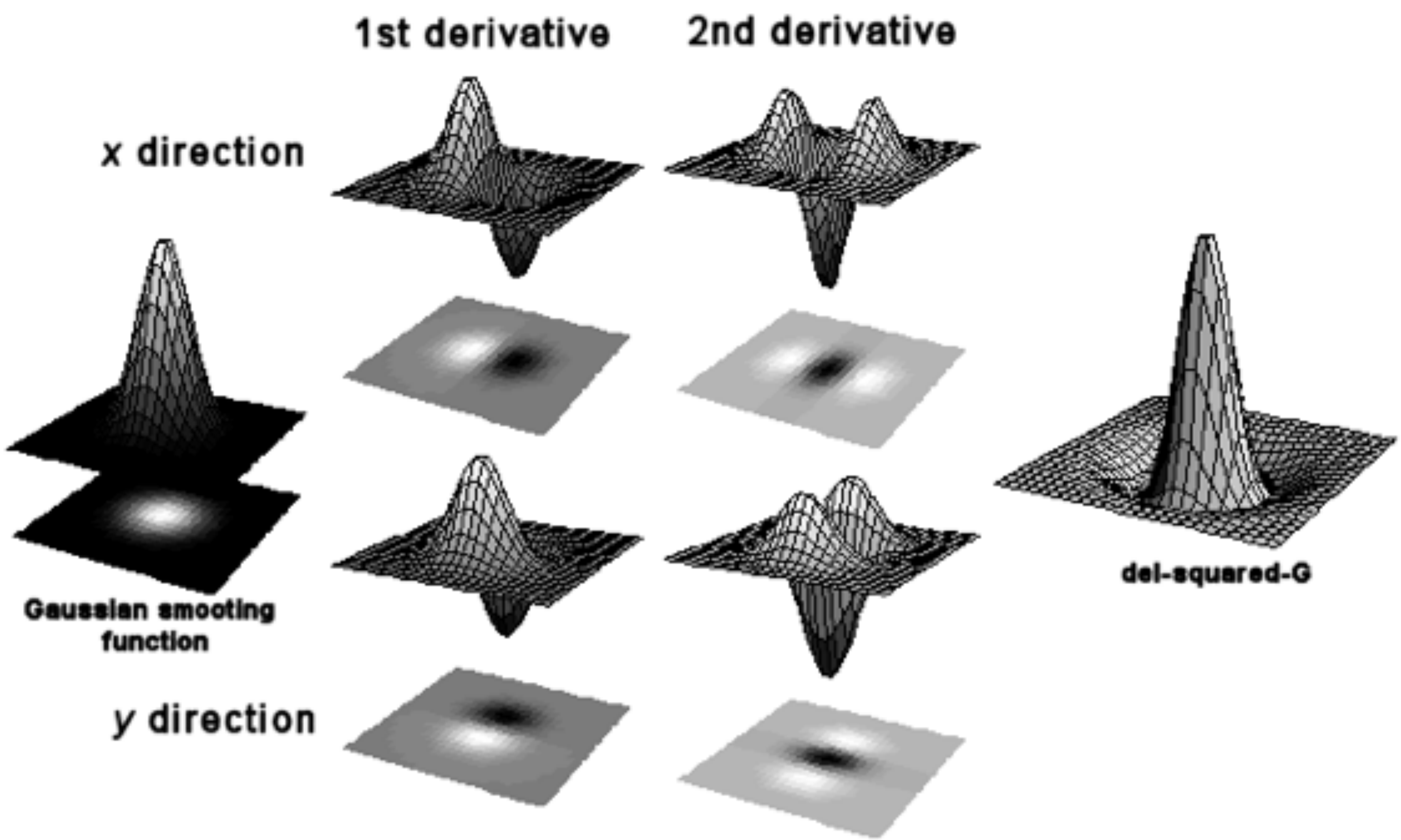
Smoothing with a Gaussian

Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.



Source: K. Grauman

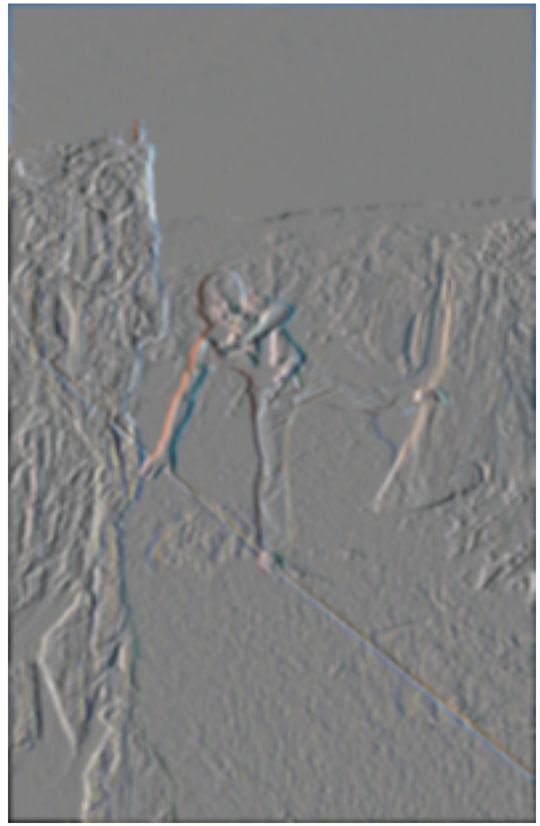
First and second derivatives



First and second derivatives

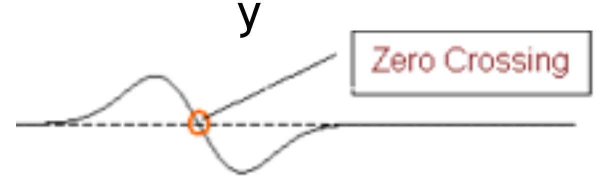
What are these good for?

Original



First Derivative x

Second Derivative x, y



Subtracting filters

$$Sharpen(x,y) = f(x,y) - \alpha(f * \nabla^2 \mathcal{G}_\sigma(x,y))$$


Original

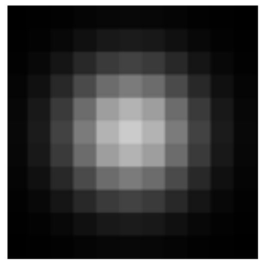

Second Derivative


Sharpened

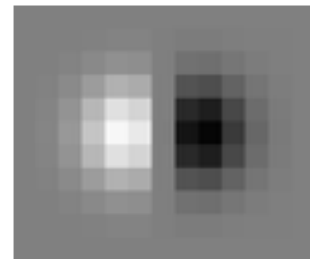
Combining filters

$$f * g * g' = f * h \text{ for some } h$$

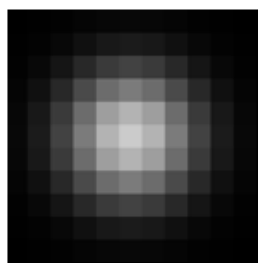
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} *$$



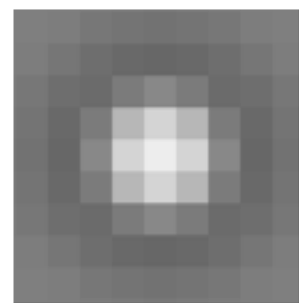
$=$



$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} *$$



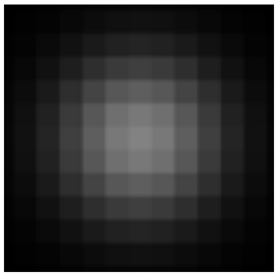
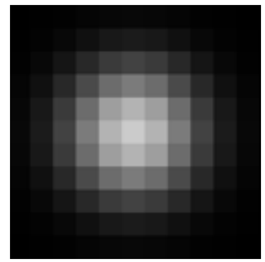
$=$



It's also true: $f * (g * h) = (f * g) * h$

$$f * g = g * f$$

Combining Gaussian filters

 $*$  $=$?

$$f * \mathcal{G}_\sigma * \mathcal{G}_{\sigma'} = f * \mathcal{G}_{\sigma''}$$

$$\sigma'' = \sqrt{\sigma^2 + \sigma'^2}$$

More blur than either individually (but less t$\sigma'' = \sigma + \sigma'$
)

Separable filters

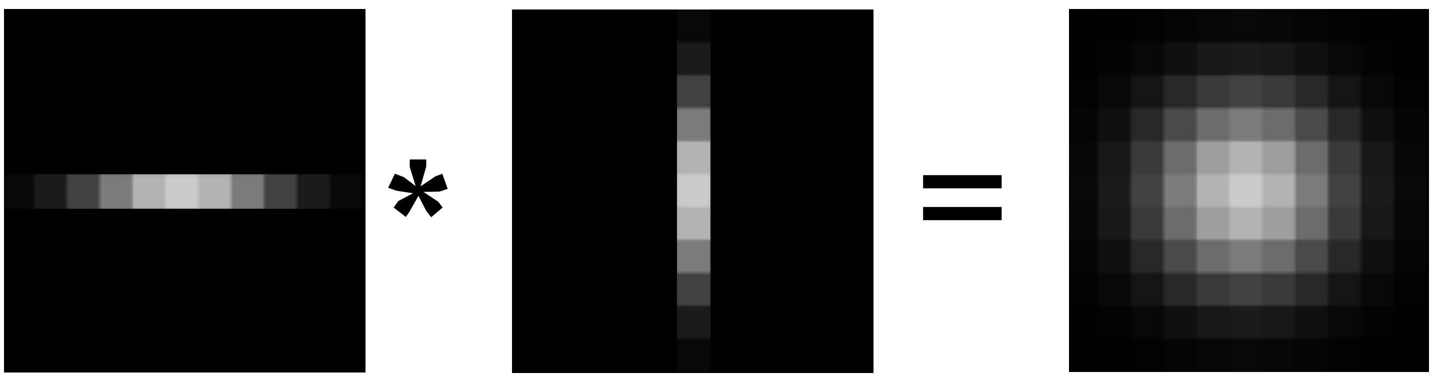
$$\mathcal{G}_\sigma = \mathcal{G}_\sigma^x * \mathcal{G}_\sigma^y$$

$$\mathcal{G}_\sigma^x(x,y) = \frac{1}{Z} e^{\frac{-(x^2)}{2\sigma^2}}$$

$$\mathcal{G}_\sigma^y(x,y) = \frac{1}{Z} e^{\frac{-(y^2)}{2\sigma^2}}$$

Compute Gaussian in horizontal direction, followed by the vertical direction.

Much faster!



Not all filters are separable.

Freeman and Adelson, 1991

Sums of rectangular regions

How do we compute the sum of the pixels in the red box?

After some pre-computation, this can be done in constant time for any box.

This “trick” is commonly used for computing Haar wavelets (a fundemental building block of many object recognition approaches.)

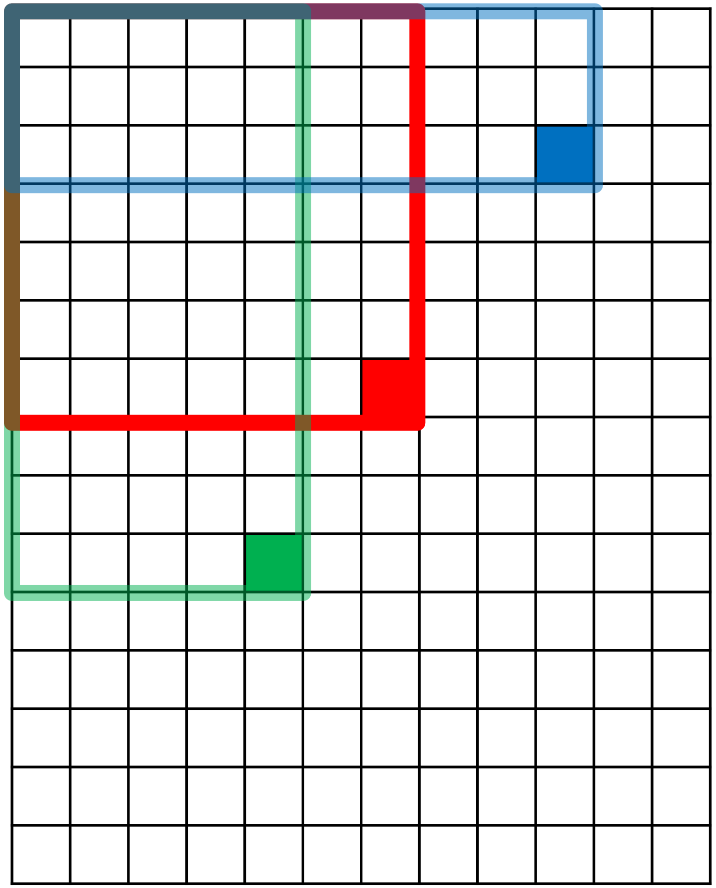
243	239	240	225	206	185	188	218	211	206	216	225
242	239	218	110	67	31	34	152	213	206	208	221
243	242	123	58	94	82	132	77	108	208	208	215
235	217	115	212	243	236	247	139	91	209	208	211
233	208	131	222	219	226	196	114	74	208	213	214
232	217	131	116	77	150	69	56	52	201	228	223
232	232	182	186	184	179	159	123	93	232	235	235
232	236	201	154	216	133	129	81	175	252	241	240
235	238	230	128	172	138	65	63	234	249	241	245
237	236	247	143	59	78	10	94	255	248	247	251
234	237	245	193	55	33	115	144	213	255	253	251
248	245	161	128	149	109	138	65	47	156	239	255
190	107	39	102	94	73	114	58	17	7	51	137
23	32	33	148	168	203	179	43	27	17	12	8
17	26	12	160	255	255	109	22	26	19	35	24

Sums of rectangular regions

The trick is to compute an "integral image." Every pixel is the sum of its neighbors to the upper left.

Sequentially compute using:

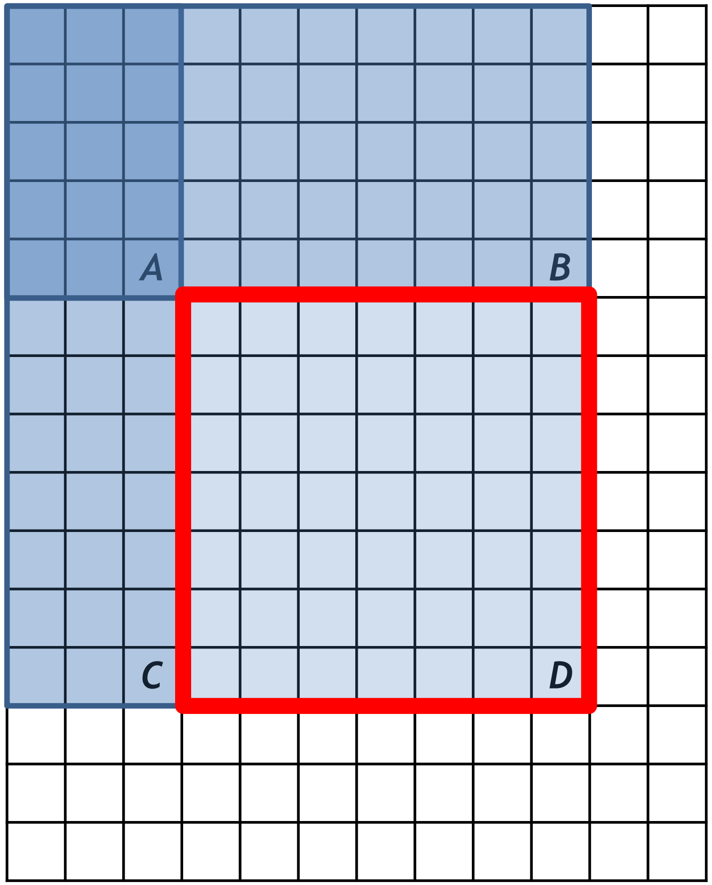
$$I(x,y) = I(x,y) + I(x-1,y) + I(x,y-1) - I(x-1,y-1)$$



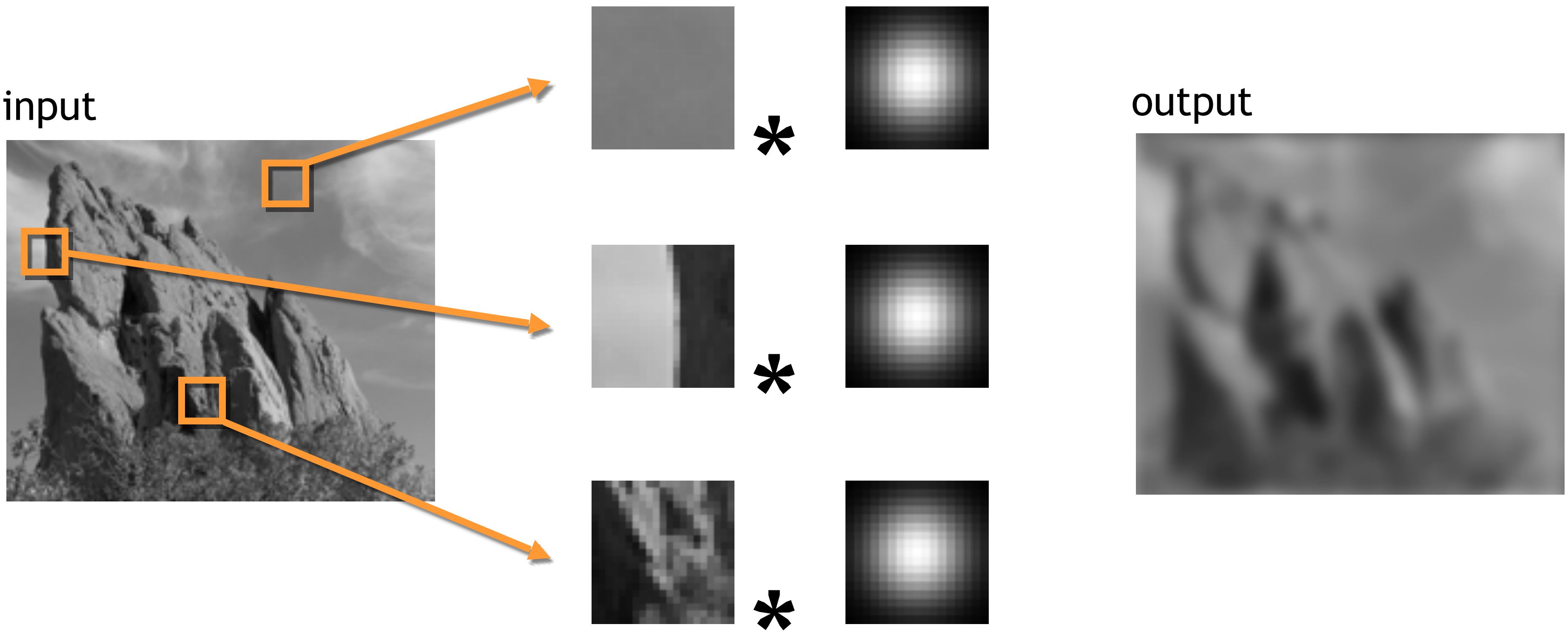
Sums of rectangular regions

Solution is found using:

$$A + D - B - C$$



Constant blur



Same Gaussian kernel everywhere.

Slides courtesy of Sylvian Paris

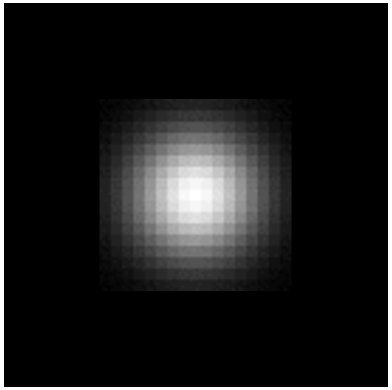
Bilateral Filter Definition: an Additional Edge Term

Same idea: **weighted average of pixels.**

$$BF[I]_{\mathbf{p}} = \underbrace{\frac{1}{W_{\mathbf{p}}}}_{\text{normalization factor}} \sum_{\mathbf{q} \in S} \underbrace{G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|)}_{\textit{space}\text{ weight}} \underbrace{G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |)}_{\textit{range}\text{ weight}} I_{\mathbf{q}}$$

normalization factor

space weight



range weight

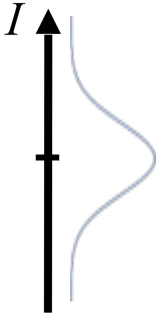
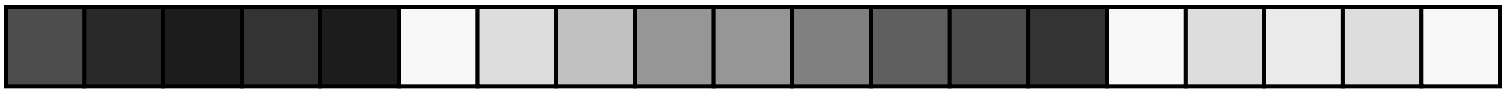
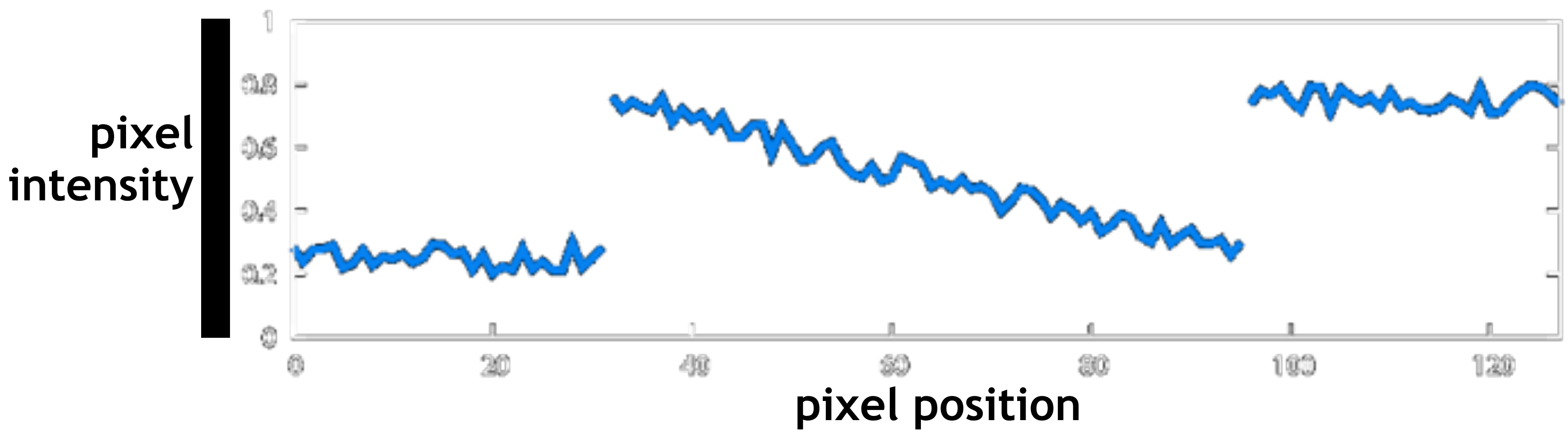


Illustration a 1D Image

- 1D image = line of pixels

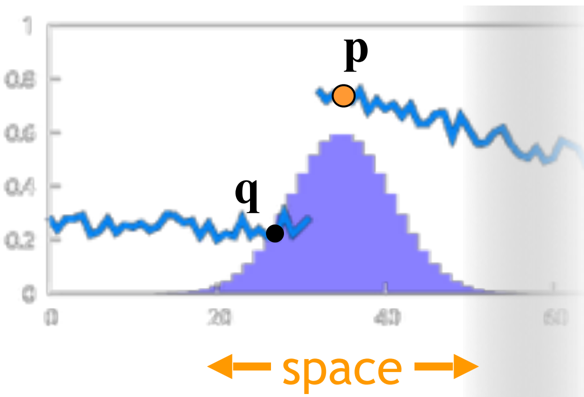


- Better visualized as a plot



Gaussian Blur and Bilateral Filter

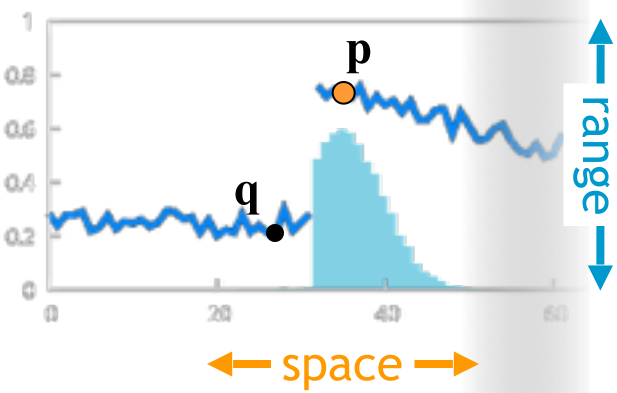
Gaussian blur



$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} \underbrace{G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} I_{\mathbf{q}}$$

Bilateral filter

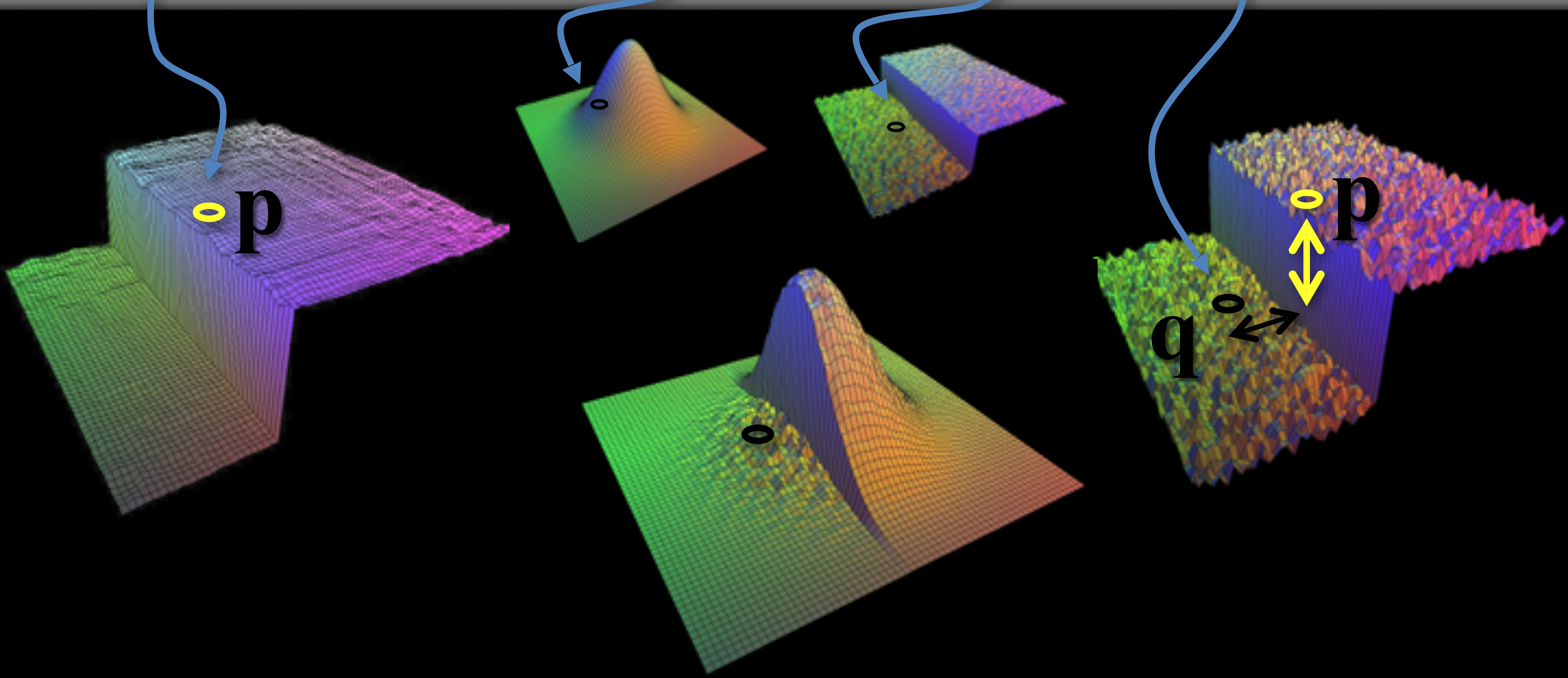
[Aurich 95, Smith 97, Tomasi 98]



$$BF[I]_{\mathbf{p}} = \underbrace{\frac{1}{W_{\mathbf{p}}}}_{\text{normalization}} \sum_{\mathbf{q}\in S} \underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} \underbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}_{\text{range}} I_{\mathbf{q}}$$

Bilateral Filter on a Height Field

$$\underbrace{BF[I]_{\mathbf{p}}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} \underbrace{G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|)} \; \underbrace{G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |)} \; I_{\mathbf{q}}$$



Space and Range Parameters

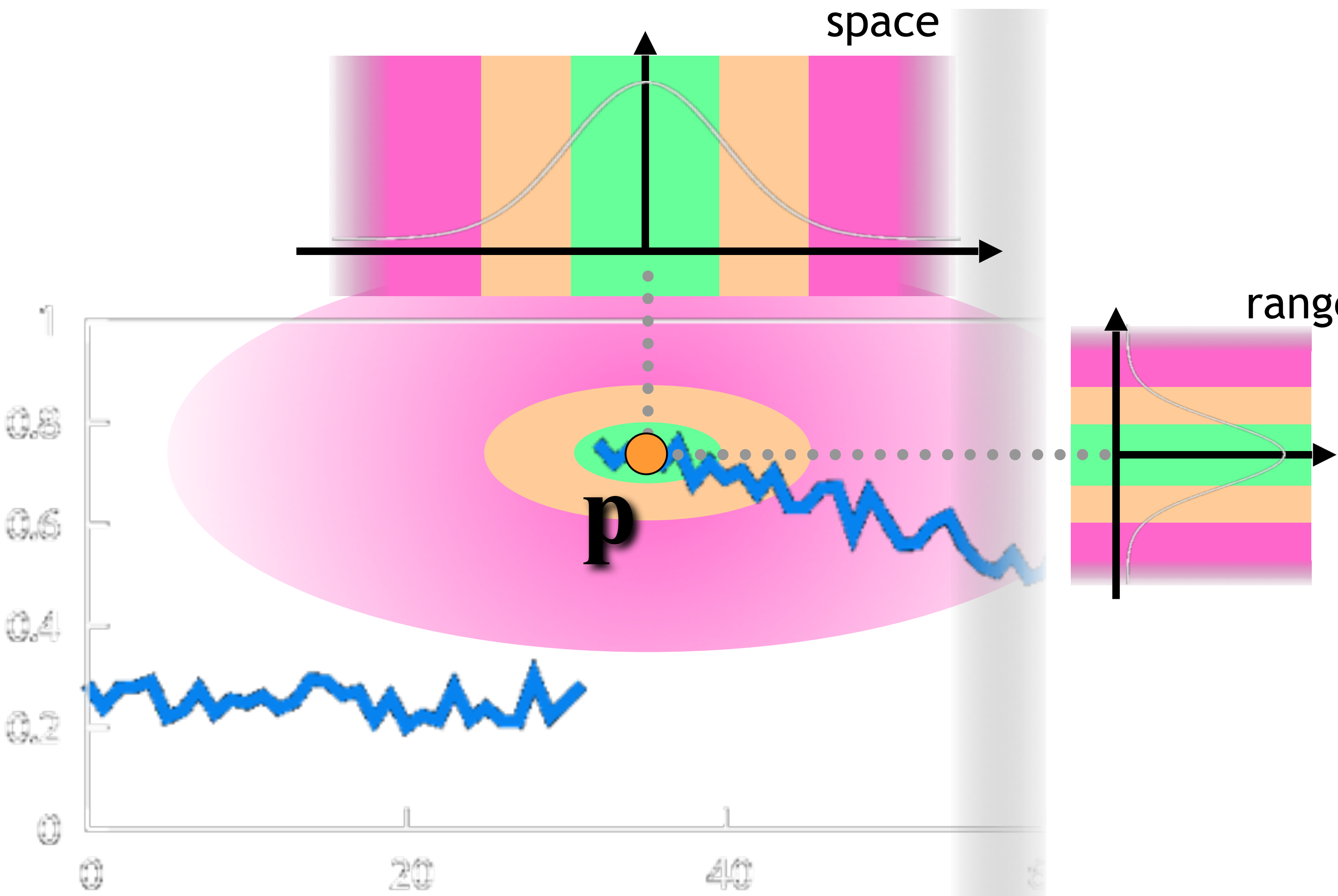
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\| \mathbf{p} - \mathbf{q} \|) G_{\sigma_r}(| I_{\mathbf{p}} - I_{\mathbf{q}} |) I_{\mathbf{q}}$$

- space σ_s : spatial extent of the kernel, size of the considered neighborhood.

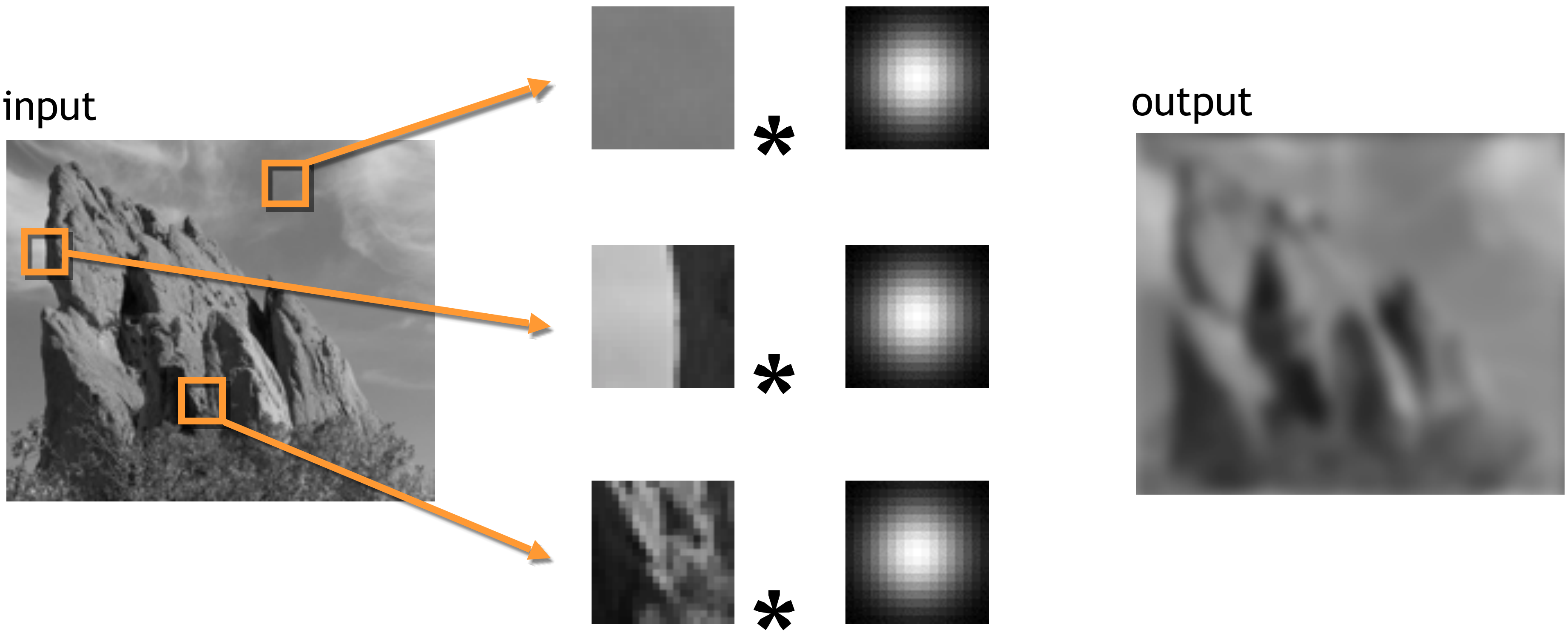
- range σ_r : “minimum” amplitude of an edge

Influence of Pixels

Only pixels close in space and in range are considered.



Constant blur

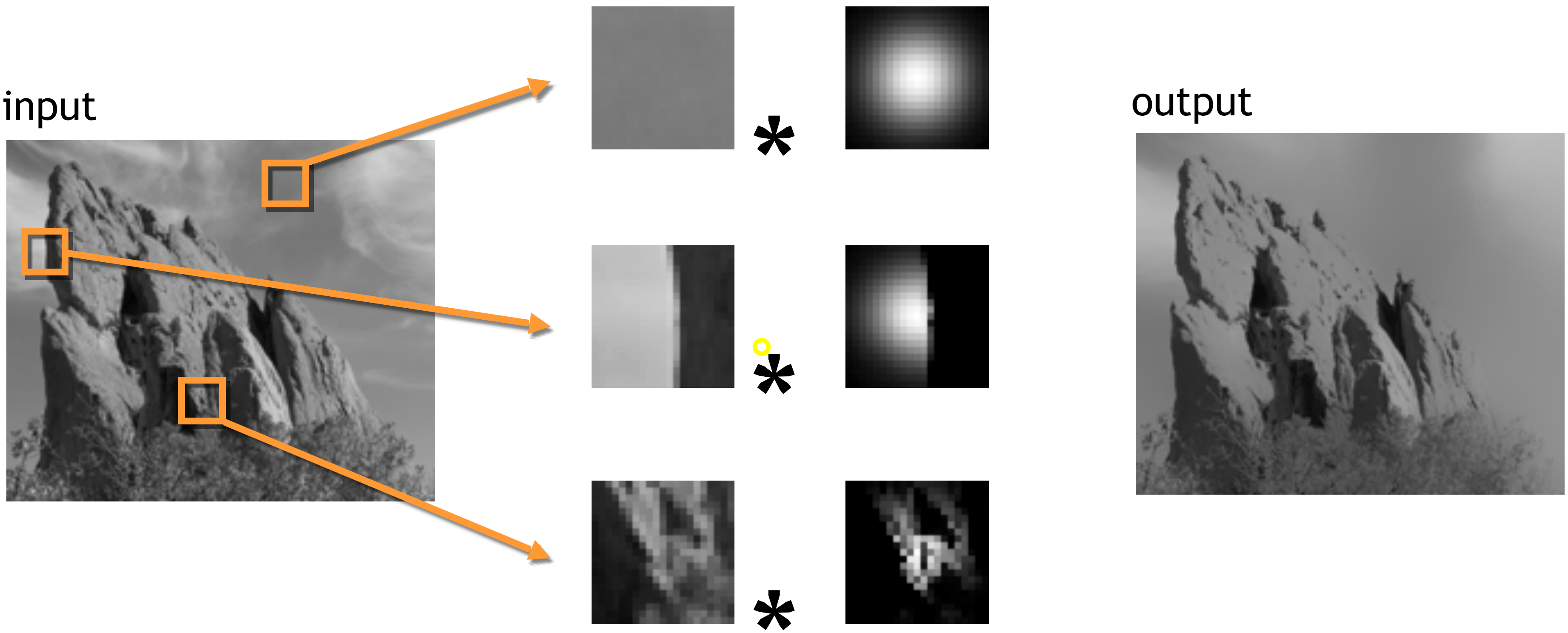


Same Gaussian kernel everywhere.

Slides courtesy of Sylvian Paris

Bilateral filter

Maintains edges when blurring!



The kernel shape depends on the image content.

Slides courtesy of Sylvian Paris

Borders

What to do about image borders:

black

fixed

periodic

reflected