21 Gravitational Fields

21.1 Gravitational Field Strength

- The **force of gravitational attraction** exists between any two masses.
- The force field around a mass is called a **gravitational field**.
  - Any other mass placed in the field is attracted towards the object.

The **gravitational field strength** g is the **force per unit mass** on a small test mass placed in the field.

$$g = \frac{F}{m}$$

The unit of gravitational field strength is $\mathrm{N\,kg^{-1}}$.

The path which the small mass would follow is called a **field line**.

- A **radial field** is where field lines are **always directed to the centre**.
- a **uniform field** is where the gravitational field strength is the same in direction and magnitude throughout the field.

21.2 Gravitational Potential

- **Gravitational potential energy** is the energy of an object due to its position in a gravitational field, the position for zero GPE is at infinity.
- The **gravitational potential** at a point is the **work done per unit mass** to move a small object from infinity to that point.

$$V = \frac{W}{m}$$

The unit of gravitational potential is $\mathrm{J\,kg^{-1}}$.

$$\Delta E_p = m\Delta V$$

Potential Gradients

- **Equipotentials** are surfaces of constant potential.
  - No work is done when moving along an equipotential surface.
  - In a small region, the equipotentials are **parallel to the ground**, as the gravitational field over a small region is uniform.
- The **potential gradient** at a point in a gravitational field is the **change in potential per metre** at that point.

Gravitational field strength is the negative of the potential gradient.

$$g = -\frac{\Delta V}{\Delta r}$$

The minus sign shows g acts in the opposite direction to the potential gradient.

21.3 Newton's Law of Gravitation

Kepler's third law states that the value of $\frac{r^3}{T^2}$ is the same for all planets.

Newton's law of gravitation states that the gravitational force between any two **point objects** is

- Always an **attractive** force.
- Proportional to the mass of each object.
- Proportional to $\frac{1}{r^2}$.

$$F_g = \frac{Gm_1m_2}{r^2}$$

G is the **universal constant of gravitation**.

$$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$$

21.4 Planetary Fields

The field for a spherical mass is the same as if the mass were concentrated at its centre.

For a **point mass** (or spherical mass) M

$$g = \frac{GM}{r^2} = \frac{g_s R^2}{r^2}$$

where g_s is g at the surface with radius R.

Inside the planet, $g \propto r$.

Gravitational Potential

At or beyond the surface of a spherical planet.

$$V = -\frac{GM}{r}$$

The **escape velocity** from a planet is the minimum velocity an object must be given to escape from the planet.

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

21.5 Satellite Motion

The force of gravitational attraction between each planet and the sun is the **centripetal force** that keeps the planet on hits orbit.

$$\begin{aligned}
\frac{GMm}{r^2} &= \frac{mv^2}{r} \\
v^2 &= GMr^3 \\
\left(\frac{2\pi r}{T}\right)^2 &= \frac{GM}{r} \\
\frac{r^3}{T^2} &= \frac{GM}{4\pi^2}
\end{aligned}$$

Because $\dfrac{GM}{4\pi^2}$ same for all planets, $\dfrac{r^3}{T^2}$ same for all planets.

A **geostationary satellite** orbits the Earth directly above the equator, and has a time period of 24h.

For a satellite in a circular orbit of radius r, its **total energy**

$$E = -\frac{GMm}{2r}$$