

MGSC 659 Winter 2022

Decision analysis and modelling for Operations

Problem Set 1-Individual Assignment

Handout date: March 16, 2022

Due date: March 26, 2022

Please submit your JuPyter Notebook with your code and answers to the problems.

Problem 1. (The Advertiser Optimization Problem) (26 pts)

We have modelled and solved the Google Ad Budget allocation problem in class. Let's try to model the advertiser's optimization problem, for example for the Residence Inn Hotel.

The Residence Inn Hotel wants to maximize traffic to their website, namely **maximize number of clicks from their ads to their site**. Advertiser faces three decisions: (i) Ad budget, (ii) Which queries to bid on, (iii) What to bid for each query. We will assume the budget and queries have already been determined. For example, the Residence Inn Hotel Budget is \$10/day. The Residence Inn can bid on three queries: "Hotel near McGill", "McGill hotel", and "Downtown Montreal hotel".

Residence Inn needs to decide which bids to make on each query b_1 , b_2 , b_3 . Assume the bids from the other three hotels and all quality scores are as before (see Figure 1).

Figure 1: Bids and Quality Scores

Hotel	Bid X QS (and order)					
	"hotel near McGill"		"McGill hotel"		"Downtown Montreal hotel"	
	Bid xQS	Order	Bid xQS	Order	Bid xQS	Order
Residence Inn	9 x b_1	TBD	9 x b_2	TBD	9 x b_3	TBD
Le St Martin	125	1	75	2	175	1
Le Germain	90	3	0	-	120	2
Sofitel	120	2	160	1	100	3

Like in the context of Session1, Google runs a modified 2nd price auction, where advertisers' positions are determined by the rank of their $bid \times quality\ score$. Residence Inn's bids will determine for each query

will determine which position it will land in.

Assume 4 positions on results page, and that the CTR, PPC, and costs (to Residence) will depend on bid position. See Figure 2 for how CTR changes with position on the page.

Figure 2: The click through rates depend on the position on the page

Position on Page	Click-Through-Rate
First	0.08
Second	0.05
Third	0.025
Fourth	0.01
Fifth	0.005

1. For each query, determine the possible range of bids that Residence Inn can submit to land in each position. (3pts)
2. For each query and position, what is the PPC (pay per click) for the Residence Inn? (3pts)
3. Residence Inn's bid optimization problem is reduced to deciding which position they want to be in for every query. Residence Inn will always bid for one position only per query. Define your variables. Write the constraint that your variables need to verify. (3pts)
4. Residence Inn has set a maximum number of times (Max Displays, see Figure 3) they would bid for any position for every query. Assume if Residence Inn bids for a particular position for a query, they will bid that maximum number of displays.

Figure 3: Maximum Number of Displays per Query per Position

Bid Position	Max displays		
	"hotel near McGill"	"McGill hotel"	"Downtown Montreal hotel"
First	8	7	4
Second	14	20	11
Third	15	20	25
Fourth	15	-	25
None	-	-	-

- (a) Write the budget constraint for Residence Inn. (3pts)
- (b) Write the Objective function for Residence Inn. (3pts)

5. Using Jupyter Notebook and Gurobi, write the Residence Inn's optimization problem that you formulated in the previous questions. What is the optimal bid position decisions for each query? What should the actual bid ranges for b_1 , b_2 and b_3 be? What is the optimal expected click-through rate for the Residence Inn? (Please submit as well your Jupyter notebook with the appropriate code). (6pts)
6. Suppose instead that Residence Inn is allowed to vary its bids to have an array of positions per query (They no longer only bid on one position only), but are still subjected to a maximum number of displays per position per query they cannot surpass. Reformulate the Residence Inn Optimization problem (You don't need to solve it using Gurobi). (5pts)

Problem 2. (Revenue Management at Spirit Airways)(19pts)

Spirit Airways is a small airline that offers passenger air transportation between two major east coast cities, namely Boston and New York; two major west coast cities, namely San Francisco and Los Angeles; and one major midwest city, namely Chicago. Spirit Airways operates a hub in Chicago, at which passengers can change planes to their final destination. Spirit Airways Airlines owns and operates two identical Boeing 757 aircraft, each with a capacity of 140 seats. The daily schedule of these aircraft is shown in Figure 4.

Figure 4: The daily westbound flight schedule for Spirit Airways

<i>Aircraft Number</i>	<i>Departure City</i>	<i>Departure Time</i>	<i>Arrival City</i>
1	Boston	8am EST	Chicago
1	Chicago	10:45 am CST	San Francisco
2	New York	7:45 am EST	Chicago
2	Chicago	10:45 am CST	Los Angeles

Spirit Airways offers both discounted (Q-Class) and unrestricted (Y-Class) fares. Spirit Airways wants to focus on their westbound operations. Figure 5 shows the westbound itineraries that Spirit Airways offers, their current prices, as well as the expected daily demand for seats for both Q-Class and Y-Class fares for the four Westbound flight legs of Spirit Airways. **This data is also stored in the csv file "Spirit Airways.csv".**

Spirit Airways want to decide the number of passenger seat to reserve for each flight (8 routes portrayed in Figure 2) for each class (Q and Y). We will use the letters Q_1, Q_2, \dots, Q_8 (respectively, Y_1, Y_2, \dots, Y_8) for the decisions variables related to the 8 routes (in the order they are shown in Figure 2) for the Q-Class (Y-class respectively).

1. Formulate the linear program of maximizing Spirit Airways revenue on Mondays clearly stating the decision variables, the objective function and the constraints.(7pts)

Figure 5: Prices and the expected daily demand for the Q-Class and Y-Class fares for the westbound itineraries of Spirit Airways

<i>From</i>	<i>To</i>	<i>Q-Class Fare Price</i>	<i>Y-Class Fare Price</i>	<i>Expected demand for Q-class</i>	<i>Expected demand for Y-class</i>
Boston	Chicago	\$200	\$230	25	20
Boston	San Francisco	\$320	\$420	55	40
Boston	Los Angeles	\$400	\$490	65	25
New York	Chicago	\$250	\$290	24	16
New York	San Francisco	\$410	\$550	65	50
New York	Los Angeles	\$450	\$550	40	35
Chicago	San Francisco	\$200	\$230	21	20
Chicago	Los Angeles	\$250	\$300	25	14

2. Using Jupyter Notebook, please run the file 'HW1-part2.ipynb' to load the data for this problem, which is stored in the csv file "Spirit Airways.csv". Then, using Gurobi, solve the LP problem formulated in the previous question (please assume all variables to be continuous). What is the optimal allocation decisions for each flight and Class? What is the optimal revenue for Spirit Airways? (Please submit as well your Jupyter notebook with the appropriate code). (4pts)
3. **Analysis of a New Lease Option.** Spirit Airways is thinking of leasing a small jet from Canadair (with a capacity of 50 seats) to increase the capacity of their flight Chicago-San Fransisco, for a cost of \$2.5 million per year. Should Spirit Airways accept Canadair's offer? Why? (Note: Assume that the expected daily demand and prices are the same all year long and the shadow prices don't change by changing the right hand side of the constraints)(3pts)
4. **Managing fare reservation requests.** On a Friday, Spirit Airways received two simultaneous ticket requests:
 - (a) **Request 1:** Two tickets, one for Elon Musk (Y-Class ticket) and another for his assistant (Q-Class ticket). After attending a MIT conference, they are flying back from Boston to San Francisco.
 - (b) **Request 2:** One ticket (Q-Class ticket) for Layla Abbot, a student at the university of Chicago, who is flying back to her hometown San Francisco for the weekend.

If Spirit Airways was able to accept one request only, which request should that be? Please explain your reasoning.(5pts)