6 Forces in Equilibrium

6.1 Vectors and Scalars

- A **vector** is any physical quantity that has a direction as well as a magnitude.
  - Displacement, velocity, acceleration, force.
- A **scalar** is any physical quantity that is not directional.
  - Mass, density, volume, energy.

A vector can be **represented as an arrow** - the length of the arrow represents the magnitude of the vector quantity, the direction arrow gives the direction of the vector.

Distance travelled depends on the route, whereas the **direct distance** is always the same.

- **Displacement** is distance in a given direction.
- **Velocity** is speed in a given direction.

Vector Addition

Vectors can be added using a **scale diagram**.

$$OB = OA + AB$$

Vector addition gives the **overall effect** of the vectors. Adding two forces gives the **resultant** of the forces.

- The **resultant** is the combined effect of two forces.

Vectors can also be added using a **calculator**.

In general, if the two perpendicular forces are F_1 and F_2

- The **magnitude** of the resultant $F = \sqrt{{F_1}^2 + {F_2}^2}$
- The angle θ between the resultant and F_1 is given by $\tan\theta = F_2/F_1$.

Resolving a Vector into Two Perpendicular Components

Is the process of working out the **components of a vector** in two perpendicular directions given the magnitude and direction of the vector.

A force F can be resolved into two perpendicular components

- $F\cos\theta$ parallel to a line at angle θ to the line of action of the force.
- $F\sin\theta$ perpendicular to the line.

6.2 Balanced Force

When two forces act on a **point object**, the object is in **equilibrium** (at rest or moving at constant velocity) only if the two forces are equal and opposite to each other.

- The **resultant** of the two forces is zero.
- The two forces are said to be **balanced**.

When three forces act on a point object, their **resultant** is zero only if the resultant of any two of the forces is equal and opposite to the third force.

6.3 The Principle of Moments

The **moment of a force** about any point is defined as the force $\times$ perpendicular distance from the line of action of the force to the point.

- The unit of the moment of a force is the newton metre (Nm).

$$\text{The moments of a force} = F \times d$$

- An object that is not a point object is referred to as a **body**.
- Such object turns if a force is applied to it anywhere other than through its **centre of mass**.
- The **centre of mass** of a body is the point through which a single force on the body has no turning effect.
  - It is the point where we consider the weight of the body to act when study the effect of forces on the body.

The **principle of moments** states if a body is acted on by more than one force and it is in equilibrium, the **turning effects of the forces** must balance out. Consider the moments of the forces about any point

$$\text{sum of clockwise moments} = \text{sum of anticlockwise moments}$$

6.4 More on Moments

A **couple** is a pair of equal and opposite forces acting on a body, but not along the same line.

The moment of a couple = force $\times$ perpendicular distance between the lines of action of the forces

Note that the total moment is the same, regardless of the point about which the moments are taken.

6.5 Stability

If a body in **stable equilibrium** is displaced then released, it returns to its equilibrium position. E.g. a clothes hanger hanging from a support swings back to its equilibrium position.

1. The **centre of mass** of the object is directly below the point of support when the object is at rest.

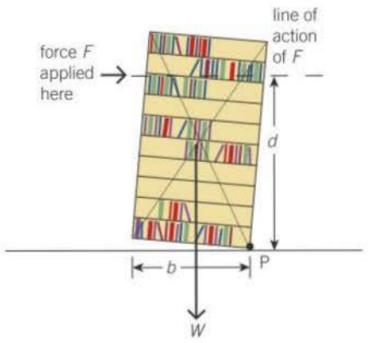
2. The **weight of the object** is considered to act at its centre of mass.
3. Thus the support force and the weight are directly equal and opposite to each other when the object is in equilibrium.
4. When the object is displaced, the line of action of the weight no longer passes through the point of support.
5. So the weight returns the object to equilibrium.

If a body is in **unstable equilibrium** is displaced slightly, it will experience a force in the direction of displacement (away from equilibrium). E.g. a plank balanced on a drum will roll off the drum when displaced slightly then released.

1. The **centre of mass** of the object is directly above the point of support when it is in equilibrium.
2. The support force is equal and opposite to the weight.
3. If displaced slightly, the centre of mass is **no longer above the point of support**.
4. The weight will therefore acts to turn it further away from the equilibrium position.

Tilting and Toppling

Tilting is where an object at rest on a surface is acted on by a force that **raises it up** on one side. To make a bookcase tilt, the force must turn it clockwise about point P. The weight of the bookcase provides an anticlockwise moment about P.



- The clockwise moment about of F about P is Fd, where d is the perpendicular distance between the line of action of the force and the pivot.
- The anticlockwise moment of W about P is $Wb/2$ where b is the width of the bookcase.

For tilting to occur

$$Fd > Wb/2$$

A tilted object will **topple over** if it is tilted too far.

- If the object is tilted so much that the line of action of **its weight passes beyond the pivot**, the object will topple over if allowed to.
- The position where **the line of action of the weight passes through the pivot** is the furthest it can be tilted without toppling.

6.6 Equilibrium Rules

When two object interact, they always exert equal and opposite forces on one another. A **free body force diagram** shows only the forces acting on the object.

For a point object acted on by three forces to be in equilibrium.

- The three forces as vectors should **form a triangle**.
- So their **overall resultant** is zero.

Condition for Equilibrium

An object in equilibrium is either **at rest** or **moving with a constant velocity**.

- The **resultant force** must be zero.
- The **principle of moments** must apply - the moments of the forces about the same point must balance out.