21 Gravitational Fields

21.1 Gravitational Field Strength

Any two masses exerts a gravitational pull on each other, but usually the force is too weak to be noticed unless at least one of the masses is very large.

The force field around a mass is called a **gravitational field strength**.

1. The mass of an object **creates a force field** around itself.
2. Any other mass placed in the field is **attracted towards the object**.
3. The second mass also has a force field around itself that **pulls on the first object** with an equal force in the opposite direction.

The **magnetic field strength** g is the force per unit mass on a small test mass placed in the field. The path which the smaller mass would follow is called a **field line** or **line of force**.

$$g = \frac{F}{m}$$

The unit of gravitational field strength is the **newton per kilogram**.

The test mass needs to be small, otherwise it might pull so much on the other object that it changes its position and alters the field.

Free Fall

The weight of an object is the force of gravity on it, $F = mg$ for an object of mass m in a gravitational field.

$$a = \frac{F}{m} = \frac{mg}{m} = g$$

Therefore g is the acceleration of a freely falling object.

The object is described as **unsupported** because it is acted on by the force of gravity alone.

Field Patterns

- A **radial field** is where the field lines are **always directed to the centre**, since the force of gravity on a small mass near a much bigger spherical mass is always directed to the centre of the larger mass, regardless of position.

  The magnitude of g in a radial field decreases with increasing distance from the massive body.

- A **uniform field** is where the gravitational field strength is the **same in magnitude and direction throughout the field**.

  The field lines are therefore **parallel** to one another and **equally spaced**.

The gravitational field strength of the Earth is radial because it falls with increasing distance. But **over small distances** compared to the Earth's radius, the change in gravitational field strength is insignificant so the field can be **considered uniform**.

21.2 Gravitational Potential

Gravitational potential energy is the energy of an object due to its position in a gravitational field.

- The position for zero GPE is at infinity - where the object is so far away that that **gravitational force on it is negligible**.
- At the surface, the GPE is negative as **work needs to be done** to escape from the field completely.

The **gravitational potential** at a point is the **work done per unit mass** to move a small test mass from infinity to that point.

$$V = \frac{W}{m}$$

The unit of gravitational potential is $\mathrm{J\,kg^{-1}}$.

The work done to move a mass from V_1 to V_2 is equal to its **change of gravitational potential energy**.

$$\Delta W = m\Delta V$$

- $\Delta E_p = mg\Delta h$ can only be applied for values of Δh that are very small compared with the Earth's radius.
- $\Delta E_p = m\Delta V$ can always be applied.

Potential Gradients

Equipotentials are surfaces of **constant potential**, so **no work needs to be done** to move along an equipotential surface.

- The equipotentials near the Earth are **circles**.
- At increasing distance from the surface, the gravitational field becomes weaker, so the **gain of GPE per metre height** becomes less.
- Away from the Earth's surface, the equipotentials for equal increases of potential are **spaced further apart**.

But near the surface **over a small region**, the equipotentials are horizontal. This is because the gravitational field over a small region is uniform.

The **potential gradient** at a point in a gravitational field is the **change of potential per metre** at that point.

For a change of potential ΔV over a small distance Δr

$$\text{Potential gradient} = \frac{\Delta V}{\Delta r}$$

Consider a test mass m being moved away from a planet. To move m by a small distance Δr in the opposite direction to the gravitational force F_{grav} on it, its gravitational potential energy is increased

- By an equal and opposite force F acting through the distance Δr.
- By an equal amount of energy equal to the work done by F

$$\Delta W = F\Delta r$$

$$\begin{aligned}
\Delta V &= \frac{F\Delta r}{m} \\
F &= \frac{m\Delta V}{\Delta r} \\
F_{\text{grav}} &= -F \\
g &= \frac{F_{\text{grav}}}{m} = -\frac{\Delta V}{\Delta r}
\end{aligned}$$

So gravitational field strength is the **negative of the potential gradient**. Where the minus sign shows that g acts in the opposite direction to the potential gradient.

- The closer the equipotentials are, the greater the potential gradient and the **stronger the field** is.
- Where the equipotentials show equal changes of potential for equal changes of spacing, the **potential gradient is constant**, so the gravitational field strength is constant and the field is **uniform**.
- The gradient is always at **right angles to the equipotentials**, so the field lines are always perpendicular to equipotentials.