18 Simple Harmonic Motion

18.1 Oscillations

- The **equilibrium** position is where an oscillation will eventually come to a standstill.
- An **oscillating object** moves repeatedly one way then in the opposite direction through its equilibrium position.
- The **displacement** of the object is the distance and direction from equilibrium.
- The **amplitude** of oscillations is the maximum displacement of the oscillating object from equilibrium.
- **Free vibrations** are oscillations where the amplitude is constant and no frictional forces are present.
- The **time period** is the time for one complete cycle of oscillation.
- One **full cycle** after passing through any position, the object passes through that same position in the same direction.
- The **frequency** of oscillations is the number of cycles per second.

The unit for frequency is the **hertz** - one cycle per second.

$$\text{Time period } T = \frac{1}{f}$$
$$\text{Angular velocity } \omega = \frac{2\pi}{T}$$

$$\textbf{Phase difference} = \frac{2\pi \Delta t}{T}$$

where Δt is the time between successive instants when the two objects are at maximum displacement in the same direction.

18.2 The Principles of Simple Harmonic Motion

Velocity is given by the **gradient of the displacement-time graph**.

- The **magnitude of velocity is greatest** at zero displacement when the object passes through equilibrium.
- The **velocity is zero** at maximum displacement in either direction.

Acceleration is given by the **gradient of the velocity-time graph**.

- The **acceleration is greatest** when velocity is zero at maximum displacement.

- The **acceleration is zero** when the velocity is a maximum displacement is zero.

Conditions for SHM

The acceleration is

- Proportional to displacement.
- In the opposite direction as displacement.

$$a \propto -x$$

Therefore $a = -kx$

$$a = -\omega^2 x$$

18.3 More about Sine Waves

For a object P in **uniform circular motion**, is coordinates are given by $P(r\cos\theta, r\sin\theta)$.

1. Because the ball is in uniform circular motion, the acceleration is **towards the centre** and
$$a = -\omega^2 r$$
2. Consider the shadow of the ball, the acceleration of its shadow is
$$a = -\omega^2 r\cos\theta$$
3. But since $x = r\cos\theta$
$$a = -\omega^2 x$$

For any object oscillating at frequency f in a simple harmonic motion

- Its **acceleration** a **at displacement** x
$$a = -\omega^2 x$$
- Its **displacement** x **at time** t
$$x = A\cos\omega t$$

18.4 Applications of Simple Harmonic Motion

The frequency of oscillation is reduced by

- Adding **extra mass**.
- Use **weaker springs**.

The acceleration is also given by

$$a = -\omega^2 x \qquad \text{where } \omega^2 = \frac{k}{m}$$

The Simple Pendulum

For a simple pendulum where a mass is attached to a thread of length L, and angle does not exceed $10°$

$$a = -\frac{g}{L}s = -\omega^2 s \qquad \text{where } \omega^2 = \frac{g}{L}$$

18.5 Energy of Simple Harmonic Motion

A freely oscillating object **oscillates with a constant amplitude** because no friction is acting on it.

1. The **potential energy** changes with displacement x from equilibrium.

$$E_P = \frac{1}{2}kx^2$$

2. The **total energy** of the system is therefore

$$E_T = \frac{1}{2}kA^2$$

3. So the **kinetic energy** of the system is

$$E_K = \frac{1}{2}k(A^2 - x^2)$$

Equating $E_K = \frac{1}{2}mv^2$, we also have

$$v \pm \omega\sqrt{A^2 - x^2}$$

showing speed is maximum when $x = 0$ and $v = \omega A$.

Damping

In any oscillating system where **friction or air resistance is present**, the amplitude decreases.

Dissipative forces dissipate the energy of the system to the surroundings as thermal energy. The motion is said to be **damped** if dissipative forces are present.

- **Light damping -** the time period is independent of the amplitude, so each cycle takes the same length of time.
- **Critical damping -** the oscillating system returns to equilibrium in the **shortest possible time** without overshooting.
- **Heavy damping -** the displaced object returns to equilibrium much more slowly than if the system is critically damped, and **no oscillating motion** takes place.

18.6 Forced Vibrations and Resonance

- A **periodic force** is a force applied at regular intervals.
- The **natural frequency** is the system's frequency when it **oscillates without a periodic force**.
- A system undergoes **forced vibrations** when a period force is applied to it.

For a mass attached to two stretched springs, with the lower spring attached to a mechanical oscillator connected to a signal generator.

- The amplitude of oscillations **increases until a maximum amplitude** at a particular frequency, then decreases again.
- The **phase difference** between the displacement and the period force increases from 0 to $\frac{\pi}{2}$ at maximum amplitude, then from $\frac{\pi}{2}$ as frequency increases further.

Resonance

When a system is in resonance, the periodic force is **exactly in phase** with the velocity of the oscillating system. The frequency at the maximum amplitude is the **resonant frequency**.

The lighter the damping

- The larger the maximum amplitude at resonance.
- The closer the **resonant frequency** is to the natural frequency of the system.

As the applied frequency increases from the resonant frequency

- The **amplitude decreases**.
- The **phase difference** between the displacement and periodic force increases until they are π rad out of phase.

For an oscillating system with no damping

$$\text{Resonant frequency} = \text{natural frequency}$$