23 Capacitors

23.1 Capacitance

A **capacitor** is a device designed to store charge, two parallel metal plates placed near each other form a capacitor.

- A capacitors consists of two **conductors insulated from each other**.
- When connected to a battery, one plate gains an equal number of electrons lost by the other plate.
- When the charge stored in the capacitor is Q, one conductor store charge $+Q$ and the other $-Q$.

The capacitance of a capacitor is defined as **charge stored per unit pd**.

$$C = \frac{Q}{V}$$

The unit of capacitance is the **farad**, equal to one coulomb per volt.

23.1.1 Energy Stored in a Charged Capacitor

When a capacitor is charged, **energy is stored in it** because electrons are forced onto one of its plates and taken off the other plate.

$$\text{Energy stored by a capacitor } E = \frac{1}{2}QV$$

Other forms of the equation are $E = \frac{1}{2}CV^2$ and $E = \frac{1}{2}\frac{Q^2}{V}$.

23.2 Charging and Discharging a Capacitor through a Fixed Resistor

When a capacitor discharges through a fixed resistor, the **discharge current decreases gradually** to zero, this is because the **pd across the capacitor decreases** as it loses charge.

So the **resistor current** $= \frac{V}{R}$.

Both current, charge and pd **decreases exponentially** - they **decrease by the same factor** in equal intervals of time.

Since $V = \dfrac{Q}{C}$

$$\begin{aligned}
\frac{dQ}{dt} &= -\frac{Q}{CR} \\
\int \frac{1}{Q} dQ &= -\int \frac{1}{CR} dt \\
\ln Q &= -\frac{t}{CR} + C \\
Q &= Q_0 \exp\left(-\frac{t}{CR}\right)
\end{aligned}$$

The quantity RC is called the **time constant** of the circuit.

Charging a Capacitor through a Fixed Resistor

When a capacitor is charged by connecting it to a source of constant pd, the **charging current decreases** as the capacitor charge and the pd increase.

When the capacitor is fully charged

- The pd is equal to the source pd.
- The current is zero because no more charge flows.

$$\begin{aligned}
I &= I_0 \exp\left(-\frac{t}{RC}\right) \\
V &= V_0 \left(1 - \exp\left(-\frac{t}{RC}\right)\right)
\end{aligned}$$

23.3 Dielectrics

Dielectrics are **electrically insulating materials** that increase the ability of a parallel-plate capacitor to store charge when placed between the plates.

When a dielectric is placed between two oppositely charged parallel plates connected to a battery.

1. Dielectric substances made of **polar molecules** are already polarised, but lie in **random directions**.
2. These molecules turn when the dielectric is placed between the charged plates because their electrons are attracted slightly to the positive plates.
3. Therefore, the surface of the dielectric near the positive plate **gains negative charge**, the other surface gains positive charge.
4. As a result, more charge is stored on the plates because

The positive side of the dielectric **attracts more electrons** from the battery onto the negative plate.

Similar reasoning for the positive plate.

The effect of a dielectric is to **increase the capacitance** of the capacitor.

Relative Permittivity

The ratio of the charge stored with the dielectrics relative to the charge stored without the dielectric is defined as the relative permittivity of the dielectric substance.

$$\begin{aligned}\varepsilon_r &= \frac{Q}{Q_0} \\ &= \frac{C}{C_0}\end{aligned}$$

The capacitance for a parallel-plate capacitor

$$C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

A large capacitance can be achieved by

- Making area A as large as possible.
- Making the plate spacing d as small as possible.
- Filling the space between the plate with a dielectric which has a ε_r as large as possible.