18 Simple Harmonic Motion

18.1 Oscillations

The **equilibrium position** is the position with the **least potential energy** - it is where the oscillation will eventually come to a standstill. The motion is described as **oscillating about equilibrium**.

- An **object on a spring** moves up and down repeatedly.
- A **pendulum** moving back and forth.
- A boat rocking from side to side.

An oscillating object moves repeatedly **one way then in the opposite direction** through its equilibrium position.

The **displacement** of the object is its distance and direction from equilibrium. It **continually changes** during the motion, in **one full cycle** after being **released from a non-equilibrium position**.

1. **Decreases** as it returns to equilibrium,
2. **Reverses and increases** as it moves away from equilibrium in the opposite direction.
3. Decreases as it returns to equilibrium.
4. Increases as it moves away from equilibrium towards its starting position.

- The **amplitude** of the oscillations is the **maximum displacement** of the oscillating object from equilibrium.
- If the **amplitude is constant** and **no frictional forces** are present, the oscillations are described as **free vibrations**.
- The **time period** of the oscillating motion is the time for **one complete cycle of oscillation**.
- One **full cycle** after passing through any position, the object passes through that same position in the same direction.
- The **frequency** of oscillations is the **number of cycles per second** made by an oscillating object.

  The unit of frequency is the **hertz** (Hz), which is one cycle per second.
- The **angular frequency** of the oscillating motion is defined as

$$\omega = \frac{2\pi}{T}$$

The **phase difference** of two oscillating objects stays the same if they oscillates with the same frequency.

If Δt is the time between successive instants when the two objects are at **maximum displacement in the same direction**, one object is always $\dfrac{\Delta t}{T}$ cycles ahead. So for two objects oscillating in the same frequency.

$$\text{Phase difference} = \frac{2\pi\Delta t}{T} = 2\pi\Delta t f$$

18.2 The Principles of Simple Harmonic Motion

The velocity of an object is given by the **gradient of the displacement-time graph**.

- The **magnitude of the velocity** is greatest when the object is at at **zero displacement when the object passes through equilibrium**.
- The **velocity is zero** when the object is at **maximum displacement** in either direction.

The acceleration of an object is given by the **gradient of the velocity-time graph**.

- The **acceleration is greatest** when the **velocity is zero** at maximum displacement in the opposite direction.
- The **acceleration is zero** when the displacement is zero and the velocity is a maximum.

Conditions for Simple Harmonic Motion

Simple harmonic motion is defined as oscillating motion in which acceleration is

- Proportional to the displacement, and
- Always in the **opposite direction to the displacement**

$$a \propto -x$$

or in other words, $a = -kx$.

The acceleration for simple harmonic motion is

$$a = -\omega^2 x$$

18.3 More about Sine Waves

Consider an object P in **uniform circular motion**.

1. From the centre of the circle, the coordinates of P is $x = r\cos\theta$ and $y\sin\theta$, where θ is the angle between the x-axis and the radial line OP.
2. Consider the **shadow of the ball** alongside the **shadow of a pendulum bob**, the two shadows keep up with each other exactly when their time periods are matched.
3. So they have the **same horizontal motion**, and the **same horizontal acceleration**.

$$\text{Acceleration of the ball} = -\omega^2 r$$

4. The **horizontal component of acceleration of the ball** is

$$a_x = a\cos\theta = -\omega^2 r\cos\theta$$

5. Since $x = r\cos\theta$

$$\textbf{Acceleration of the pendulum bob } a_x = -\omega^2 r$$

If $x = A$ and $v = 0$ when $t = 0$

$$x = A\cos\omega t$$