QM1-2　利用Maupertuis原理证明Newton第二定律.

证明:

已知Maupertuis原理

$$\delta\int\sqrt{E-U}\mathrm{ds}=\int((\sqrt{E-U})\delta\mathrm{ds}-\frac{\delta U}{2\sqrt{E-U}}\mathrm{ds})=0 \tag{1}$$

由

$$\delta\mathrm{ds}=\sum\frac{\mathrm{dx_i}}{\mathrm{ds}}\delta\mathrm{dx_i} \tag{2}$$

$$\delta U=\sum\frac{\partial U}{\partial x_i}\delta x_i \tag{3}$$

得

$$\begin{aligned}&\int(\sqrt{E-U}\cdot\sum\frac{\mathrm{dx_i}}{\mathrm{ds}}\mathrm{d}\delta x_i-\frac{\delta U}{2\sqrt{E-U}}\delta x_i)=0\\ \Rightarrow&\int\sum(\delta x_i\cdot\mathrm{d}(\sqrt{\mathrm{E-U}}\frac{\mathrm{dx_i}}{\mathrm{ds}})+\frac{1}{2\sqrt{E-U}}\cdot\frac{\partial U}{\partial x_i}\mathrm{ds}\cdot\delta x_i)=0\end{aligned} \tag{4}$$

由δx_i的任意性可得

$$\frac{d}{ds}(\sqrt{E-U}\cdot\frac{dx_i}{ds})=-\frac{1}{2\sqrt{E-U}}\cdot\frac{\partial U}{\partial x_i} \tag{5}$$

又由动力学基础得

$$v=\sqrt{\frac{2(E-U)}{m}}\quad,\quad\mathrm{dt}=\frac{ds}{v}=\sqrt{\frac{m}{2(E-U)}}\mathrm{ds} \tag{6}$$

将(5)(6)代入(4)可得

$$m\frac{d^2x_i}{dt^2}=-\frac{\partial U}{\partial x_i}$$

即为Newton第二定律.

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