20 Gases

20.1 The Experimental Gas Laws

The **pressure** of a gas is the force per unit area that the gas exerts **normally** on a surface. The unit of pressure is that **pascal**.

Boyle's Law

Boyle's law states for a **fixed mass** of gas at **constant temperature**.

$$pV = \text{constant}$$

Charles' Law

Charles' law states for a **fixed mass** of gas at **constant pressure**.

$$\frac{V}{T} = \text{constant}$$

The graph of **volume against temperature** in kelvins is a straight line through the origin.

- **Isobaric change** is any change at **constant pressure**.
- When work is done to **change the volume** of a gas energy must be transferred to keep the pressure constant.

$$\text{Work done} = p\Delta V$$

The Pressure Law

The pressure law shows how the pressure of a fixed mass of gas at **constant volume** changes with temperature.

$$\frac{p}{T} = \text{constant}$$

20.2 The Idea Gas Law

The molecules of a gas move at random with different speeds.

- The pressure of a gas on a surface is due to **gas molecules hitting the surface**. Each impact exerts a tiny force on the surface.
- **Brownian motion** - smoke particles wiggle around unpredictably because it is bombarded unevenly and randomly by individual molecules. The particle experiences forces due to these impacts which changes its velocity at random.

Molar Mass

Avogadro's constant N_A is defined as the number of atoms in exactly 12g of $^{12}_{6}$C.

$$N_A = 6.02 \times 10^{23}$$

- One **mole** of substance consisting of identical particles is defined as the quantity of substance that contain N_A particles.
- The number of moles in a given quantity of a substance is its **molarity**. The unit of molarity is the mol.
- The **molar mass** of a substance is the mass of 1 mol of that substance. The unit of molar mass is $\text{kg}\,\text{mol}^{-1}$.

The Ideal Gas Equation

An **ideal gas** is a gas that obeys **Boyle's law**.

The three experimental gas laws combine to give

$$\frac{pV}{T} = \text{constant}$$

Equal volumes of ideal gases at the same temperature and pressure contain equal number of moles.

For one mole of ideal gas

$$\frac{pV}{T} = 8.31\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$$

This value is called the **molar gas constant** R.

A graph of pV against T for n moles is a **straight line through absolute zero** and has a gradient equal to nR.

The combined gas law can be written as the **ideal gas equation**.

$$pV = nRT$$

Using $n = \dfrac{N}{N_A}$, the **Boltzmann constant** $k = \dfrac{R}{N_A}$ where N is the number of molecules.

$$pV = NkT$$

20.3 The Kinetic Theory of Gases

- **Boyle's law**: the pressure of a gas at constant temperature is increased by reducing volume.

  Because the gas molecules **travel less distance** between impacts at the walls due to the reduced volume. Therefore **more impacts per second** so **pressure is greater**.

- **Pressure law**: the pressure of a gas at constant volume is increased by raising its temperature.

  The average speed of the molecules is increased by raising its temperature. So impacts are **harder and more frequent**. So the pressure is raised.

Molecular Speeds

Molecules in an ideal gas have a **continuous spread of speed**. As long as the temperature does not change, the **distribution stays the same**.

The **root mean square** speed

$$c_{\mathrm{rms}} = \sqrt{\frac{{c_1}^2 + {c_2}^2 + \cdots + {c_N}^2}{N}}$$

If the temperature is raised, the **root mean square speed increases**, the distribution curve becomes **flatter and broader** because more molecules are moving at higher speeds.

The Kinetic Theory Equation

Assumptions about molecules:

- They are **point molecules**.
- They **do not attract** each other.
- They move about in **continual random motion**.
- Collisions are **elastic**.
- Collision with the container surface is a much shorter duration than the time between impacts.

Consider a molecule in a rectangular box.

1. Each impact of the molecule with a surface **reverses the x-component of velocity**, the **change in momentum** is $2mv_x$.
2. The **time between successive impacts** on this face is $\dfrac{2L_x}{v_x}$.
3. The **force on the surface** is

$$F = \frac{\Delta p}{\Delta t} = \frac{m{v_x}^2}{L_x}$$

4. The **pressure** is

$$p = \frac{F}{A} = \frac{m{v_x}^2}{L_x L_y L_z} = \frac{m{v_x}^2}{V}$$

The speed of the molecule is given by its three velocity components.

$$c^2 = {v_x}^2 + {v_y}^2 + {v_z}^2$$

5. For N molecules in the box, the total pressure is the sum of the individual pressures.

$$\begin{aligned} p &= \frac{m{v_{x1}}^2}{V} + \frac{m{v_{x2}}^2}{V} + \cdots + \frac{m{v_{xN}}^2}{V} \\ &= \frac{Nm}{v} \cdot \frac{{v_{x1}}^2 + {v_{x2}}^2 + \cdots + {v_{xN}}^2}{N} \\ &= \frac{Nm\,\bar{v_x}^2}{V} \end{aligned}$$

6. Because the motion of the molecules are random - there is **no preferred direction of motion**.

$$\begin{aligned} p &= \frac{Nm\,\bar{v_x}^2}{V} = \frac{Nm\,\bar{v_y}^2}{V} = \frac{Nm\,\bar{v_z}^2}{V} \\ 3p &= \frac{Nm}{V}({v_x}^2 + {v_y}^2 + {v_z}^2) = \frac{Nm{c_{\text{rms}}}^2}{V} \end{aligned}$$

Therefore

$$pV = \frac{1}{3}Nm{c_{\text{rms}}}^2$$

Kinetic Energy of Ideal Gas

$$\text{Mean kinetic energy} = \frac{1}{2}m{c_{\text{rms}}}^2 = \frac{3}{2}kT$$

So for n moles of ideal gas at temperature T.

$$\text{Internal energy} = \frac{3}{2}nRT$$