23 Capacitors

23.1 Capacitance

A **capacitor** is a device designed to store charge.

Two parallel metal plates placed near each other form a capacitor.

- When the plates are connected to a battery, electrons move through the battery.
  - Electrons are forced from the negative terminal of the battery onto one of the plates.
  - An equal number of electrons leave the other plate to return to the battery via its positive terminal.

  So each plate gains an **equal and opposite charge**.
- When we say the charge stored by the capacitor is Q, we mean one conductor stores charge $+Q$ and the other conductor stores charge $-Q$.

Charging at Constant Current

This can be achieved using a **variable resistor**, a switch, a microammeter, and a cell in series with the capacitor.

- When the switch is closed, the variable resistor is continually adjusted to **keep the microammeter reading constant**.
- At any given time t after the switch is closed, the charge Q on the capacitor.

$$Q = It$$

The **capacitance** C of a capacitor is defined as the **charge stored per unit pd**.

The unit of capacitance is the **farad** (F), equal to one coulomb per volt.

$$C = \frac{Q}{V}$$

23.2 Energy Stored in a Charged Capacitor

When a capacitor is charged, energy is stored in it because electrons are **forced onto one of its plates** and taken off the other plate. This energy is stored in the capacitor as **electric potential energy**.

1. To increase the charge on the plates by a small amount Δq from q to $q + \Delta q$. The energy stored ΔE in the capacitance is equal to the work done to force the extra charge onto the plate.
$$\Delta E = v\Delta q$$
2. $v\Delta q$ is represented by the area of the vertical strip of width Δq and height v under the line. Therefore the area of this strip represents the work done ΔE in this small step.

3. **Consider all the steps** from zero pd to the final pd V, the total energy stored is obtained by adding up the energy stored in each small step.

   E is represented by the total area under the line from zero pd to pd V, which is a triangle of height V and base length Q.

$$\begin{aligned}\text{Energy stored by the capacitor } E &= \frac{1}{2}QV \\ &= \frac{1}{2}CV^2 \\ &= \frac{1}{2}\frac{Q^2}{C}\end{aligned}$$

Energy in a Thundercloud

The thundercloud and the Earth below are like a pair of charged parallel plates.

1. Because the thundercloud is charged, an electric field exists between the thundercloud and the ground - the potential difference between the thundercloud and the ground is $V = Ed$.
2. For a thundercloud carrying constant charge Q.

$$E = \frac{1}{2}QV = \frac{1}{2}QEd$$

3. If the thundercloud raise up to a new height d', the new energy stored

$$E = \frac{1}{2}QEd'$$

4. The **increase in energy**

$$\Delta E = \frac{1}{2}QEd' - \frac{1}{2}QEd = \frac{1}{2}QE\Delta d$$

   where $\Delta d = d' - d$.

The energy stored increases because **work is done** by the force (of wind) to overcome the electrical attraction between the thundercloud and the ground. To make the charged thundercloud move away from the ground.

23.3 Charging and Discharging a Capacitor through a Fixed Resistor

When a capacitor discharges through a fixed resistor, the **discharge current decreases gradually to zero**. The current decreases gradually because the pd across the capacitor decreases as it loses charge.

$$\text{resistor current} = \frac{V}{R}$$

All three current, charge and pd **decreases exponentially**, meaning these quantities **decreases by the same factor in equal intervals of time**.

Since $V = \frac{Q}{C}$

$$I = \frac{V}{R} = \frac{Q}{CR}$$

Solving mathematically

$$\frac{dQ}{dt} = -\frac{Q}{CR}$$
$$\int \frac{1}{Q}\,dQ = -\int \frac{1}{CR}\,dt$$
$$\ln Q = -\frac{t}{CR} + C$$
$$Q = Q_0 e^{-\frac{t}{CR}}$$

The quantity RC is called the **time constant** of the circuit. The unit of RC is the second.

- At time $t = RC$ after the start of discharge, the charge falls to $e^{-1} = 0.37$ of its initial value.
- At $t = 5RC$, the capacitor is considered to be fully discharged.

Charging a Capacitor through a Fixed Resistor

When a capacitor is charged by connecting it to a **source of constant pd**, the **charging current decreases** as the capacitor charge and pd increase.

When the capacitor is fully charged

- Its pd is equal to the source pd.
- The **current is zero** because no more charge flows in the circuit.

The time constant for the circuit is the **time taken to reach 63%** of the final charge (37% more needed to fully charge).

- At any instance during the charging process

$$\text{Source pd } V_0 = \text{resistor pd} + \text{capacitor pd} = IR + \frac{Q}{C}$$

- The **initial current** $I_0 = \dfrac{V_0}{R}$ because the capacitor is initially uncharged.
- At time t after charging starts, $I = I_0 e^{\frac{-t}{RC}}$

Combining these equations give

$$V_0 = V_0 e^{\frac{-t}{RC}} + \frac{Q}{C}$$
$$\frac{Q}{C} = V_0\left(1 - e^{\frac{-t}{RC}}\right) = V$$

23.4 Dielectrics

The charge stored on the plates can be increased by **inserting a dielectric** between the plates.

Dielectrics are **electrically insulating materials** that increase the ability of a parallel-plate capacitor to store charge when a dielectric is placed between the plates of the capacitor.

Polythene and waxed paper are examples of dielectrics.

1. When a dielectric is placed between two oppositely charged parallel plates, each molecule of the dielectric becomes **polarised**.
2. The surface of the dielectric near the positive plate gains negative charge, the other surface gains positive charge.
3. The positive side of the dielectric **attracts more electrons** from the battery onto the negative plate.

   The negative side of the dielectric **pushes electrons back** to the battery from the positive plate.

The effect of a dielectric is to **increase the charge stored** in a capacitor for any given pd across the capacitor terminals - to **increase the capacitance** of the capacitor.

Relative Permittivity

The ratio of charge stored with the dielectric to the charge stored without the dielectric is defined as the relative permittivity of the dielectric substance.

$$\varepsilon_r = \frac{Q}{Q_0}$$

where Q is the charge stored by a parallel-plate capacitor when the place between the plates of the capacitor is **completely filled with the dielectric substance**.

Since $\frac{Q}{Q_0} = \frac{C}{C_0}$, the relative permittivity may be defined by the equation

$$\varepsilon_0 = \frac{C}{C_0}$$

The relative permittivity of a substance is also called its **dielectric constant**.

For a parallel-plate capacitor

$$C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

where A is the surface area of each plate.

High capacitance can be achieved by

- Making the area A as large as possible.
- Making the plate spacing d as small as possible.
- Filling the space between the plates with a dielectric which as a relative permittivity ε_r as large as possible.

Polarisation Mechanisms

The relative permittivity of a dielectric in a constant electric field is due to three different polarisation mechanisms.

- **Orientation polarisation** occurs in substances which contains molecules where covalent bonds are formed between atoms of different elements. The electrons in each covalent bond are **shared unequally between the two atoms** joined by the bonds.

  The two atoms form a **permanent electric dipole** which one atom is positively charged and the other negatively charged.

  When the electric field is applied, the two atoms in each bond are displaced in the opposite direction so the dipole **align with the field** by turning slightly.

- **Ionic polarisation** occurs in substances where ions are held together by ionic bonds.

  The oppositely-charged ions of each ionic bond are **displaced in opposite directions** when an electric field is applied. The ions of each bond forms a dipole to **align with the field**.

- **Electronic polarisation** occurs where the electrons of each atom are **displaced relative to the nucleus** of the atom when an electric field is applied.

  The electron distribution and the nucleus forms a dipole that align with the field.

In an alternating electric field, the **polar dipoles rotate** and the **non-polar dipoles oscillate** one way then the opposite way as the field strength increases and decreases.

- At **low frequencies**, the three polarisation mechanisms alternate in phase with the field.

- As frequency increases, each mechanism ceases to work due to the **inertia of the particles** involved and the **resistive forces** that opposes the motion of the dipoles.

  As a result, relative permittivity decreases as the frequency of the applied field increases.

The mass of the particles being moved by the field determines which mechanism ceases first as frequency increases - the greater the inertia, the lower the frequency it ceases.

1. Orientation polarisation ceases first.

2. Ionic polarisation next.

3. Electronic polarisation last.