6 Forces in Equilibrium

6.1 Vectors and Scalars

- A **vector** is any physical quantity that has a direction and a magnitude.
- A **scalar** is any physical quantity that is not directional.

A vector can be **represented as an arrow**. The **length** represents the magnitude and the **direction** gives the direction of the vector quantity.

- **Displacement** is distance in a given direction.
- **Velocity** is speed in a given direction.
- **Force and acceleration** are also vector quantities.

Vector Addition

$$OB = OA + AB$$

Two vectors can be added together using a **scale diagram**.

- **Resultant** is the combined effects of two forces.

For two **perpendicular forces** F_1 and F_2

- The magnitude of resultant $F = \sqrt{{F_1}^2 + {F_2}^2}$
- The angle between the resultant and F_1 is given by $\tan\theta = F_2/F_1$

A force F can be resolved into **two perpendicular components**

- $F\cos\theta$ parallel to a line at angle θ to the line of action of the force.
- $F\sin\theta$ perpendicular to the line.

6.2 Balanced Forces

An object in **equilibrium** stays at rest or moves at a constant velocity.

- When two forces act on a **point object**, the object is in **equilibrium** only if the two forces are equal and opposite to each other.
  - The **resultant** of the two force is zero.
  - The two forces are said to be **balanced**.
- When three forces act on a **point object**, the **resultant** is zero only if the resultant of any two forces is **equal and opposite to the third force**.

6.3 The Principle of Moments

The **moment of a force** about any point is defined as force $\times$ perpendicular distance from the line of the force to the point.

$$\text{moment of the force} = F \times d$$

The unit of the moment of a force is the **newton metre** (Nm).

- A **body** is an object that is not a point object.
- A body turns if a force is applied to it anywhere other than through its **centre of mass**.

The **principle of moments** states if a body is acted on by more than one force and it is in equilibrium, the turning effects of the forces must balance out.

If a body is in equilibrium

$$\text{sum of clockwise moments} = \text{sum of anticlockwise moments}$$

The **centre of mass** of a body is the point through which a single force on the body has no turning effect - it is the point where we consider the **weight of the body** to act.

6.4 More on Moments

A **couple** is a pair of equal and opposite forces acting on a body, but **not along the same line**.

$$\text{moment of a couple} = F \times d$$

where d is the perpendicular distance between the lines of action of the forces.

6.5 Stability

If a body in **stable equilibrium** is displaced then released, it returns to its equilibrium position.

- The **centre of mass** of the object is directly below the point of support when the object is at rest.
- The **weight** of the object is considered to act at its centre of mass.
- So the **support force** and the weight are directly equal and opposite to each other when the object is at equilibrium.
- When it is displaced, the line of action of the weight no longer pass through the point of support.
- So the weight returns the object to equilibrium.

If an object in **unstable equilibrium** is displaced slightly from equilibrium then released, the object experiences a force in the direction of displacement.

6.6 Equilibrium Rules

When two objects interact, they exert **equal and opposite forces** on one another. A **free body force diagram** shows only the forces acting on the object.

For a body to be in **equilibrium**

- The **resultant force must be zero**.
- The **principle of momentum must apply**.