13 Direct Current Circuits

13.1 Circuit Rules

Current Rules

Kirchoff's first rule: At any junction in a circuit, the total current leaving the junction is equal to the total current entering the junction.

For component **in series**:

- The current entering a component is the **same as the current leaving** the component.
  - Components do not use the current.
- The current passing through two or more components in series is the **same through each component**.

Potential Difference Rules

The potential difference between any two points in a circuit is defined as the **energy transfer per coulomb of charge** that flows from one point to the other.

- If the charge carriers lose energy, the potential difference is a **potential drop**.
- Charge carriers gain energy when they **pass through a cell**, the potential difference is a **potential rise** equal to the pd across the cell's terminals.
- For any two components in series, the total pd across all the components is equal to the **sum of the potential differences** across each component.
  - The pd across each component is the **energy delivered per coulomb charge** to that component.
  - So the sum of pds across the components is the **total energy** delivered to the components per coulomb of charge passing through them.
- The pd across components in parallel is the same.

Kirchoff's second law: For any complete loop of a circuit, the sum of the emfs round the loop is equal to the sum of potential drops around the loop.

- The statement follows from the **conservation of energy**.
- The total emf in a loop is the **total electrical energy** per coulomb produced in the loop.
- The sum of the potential drops is the **electrical energy per coulomb delivered** around the loop.

13.2 More about Resistance

Resistors in Series

Since resistors in series **pass the same current**. For two resistors R_1 and R_2 in series

- $V_1 = IR_1$
- $V_2 = IR_2$

The total pd is given by

$$V = V_1 + V_2 = IR_1 + IR_2$$

So the total resistance is given by

$$R = \frac{V}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2$$

So the **total resistance** is equal to the sum of the individual resistances.

$$R = R_1 + R_2 + R_3 + \cdots$$

Resistors in Parallel

Since resistors in parallel have the **same pd**. For two resistors R_1 and R_2 in parallel.

- $I_1 = \dfrac{V}{R_1}$
- $I_2 = \dfrac{V}{R_2}$

The **total current** through the combination is

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$

Since total current is $I = \dfrac{V}{R}$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

So the **total resistance** is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Resistance Heating

The **heating effect of an electric current** in any component is due tot he resistance of the component.

- The charge carriers **repeatedly collide** with the positive ions of the conducting material.
- There is a **net transfer of energy** from the charge carriers to the positive ions as a result of these collisions.

- The force due to the pd across the material accelerates the charge carrier until it collides with another positive ions.

Since $V = IR$, $P = IV = I^2R = \dfrac{V^2}{R}$

- So the **energy transferred to the object** by electric current in time Δt

$$\Delta E = I^2 R \Delta t$$

- The energy transfer per second to the component **does not depend on the direction** of the current.

13.3 Electromotive Force and Internal Resistance

The internal resistance of a source is the **loss of potential difference per unit current** in the source when current passes through the source.

- The internal resistance of a source is due to the **opposition to the flow of charge** through the source. This causes electrical energy produced by the source to be dissipated inside the source when the charge flows through it.
- The **electromotive force** of a source is the **electrical energy per unit charge** produced by the source.

$$\epsilon = \frac{\Delta E}{\Delta Q}$$

- The **terminal pd** of the source is the **electrical energy per unit charge** delivered by the source when it is in a circuit.

  The terminal pd is **less than the emf** whenever current passes through the source, the difference is due to the **internal resistance** of the source.

$$\epsilon = IR + Ir$$

- The **lost volts** inside the cell is equal to the difference between the cell emf and the terminal pd.
- The lost volts is the **energy per coulomb** dissipated inside the cell due to internal resistance.

Power

$$P = I\epsilon = I^2R + I^2r$$

- The peak of the **power curve** is at $R = r$.
- When a source delivers power to a load, the **maximum power** is delivered when the load resistance is equal to the internal resistance of the source.

Measurement of Internal Resistance

- Connect a **voltmeter directly across the terminals** of the cell to measure the **terminal pd**.
- A **ammeter in series** to measure the **cell current**.
- A **variable resistor** to adjust the current.
- A **fixed resistor** to limit the **maximum current** that can pass through the cell.

Measurements of terminal pd and current for a cell can be plotted as graph.

- The terminal pd is **equal to the cell emf at zero current**. Because lost volts is zero at zero current.
- The graph is a **straight line with negative gradient**.

$$V = \epsilon - Ir$$

By **comparison with the standard equation for a straight line** $y = mx + c$.

- The **y-intercept** is ϵ.
- The gradient of the line is $-r$.
- For current I_1, $V_1 = \epsilon - I_1 r$
- For current I_2, $V_2 = \epsilon - I_2 r$

$$\begin{aligned} V_1 - V_2 &= (\epsilon - I_1 r) - (\epsilon - I_2 r) \\ &= (I_2 - I_1) r \\ r &= \frac{V_1 - V_2}{I_2 - I_1} \end{aligned}$$

13.4 The Potential Divider

A potential divider consists of

- Two or more resistors in series with each other.
- A source of **fixed potential difference**.

The potential difference of the source is **divided between components**.

A potential divider can

- Supply a pd which is fixed at any value between zero and source pd.
- Supply a variable pd.
- Supply a pd that **varies with physical condition**.

Supply a Fixed PD

Consider two resistors $R_1 + R_2$ in series connected to a source of fixed pd V_0.

$$I = \frac{V_0}{R_1 + R_2}$$
$$V_2 = \frac{V0R_2}{R_1 + R_2}$$

The **ratio of pds** across each resistor is equal to the **resistance ratio** of the two resistors.

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

Supply a Variable PD

- A source pd connected to a **fixed length of uniform resistance wire**.
- Giving a variable pd between the **contact and one end of the wire**.

A **variable potential divider** is used for

- a **simple audio volume control** to change the loudness of the sound of a loudspeaker, where the **audio signal is supplied as a potential difference**.
- To vary the **brightness of a light bulb** between zero and normal brightness.

Sensor Circuits

A sensor circuit produces an output pd which **changes as a result of a change of physical variable**.

- A **temperature sensor** is a potential divider made using a thermistor and a variable resistor.
- A **light sensor** is a potential divider made using a light-dependent resistor and a variable resistor.