17 Motion in a Circle

17.1 Uniform Circular Motion

An object rotating at a steady rate is said to be in **uniform circular motion**.

Consider a point on the perimeter rotating at a steady speed.

$$\begin{aligned} f &= \frac{1}{T} \\ v &= \frac{2\pi r}{T} \\ &= 2\pi r f \end{aligned}$$

- The **angular displacement** of an object in time Δt is

$$\Delta\theta = \frac{2\pi\Delta t}{T} = 2\pi\Delta t f$$

- The **angular speed** is defined as the angular displacement per second.

$$\omega = \frac{\Delta\theta}{t} = \frac{2\pi}{T} = 2\pi f$$

The unit of ω is the **radian per second**.

17.2 Centripetal Acceleration

- The velocity of an object in **uniform circular motion** at any point along its path is **tangent to the circle at that point**.
- The **direction of velocity changes** continually as the object moves along the circular path.
- This change in direction of velocity is **towards the centre of the circle**.

This acceleration is called **centripetal acceleration**.

$$a = \frac{v^2}{r} = \omega^2 r$$

Proof

1. Consider an object in circular motion at speed v moving in a short time interval δt from position A to B along the perimeter of a circle of radius r.

$$\mathrm{AB} = \delta s = v\delta t$$

2. The line from the object to the centre of the circle at C turns through angle $\delta\theta$ when the object moves from A to B, the velocity direction of the object turns through the same angle $\delta\theta$.

3. The change of velocity δv is $v_B - v_A$.
4. The triangle ABC and the velocity vector triangle have the same shape because they both have two sides of equal length with the same angle $\delta\theta$ between the two sides.

When δv is small

$$\begin{aligned} \frac{\delta v}{v} &= \frac{\delta s}{r} \\ &= \frac{v\delta t}{r} \\ \frac{\delta v}{\delta t} &= \frac{v^2}{r} \end{aligned}$$

Centripetal Force

To make an object move on a circular path, it must be acted on by a resultant force that **changes its direction of motion**.

The **resultant force** on an object moving around a circle at constant speed is called the **centripetal force**, it acts in the same direction as the centripetal acceleration - towards the centre of the circle.

- For an object swung around the end of a string, the **tension in the string** provides the centripetal force.
- For a satellite moving around the Earth, the **force of gravity** between the satellite and the Earth is the centripetal force.
- For a planet moving around the sun, the **gravity on the planet** due to the sun is the centripetal force.
- For a capsule on a ferris wheel, the **resultant of the support force and its weight** is the centripetal force.
- For charged particles travelling through a magnetic field in a circular path, the **magnetic force** on the moving charged particles is the centripetal force.

$$F = \frac{mv^2}{r} = m\omega^2 r$$