11 Materials

11.1 Density

The density of a substance is defined as mass per unit volume.

$$\rho = \frac{m}{V}$$

for a certain amount of substance of mass m and volume V.

$$[\rho] = \text{kgm}^{-3}$$

Density Measurements

- A **regular solid**
  1. **Measure its mass** using a top pan balance.
  2. **Measure its dimensions** using vernier calipers.
  3. **Calculate its volume** using the appropriate equation.
  4. **Calculate its density** from mass/volume.
- A **liquid**
  1. **Measure the mass** of an empty measuring cylinder.
  2. **Pour some liquid** into the cylinder and **measure the volume** of the liquid.
  3. **Measure the mass** of the cylinder and liquid.
  4. **Calculate the mass** of the liquid.
  5. **Calculate the density** from mass/volume.
- An **irregular solid**
  1. **Measure the mass** of the object.
  2. **Immerse the object** in liquid in a measuring cylinder.
  3. **Observe the increase** of liquid level, this is the volume of the object.
  4. **Calculate the density** from mass/volume.

11.2 Springs

A **stretched spring** exerts a pull on the object holding each end of the spring, this full is referred to as the **tension** of the spring.

- The tension in the spring is equal and opposite to the force needed to stretch the spring.

Hook's law states that the force needed to stretch a spring is directly proportional to the extension of the spring from its natural length.

$$F = k\Delta L$$

where k is the **string constant** and ΔL the extension from its natural length L.

- $[k] = \mathrm{Nm}^{-1}$, the greater the value of k, the stiffer the spring is.
- The graph of F against ΔL is a straight line of gradient k through the origin.

If a spring is stretched beyond its **elastic limit**, it will not regain its initial length when the force applied to it is removed.

Springs in Parallel

If weight is supported by two springs P and Q in parallel, where the extension ΔL of each spring is the same.

- $F_P = k_P \Delta L$
- $F_Q = k_Q \Delta L$

Since the weight is supported by both springs.

$$W = F_P + F_Q = k_P \Delta L + k_Q \Delta L = k \Delta L$$

giving **effective spring constant** $k = k_P + k_Q$

Springs in Series

If a weight is supported by two springs joined end-on in series with each other, the tension in each spring is the same and equal to W.

- $\Delta L_P = \dfrac{W}{k_P}$
- $\Delta L_Q = \dfrac{W}{k_Q}$

Therefore total extension

$$\Delta L = \Delta L_P + \Delta L_Q = \frac{W}{k_P} + \frac{W}{k_Q} = \frac{W}{k}$$

giving **effective spring constant**

$$\frac{1}{k} = \frac{1}{k_P} + \frac{1}{k_Q}$$

Elastic Potential Energy

Elastic potential energy is the energy stored in a stretched spring.

- If a spring is released, the elastic energy stored in it is **transferred into kinetic energy** of the spring.

The **work done** to stretch a spring by extension ΔL from its unstretched length is $\frac{1}{2} F \Delta L$. Giving

$$E_P = \frac{1}{2} F \Delta L = \frac{1}{2} k \Delta L^2$$

11.3 Deformation of Solids

The **elasticity** of a solid material is its ability to regain its shape after it has been deformed or distorted and the forces that deformed it have been released.

- Deformations that stretches an object is **tensile**.
- Deformations that compresses an object is **compressive**.

Measurements for **tensile-extension graphs**.

1. A material is held at its upper end and **loaded by hanging weights** at its lower end.
2. A set square attached to the bottom of the weights to **measure extension** of the material.
3. The weight of the load is **increased in steps** then **decreased to zero**.
4. The **extension** of the strip of material at each step is its increase of length from its unloaded length.

The measurements can be plotted as a **tension-extension graph**.

- A **steel spring** gives a straight line in accordance with Hooke's law.
- A **rubber band** extends easily when stretched, but becomes **fully stretched** and very difficult to stretch further when it has been lengthened considerably.
- A **polythene strip** stretches easily after its initial stiffness is overcome. But after 'giving' easily, it extends little and becomes difficult to stretch.

For a wire of length L and cross section area A under tension.

$$\text{Tensile stress } \sigma = \frac{T}{A} \qquad [\sigma] = \text{Nm}^{-2}$$
$$\text{Tensile strain } \varepsilon = \frac{\Delta L}{L} \qquad \varepsilon \text{ is a ratio and has no unit}$$

1. From 0 to the **limit of proportionality**, tensile stress is proportional to tensile strain - the value of stress/strain is constant and known as the **Young's modulus** of the material.

$$\text{Young modulus } E = \frac{\sigma}{\varepsilon} = \frac{TL}{A\Delta L}$$

2. Beyond the limit of proportionality, the line curves and continue beyond the **elastic limit** to the **yield point** where the wire weakens temporarily.
    - The **elastic limit** is the point beyond which the wire is permanently stretched and suffers **plastic deformation**.
3. Beyond the yield point, a small increase in tensile stress causes a large increase in tensile strain as the material of the wire undergoes **plastic flow**.
4. Beyond the **ultimate tensile stress**, the wire loses its strength, extends and **becomes narrower** at its weakest point. Increase of tensile stress occurs due to the reduce cross section area at this point until the wire breaks.

The **ultimate tensile stress** is the maximum tensile stress, also called the **breaking stress**.

Stress-strain Curves

The **stiffness** of different materials can be compared using the gradient of the stress-strain line - equal to the **Young's modulus** of the material.

- A **brittle** material snaps without giving any noticeable yield.
- A **ductile** material can be drawn into a wire.

11.4 More about Stress and Strain

Loading/unloading Curves

Loading/unloading curves are used to study how the strength of a material changes as a result of being stretched.

1. The tension in a strip of material is increased by **increasing the weight** it support in steps.
2. At each step, the extension of the material is measured.
3. The measurements can be plotted as loading/unloading curves.

- For a **metal wire** - the loading and unloading curves are the same, provided its **elastic limit** is not exceeded, so the wire **returns to its original length** when unloaded.
  - Beyond its elastic limit, the unloading line is **parallel to the loading line**.
  - In this case the wire is slightly longer when unloaded - it has a **permanent extension**.
- For a **rubber band** - the rubber band returns to the same unstretched length, but the unloading curve is **below the loading curve** except at zero and maximum extensions.
  - The rubber band **remains elastic** as it regains its initial length.
  - But it has a **low limit of proportionality**.
- For a **polythene strip**, the extension during unloading is **greater than during loading** - the strip does not return to the same initial length when it is completely unloaded.
  - The polythene strip has a **low limit of proportionality**.
  - It suffers **plastic deformation**.

Strain Energy

The work done to deform an object is referred to as strain energy. The area under the line of a force-extension graph is equal to the work done to stretch the wire.

Metal Wire

Provided the **limit of proportionality** is not exceeded, the stretch a wire to an extension ΔL, the work done is $\frac{1}{2}T\Delta L$

$$\text{Elastic energy stored in a stretched wire} = \frac{1}{2}T\Delta L$$

Because the graph of tension against extension is the same for loading and unloading, **all energy stored in the wire can be recovered** when the wire is unloaded.

Rubber Band

- Work done to stretch the rubber band is represented by the area under the loading curve.
- Work done by the rubber band when it is unloaded is represented by the area under the unloading curve.

The **area between the loading and unloading curve** represents the difference between the energy stored in the rubber band when it is stretched and the useful energy recovered from it when it is unstretched.

The difference occurs because some of the energy stored in the rubber band becomes the **internal energy of the molecules** when the rubber band unstretches.

Polythene

As it does not regain its initial length, the areabetween the loading and unloading curves represents the internal enregy retained by the polythene when it unstretches.