20 Gases

20.1 The Experimental Gas Laws

The **pressure** of a gas is the force per unit area that the gas exerts normally on a surface.

The unit of pressure is the **pascal**, $1\text{Pa} = 1\text{N m}^{-2}$.

The pressure of gas depends on

- Its **temperature**.
- The **volume** of the gas container.
- The **mass** of gas in the container.

Boyle's Law

Boyle's law states that for a **fixed mass** of gas at **constant temperature**.

$$pV = \text{constant}$$

For a constant temperature, the measurements plotted on a graph of **pressure against** $\dfrac{1}{\textbf{volume}}$ is a straight line through the origin.

Any change at constant temperature is called an **isothermal change**.

Charles' Law

For a **fixed mass** of gas at **constant pressure**. Charles' law states the relation between volume and temperature in kelvins can be written as

$$\frac{V}{T} = \text{constant}$$

For a constant pressure, the measurements plotted on a graph of **volume against temperature** is a straight line through the origin - no matter how much gas is used, the volume of an ideal gas is zero at **absolute zero**.

Any change at constant pressure is called an **isobaric change**.

- When work is done to change the volume of gas, energy must be **transferred by heating** to keep the pressure constant.

$$\text{Work done} = p\Delta V$$

The Pressure Law

For a **fixed mass** of gas at **constant volume**, the pressure law states the relation between pressure and temperature can be written as

$$\frac{p}{T} = \text{constant}$$

20.2   The Ideal Gas Law

The molecules of a gas **move at random with different speeds**, when a molecule collides with another molecule or with a solid surface, it bounces off **without losing speed**.

The pressure of a gas on a surface is due to the gas molecules hitting the surface.

1. Each impact causes a tiny force on the surface.

2. Because there is a very large number of impacts each second.

3. The overall result is that the gas exerts a measurable pressure on the surface.

Smoke particles **wriggle about unpredictably**

1. Because it is **bombarded unevenly** and randomly by individual molecules.

2. The particle therefore experiences forces due to these impacts.

3. Which **change its magnitude and direction** at random.

This type of motion is called **Brownian motion**, it showed the existence of molecules and atoms.

Molar Mass

- The **Avogadro constant** N_A is defined as the number of atoms in exactly 12g of the carbon isotope $^{12}_{6}C$.

$$N_A = 6.02 \times 10^{23}$$

- One **atomic mass unit** u is $\dfrac{1}{12}$ of the mass of a $^{12}_{6}C$ atom.

$$1u = 1.66 \times 10^{-27} \text{kg}$$

- One **mole** of substance consists of identical particles is defined as the quantity of substance that contains N_A particles.

- The number of moles in a given quantity is its **molarity**.

  The unit of molarity is the **mol**.

- The **molar mass** of a substance is the mass of 1 mol of that substance.

  The unit of molar mass is kg mol^{-1}

$$\text{Number of moles } n = \frac{\text{mass of substance } M_S}{\text{molar mass of substance } M}$$
$$\text{Number of molecules} = nN_A$$

The Ideal Gas Equation

An ideal gas is a gas that obeys Boyle's law.

The three experimental gas laws can be combined to give the equation

$$\frac{pV}{T} = \text{constant}$$

where T is the absolute temperature.

Equal volumes of ideal gases at the same temperature and pressure contains **equal number of moles**.

For 1 mol of any ideal gas, the value of $\frac{pV}{T} = 8.61\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$, this value is called the **molar gas constant** R - the graph of pV against against T for n moles is a **straight line through absolute zero** and has a gradient equal to nR.

For n moles of ideal gas

$$pV = nRT$$

This equation is called the **ideal gas equation**.

The **Boltzmann constant** $k = \dfrac{R}{N_A} = 1.38 \times 10^{-23}\,\text{JK}^{-1}$

$$pV = nRT$$
$$= \frac{N}{N_A}RT$$
$$= N\frac{R}{N_A}T$$
$$= NkT$$

Density of Ideal Gas

Since the mass of a substance $M_S = $ molar mass $M \times n$, and $n = \dfrac{pV}{RT}$.

$$M_S = M \times \frac{pV}{RT}$$

And the **density** of an ideal gas with molar mass M

$$\rho = \frac{M_S}{V} = \frac{pM}{RT}$$

Showing for an ideal gas at **constant pressure**

$$\rho \propto \frac{1}{T}$$

20.3   The Kinetic Theory of Gases

Experimental laws can be explained by assuming the gas consists of **point molecules** moving about at random, continually colliding with the container walls.

- **Boyle's law**: The pressure of gas at constant temperature is increased by reducing volume because the gas molecules **travel less distance between impacts** at the wall due to reduced volume.

  There are more impacts per second, so pressure is greater.

- **Charles' law**: The pressure of gas at constant volume is increased by raising temperature. The **average speed of the molecules** is increased by raising the gas temperature.

  The impacts of molecules on the container walls are **harder and more frequent**, so the pressure is raised as a result.

Molecules in an ideal gas have a **continuous spread of speeds**.

- The speed of an individual molecule changes when it collides with another gas molecule.

- But the **distribution stays the same**, as long as the temperature doesn't change.

The **root mean square** speed of the molecules

$$c_{\text{rms}} = \sqrt{\frac{c_1{}^2 + c_2{}^2 + \cdots + c_N{}^2}{N}}$$

Where $c_1, c_2, \ldots$ represent the speeds of the individual molecules.

If the **temperature is raised**.

- The molecules **move faster on average**, so the root mean square speed increases.

- The distribution curve becomes **flatter and broader** because more molecules are moving at higher speeds.

The Kinetic Theory Equation

Assumptions made about the molecules in a gas

- Molecules are **point particles** - the volume of each molecule is negligible compared with the volume of the gas.

- They **do not attract each other** - if they did the force of their impacts on the container surface would be reduced.

- They move about in **continual random motion**.

- The collisions they undergo with each other and with the container surface are **elastic collisions**.

- Each collision with the container surface is of much shorter duration than the time between impacts.

Consider **one molecule** of mass m in a rectangular box.

1. Each impact of the molecule with the (shaded) surface **reverses the x-component of velocity**. So the change in momentum is

$$\Delta p = 2mv_x$$

2. The **time between successive impacts** on this face is

$$\Delta t = \frac{2L_x}{v_x}$$

3. The **force on the molecule**, and by Newton's third law - on the surface is

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_x^2}{L_x}$$

4. The pressure of the molecule on the surface is

$$p = \frac{F}{A} = \frac{mv_x^2}{L_x L_y L_z} = \frac{mv_x^2}{V}$$

5. For N molecules in the box moving at different velocities, the **total pressure** is the sum of the individual pressures $p_1, p_2, \cdots, p_N$.

$$\begin{aligned}
p &= \frac{mv_{x1}^2}{V} + \frac{mv_{x2}^2}{V} + \cdots + \frac{mv_{xN}^2}{V} \\
&= \frac{m}{V}(v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2) \\
&= \frac{Nm\bar{v}_x^2}{V}
\end{aligned}$$

where $\bar{v}_x = \dfrac{v_{x1}^2 + v_{x2}^2 + \cdots + v_{xN}^2}{N}$

6. Because the motion of the molecules is random, there is **no preferred direction** of motion. The equation above is also true for

$$p = \frac{Nm\bar{v}_y^2}{V} = \frac{Nm\bar{v}_z^2}{V}$$

7. Since $c_{\text{rms}}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2$

$$3p = \frac{Nm}{V}(\bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2)$$

$$pV = \frac{1}{3}Nmc_{\text{rms}}^2$$

Temperature and Internal Energy

The **mean kinetic energy** of a molecule of a gas

$$\bar{E}_K = \frac{1}{2}mc_{\mathrm{rms}}{}^2$$

The higher the temperature of a gas, the greater the mean kinetic energy of a molecule of the gas.

Assuming the **mean kinetic energy** of a molecule is given by

$$\frac{1}{2}mc_{\mathrm{rms}}{}^2 = \frac{3}{2}kT$$

Then

$$mc_{\mathrm{rms}}{}^2 = 3kT$$

Substitute into

$$pV = \frac{1}{3}Nmc_{\mathrm{rms}}{}^2$$
$$= NkT$$

For an ideal gas at temperature T, the **mean kinetic energy** of a molecule of an ideal gas is $\frac{3}{2}kT$.

The **total kinetic energy** for an ideal gas is its **internal energy**

$$\text{Internal energy} = \frac{3}{2}NkT = \frac{3}{2}nRT$$