

QM2 - 将 $\psi$ 代入 $\tilde{F}\psi = 0$ ,验证相对论能量动量关系式.

已知

$$\tilde{F} = (i\hbar \frac{\partial}{\partial t})^2 - (-i\hbar \nabla)^2 c^2 - m^2 c^4$$

将 $\psi = e^{-i(\vec{p} \cdot \vec{x} - Et)/\hbar}$  代入得

$$\frac{\partial \psi}{\partial t} = \frac{i \cdot E}{\hbar} \psi \quad , \quad \nabla \psi = -\frac{i \cdot p}{\hbar} \psi$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\frac{E^2}{\hbar^2} \psi \quad , \quad \nabla^2 \psi = \frac{p^2}{\hbar^2} \psi$$

故由

$$\begin{aligned} \tilde{F}\psi &= (i\hbar)^2 \cdot \left(\frac{\partial^2 \psi}{\partial t^2}\right) - (-i\hbar)^2 \cdot (\nabla^2 \psi) - m^2 c^4 \cdot \psi \\ &= (E^2 - p^2 c^2 - m^2 c^4) \psi = 0 \end{aligned}$$

得

$$E^2 = p^2 c^2 + m^2 c^4.$$

PB12203077 吴奕涛  
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