MGSC 659, Winter 2024

Decision analysis and modelling for Operations

Problem Set 2-Individual Homework

Handout date: March 31, 2024
Due date: April 10, 2024

Problem 1. (32 pts)

In this problem, we will investigate the case "APLO: Optimal Supply of Street Lights".

1. What are APLO's aims in undertaking this project? For this project to be a showcase for the company, what would you recommend (in non-math terms) that they should try to optimize? (2 pts)

2. How should we evaluate the total expected cost of a set of components over the five-year guarantee period? Explain any assumptions or approximations that you have made in doing so. (2 pts)

3. We will examine two possible ways of approaching this problem. First, write down the model that minimizes total expected cost over the five-year guarantee period, assuming that each component fails at most once over five years. (For incompatibility issues and complementary features, use only those mentioned in the case.) (6 pts)

4. Implement and solve this model in Jupyter Notebook using Gurobi. **You may use the template APLO-template.ipynb provided to you**. We call the resulting output Solution 1. What is the cost of Solution 1? What is the probability that no component in Solution 1 fails over the five-year guarantee period? What is the probability that at most one of the components in Solution 1 fails over the five-year guarantee period? (8 pts)

5. Write down the non-linear model that maximizes the probability that no components fail over the five-year guarantee period, subject to budget constraints. Why will this be more difficult to implement? (5 pts)

6. To implement this, we approximate the objective function by the sum of probability of failure for all selected components. Implement and solve this model in Jupyter Notebook. We call the resulting

output Solution 2. What is the cost of Solution 2? What is the optimal value of our objective function? Given the optimal decisions in Solution 2, what is the probability that no component in Solution 2 fails over the five-year guarantee period? What is the probability that at most one of the components in Solution 2 fails over the five-year guarantee period? (5 pts)

7. Which of the two models would you recommend that APLO use, and why? (4 pts)

Problem 2. (18 pts)

A project manager in a company is considering a portfolio of 10 possible large project investments. These investments differ in their estimated long--run profit (net present value) they will generate as well as in the amount of initial capital required.

As per the table below, let P_j and C_j denote the estimated long--run profit and initial capital required (in millions of dollars) for each of the projects $j = 1, ..., 10$.

Project (i)	P_i: Estimated profit (Million \$)	C_i: Initial Capital Required (Million \$)
1	92	23
2	57	31
3	49	29
4	68	44
5	60	53
6	43	38
7	67	63
8	84	85
9	87	89
10	72	82

The project manager would like to determine which projects to undertake in order to maximize the total estimated long--run profit (**Hint**: note that the profits already take into account the initial capital required) subject to two initial constraints: a) that the total initial capital required for the projects cannot exceed \$175 million, b) that they should choose at least 4 projects.

1. Formulate this problem using a linear discrete optimization model. (You do not need to actually solve the model but only write down its formulation). That is, write down the decision variables, the objective function, and the initial constraints. including the non-negativity or other restrictions on the decision variables. (5pts)

2. In addition, the following constraints have to be met. Please formulate each one independently:

   (a) If they invest in 3 or more projects with an even subscript (i.e. $\in \{2, 4, 6, 8, 10\}$), they need to invest in at least one project with an odd subscript (i.e. $\in \{1, 3, 5, 7, 9\}$). (3pts)

(b) If they choose to invest in all of the projects, it is estimated that they will lose profit on the long run of $50 Millions. (3pts)

(c) If the total initial capital required for the projects exceeds $150 Millions, then the expected long-run profit needs to be at least $300 Millions. (3pts)

(d) The total initial capital required for the projects takes discrete values: it needs to be either $100, $125, $150 or $175 millions. (2pts)

3. Assume that you are instead minimizing the initial capital required across chosen projects, with some generic constraints. After you solve the model using an optimization solver, your code shows an optimal solution that suggests to undertake projects 1, 2, 3, 4, and 6. If there is another (different) optimal solution that yields the same objective function value, what constraint would you add to the problem in order to find this alternate solution? (2pts)