

Digital Electronics

Definition

Abstraction is the hiding of data/complexity when they are not important, only expose the things needed to implement a particular task.

Layers of Abstraction

Layer	Examples
Application software	Programs
Operating systems	Device drivers
Architecture	Instructions, registers
Microarchitecture	The way the processor implements the architecture
Logic elements	Adders, memories
Digital circuits	Gates and stuff
Devices	Transistors
Physics	Electrons, quantum mechanics, Maxwell's equations

This course starts at the **devices** layer.

To design something on the stack, you need to know the layer below to implement it, and the layer above to see how the thing you design is used.

Combinatory Logic

Definition

Combinatory output depends only on current input.

- It has no memory.
- The same input always gives the same output, so is entirely predictable.

Boole algebra is used for simplifying logic so less gates are used, so is cheaper.

Logic Gates

Definition

Logical variables can only take on two values.

In electronic circuits, they are represented by

- High voltage for 1
- Low voltage for 0

Using only 2 voltages allows circuits to have greater immunity to signal corruption caused by interference.

Definitions

- **Logical circuits** have one or more inputs.
- Basic logic circuits are known as **gates**.

Logic gates are represented in **symbol**, **truth tables** and **algebra**.

- NOT \bar{a}

- The triangle means **buffer**.
- The circle means **complement**.
- AND $a \cdot b$.
 - A lot of these gates can be extended to have more than 2 inputs.
- OR $a + b$
- XOR $a \oplus b$
- NAND $\overline{a \cdot b}$
- NOR $\overline{a + b}$

Boolean Algebra Laws

AND takes precedence over OR.

OR	AND
$a + 0 = a$	$a \cdot 0 = 0$
$a + a = a$	$a \cdot a = a$
$a + 1 = 1$	$a \cdot 1 = a$
$a + \bar{a} = 1$	$a \cdot \bar{a} = 0$

Commutation

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

Association

$$(a + b) + c = a + (b + c)$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Distribution

$$a \cdot (b + c + \dots) = a \cdot b + a \cdot c + \dots$$

$$a + (b \cdot c \cdot \dots) = (a + b) \cdot (a + c) \cdot \dots$$

Absorption

$$a + (a \cdot b) = a$$

$$a \cdot (a + b) = a$$

Consensus Theorem

$$a \cdot b + \bar{a} \cdot c + b \cdot c = a \cdot b + \bar{a} \cdot c$$

$$(a + b) \cdot (\bar{a} + c) \cdot (b + c) = (a + b) \cdot (\bar{a} + c)$$

Technique

1. Expand each term until it includes one instance of each variable.
2. Then simplify the terms by cancelling terms.

$$x \cdot y \rightarrow x \cdot y \cdot \bar{z} + x \cdot y \cdot z \text{ to include } z.$$

DeMorgan's Theorem

$$\overline{a + b + c + \dots} = \bar{a} \cdot \bar{b} \cdot \bar{c} \cdot \dots$$

$$\overline{a \cdot b \cdot c \cdot \dots} = \bar{a} + \bar{b} + \bar{c} + \dots$$

Proof

First prove for 2 variables by truth table.

a	b	$\overline{a + b}$	$\overline{a} \cdot \overline{b}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Extend to more variables by induction:

$$\begin{aligned}\overline{(a + b) + c} &= \overline{a + b} \cdot \overline{c} \\ &= \overline{a} \cdot \overline{b} \cdot \overline{c}\end{aligned}$$

Sometimes we wish to use NAND or NOR gates since they are usually simpler and faster. We can use DeMorgan's law to replace AND and OR to NAND and NOR.

Technique: Bubble Logic

- 2 consecutive bubble operations cancel.
- Bubbles change an AND into an OR when they pass through a gate (vice versa).

Logic Minification**Normal Forms**

A **minterm** contains all input variables of boolean function f so f and itself both evaluate to 1.

$x y z$	f	minterm
0 0 0	1	$\overline{x} \cdot \overline{y} \cdot \overline{z}$
0 0 1	0	
0 1 0	0	
0 1 1	1	$\overline{x} \cdot y \cdot z$
...

Definition

A boolean function f can be written in **disjunctive normal form** by expressing it as the *disjunction* (OR) of its minterms. (Sum of products)

$$f = \overline{x} \cdot \overline{y} \cdot \overline{z} + \overline{x} \cdot y \cdot z + \dots$$

As an inverse to minterms, **maxterm** is all the input variables where f evaluates to 0.

$x y z$	f	maxterm
0 0 0	1	
0 0 1	0	$\overline{x} \cdot \overline{y} \cdot z$
0 1 0	0	$\overline{x} \cdot y \cdot \overline{z}$
0 1 1	1	

x	y	z	f	maxterm
...

Definition

Conjunctive normal form is where the function is written as the product of sums.

$$f = (x + y + z) \cdot (x + \bar{y} + z) \cdot \dots$$

The CNF for f can be found by writing the DNF for \bar{f} (maxterms of f), then use DeMorgan's laws.

$$\bar{f} = \bar{x} \cdot \bar{y} \cdot z + \bar{x} \cdot y \cdot \bar{z} + \dots$$

$$f = (x + y + \bar{z}) \cdot (x + \bar{y} + z) \cdot \dots$$

K-Maps

Both DNF and CNF are not simplified, K-maps are useful for simplifying logical expressions of up to 5 variables.

SOP (Sum of Power) Simplification

1. Convert the truth table to K-map by plotting the minterms on the table in **grey code** order, so adjacent cells are logically next to each other.

		yz			
x		00	01	11	10
	0	1	1	1	1
	1			1	

2. Group the minterms, the larger the groups the better.
 - Group sizes have to be a power of 2.
 - Groups can wrap around edges.

		yz			
x		00	01	11	10
	0	1	1	1	1
	1			1	

So the simplified function is $f = \bar{x} + y \cdot z$

The simplified expression is in SOP form, suitable for implementations using AND and OR gates.

POS Simplification

1. Find a SOP Simplification for \bar{f} .
2. Use DeMorgan's to find a POS simplification for f .

This is suitable for implementing using NOR gates.

Don't Care Conditions

Some combinations will never happen, we can declare those *don't care* conditions.

- They are treated as 0 or 1 depending on which gives the simpler results.
- We write them as **X** in K-map.

Definitions

Keyword	Meaning
Cover	A term covers a minterm if the minterm is part of the term.
Prime implicant	A term that cannot be further combined.
Essential prime implicant	A term that covers a minterm that no other prime implicant covers.
Covering set	A minimum set of prime implicants which covers all minterms.

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Q-M Method

1. List all minterms and *don't care* terms, group them by the number of 1s.

- Minterms: 4, 5, 6, 8, 9, 10, 13
 - *Don't care* terms: 0, 7, 15
- 0000
0100
1000

0101
0110
1001
1010

1101
0111
1111

2. If two terms differ by 1 digit (e.g. 0000 and 0100), merge them and write it on the next column (e.g. 0-00 where “-” is a wildcard), put a tick next to them. Merge all terms that can be merged, including previously ticked terms.

Repeat for the 2nd column, and write the merge terms to the 3rd column, until no more terms can be merged.

Column 1	Column 2	Column 3
0000 ✓	0-00	01--
	-000	
0100 ✓		-1-1
1000 ✓	010- ✓	
	01-0 ✓	
0101 ✓	100-	
0110 ✓	10-0	
1001 ✓		
1010 ✓	-101 ✓	
	01-1 ✓	
1101 ✓	011- ✓	
0111 ✓	1-01	
1111 ✓	11-1 ✓	
	-111 ✓	

3. The unmergable terms (no ticks beside them) are the **prime implicants**. We need to get rid of some of those terms to find the **mincover**.

Create an **implication chart** with the **prime implicants** and the **minterms**. Put an X where the minterm can be expressed as the prime implicant. (e.g. $4_{10} = 0100_2$ can be expressed as $0-00$)

	4	5	6	8	9	10	13
0-00	X						
-000				X			
100-				X	X		
10-0				X		X	
1-01					X		X
01--	X	X	X				
-1-1		X					X

4. Look for **essential prime indicants**: 6 is only covered by 01--, and 10 only by 10-0, they are essential prime indicants. Cross the two row out, also cross out the numbers they cover.

	4	5	6	8	9	10	13
0-00	X						
-000				X			
100-				X	X		
10-0				X		X	
1-01					X		X
01--	X	X	X				
-1-1		X					X

5. Cross out as few rows as possible to cover the remaining (uncovered) numbers.

	4	5	6	8	9	10	13
0-00	X						
-000				X			
100-				X	X		
10-0				X		X	
1-01					X		X
01--	X	X	X				
-1-1		X					X

The prime implicants we picked are 10-0, 1-01 and 01--, giving the simplification.

$$f = a \cdot \bar{b} \cdot \bar{c} + a \cdot \bar{c} \cdot d + \bar{a} \cdot b$$

Binary Adders

We are doing the **compositional approach**, where we put half adders and full adders together to build a **ripple carry adder**.

Half Adder

Adds two bits together.

By inspection or simplifying minterms:

$$\text{sum} = a \oplus b$$

$$c_{\text{out}} = a \cdot b$$

a	b	c_{out}	sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Full Adder

$$\text{sum} = a \oplus b \oplus c_{\text{in}}$$

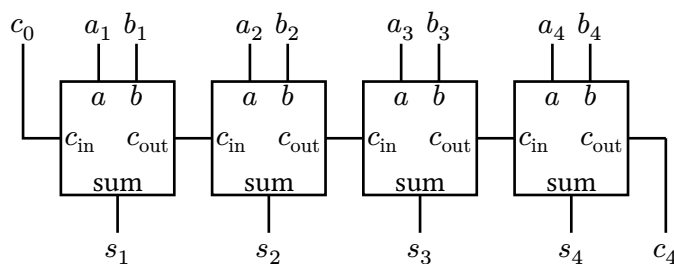
$$c_{\text{out}} = a \cdot b + c_{\text{in}} \cdot (a + b)$$

c_{in}	a	b	c_{out}	sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0

c_{in}	a	b	c_{out}	sum
1	0	0	0	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Ripple Carry Adder

Cascade multiple of these adders together to make a ripple carry adder.



A 4 bit adder has 4 full adders.

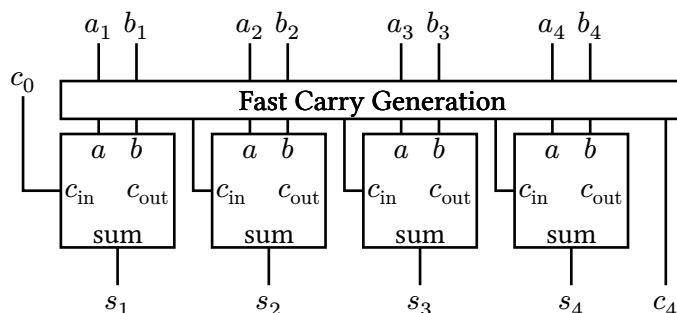
- c_0 propagates from left to right.
- Physically, they have finite propagation delay, so there is a delay between the inputs and outputs.

If we complement a then set c_0 to 1, we have $s = b - a$.

Speeding Things Up

- Design the adder as a single block of 2-level combinational logic with $2n$ inputs and n outputs.
 - Low delay.
 - Truth table is massive, and simplification is complicated. It is very complex to design.

Fast Carry Generation



c_{in}	a	b	c_{out}
?	0	0	0
?	0	1	?
?	1	0	?
?	1	1	1

- If $\overline{a_i} \cdot \overline{b_i} = 1$ then $c_{\text{out}} = 0$
- If $a_i \oplus b_i$ then $c_{\text{out}} = c_{\text{in}}$
- If $a_i \cdot b_i$ then $c_{\text{out}} = 1$

Sometimes, the carry bit may be known before c_{in} is given.

If that is not possible, the main speedup is that carry generation happens in parallel with the adders.

- Lots of gates are used in carry generation.
- Usually two 4-bit carry generators are used in an 8-bit adder.

Multilevel Logic

The ripple adder is a multilevel logic - as it cascades, it forms many levels.

- It's difficult to buy large logic gates, we can use multilevel logic to expand the input.
- But it can introduce delays and **hazards**.

Reducing Number of Gates

Simplified SOP expressions requires a lot of gates to do.

The logic below uses 9 literals, 7 gates, 2 levels.

$$\begin{aligned} z &= a \cdot d \cdot f + a \cdot e \cdot f + b \cdot d \cdot f + b \cdot e \cdot f + c \cdot d \cdot f + c \cdot e \cdot f + g \\ &= (a \cdot d + a \cdot e + b \cdot d + b \cdot e + c \cdot d) \cdot f + g \\ &= (a + b + c) \cdot (d + e) \cdot f + g \end{aligned}$$

We have recursively factored out common literals and express z in **two-level form**.

$$\begin{aligned} x &= a + b + c \\ y &= d + e \\ z &= x \cdot y \cdot f + g \end{aligned}$$

It uses 9 literals, 4 gates, 3 levels. The more levels the bigger the propagation delay.

Hazards

Definition

Hazards are brief changes in output (when there is a change in input), also called a **glitch**.

Type	Description
Static hazard	Output undergoes momentary transition when one input changes when it is supposed to remain unchanged.
Static 1 hazard	Output should stay at 1, but as signal changes it briefly drops to zero.
Static 0 hazard	The opposite of a static 1 hazard.
Dynamic hazard	Output changes more than once as opposed to just once.

This is due to the difference in **propagation delay** between layers.

Removing Hazards

1. Plot the function on a K-map.
2. If the square it is moved from and to are not in the same group, there is a hazard.
3. Add an extra term containing the two squares to remove the static 1 hazard.

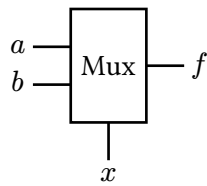
To remove a static 0 hazard, draw a K-map of the complement of the output.

Multiplexers

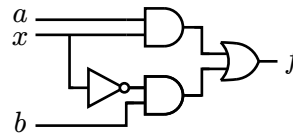
Definition

Multiplexers (mux/selector) chooses 1 of many inputs to output according to the control input.

2-to-1 Selector



a	b	x	f
X	b	0	b
a	X	0	a



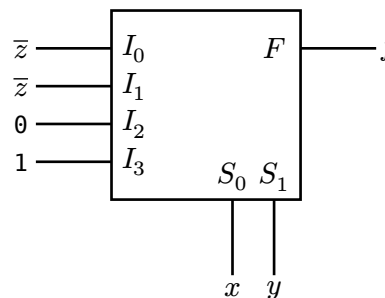
You can also express it as the sum of minterms with using a full truth table.

- n -to-1 mux is possible - an 8:1 mux requires 3 control lines.
- The mux is also called a hardware lookup table.

Sometimes it is possible to use less control lines if we use the logic variables as control input.

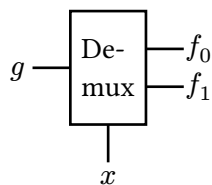
a	b	x	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

xy	f
00	\bar{z}
01	\bar{z}
10	0
11	1

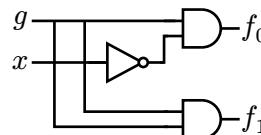


Demultiplexer

A single output is directed to exactly one of its inputs.



g	x	f_0	f_1
g	0	g	0
g	1	0	g



Larger demultiplexers are possible, e.g. a 3:8 demux.

A **decoder** is a demux where g is permanently set to 1, so only one output is 1 at any time. An **enabler** enables 1 out of n logical subsystems.

If the number of pins is limited, using a decoder/multiplexer is essential.

- Without decoder: you need 8 pins to control 8 subsystems.
- With decoder: you need 3 pins to control 8 subsystems.

We can also create any combinational logic block using the decoder. By OR-ing the outputs that corresponds to the minterms of the logic.

ROM

ROM is a storage device that can be

- Written into once
- Read at will
- Non-volatile
- Essentially a lookup table: n output lines specify the address of location holding m -bit data words.

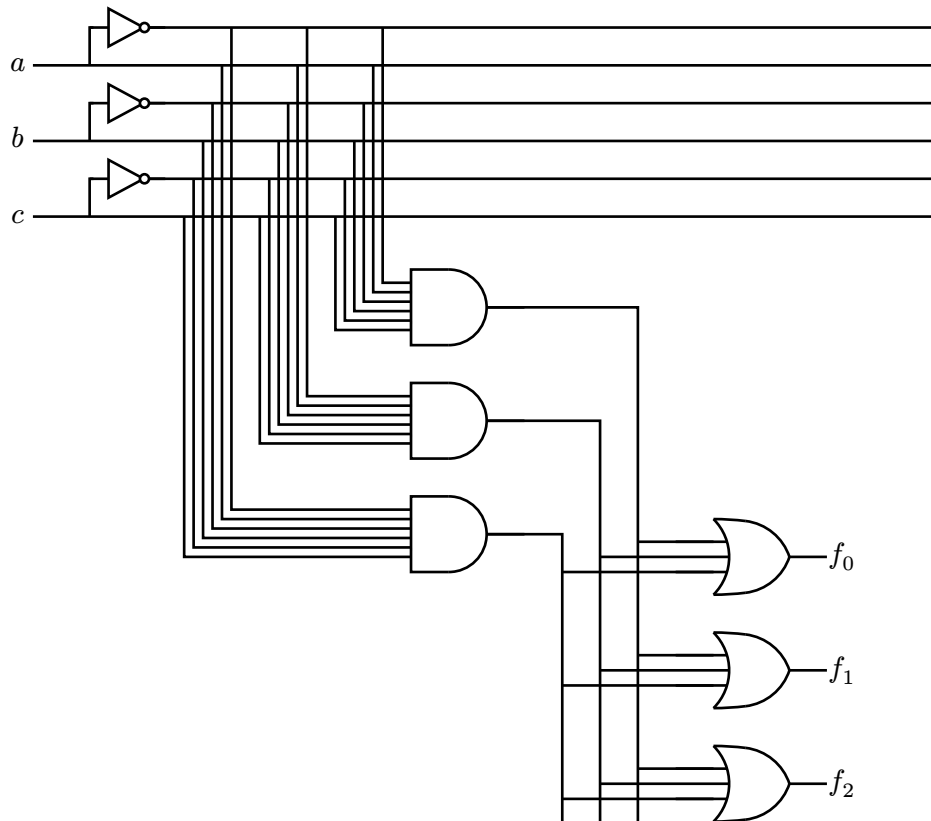
The ROM has 2^n possible locations.

We can create any combinational logic by holding all the minterms in ROM and output the stored value.

- No simplification needed.
- Reasonably efficient if lots of minterms need to be generated.
- Can be inefficiently large if many spaces are zero (there are very few minterms).

Programmable Logic Array

In a PLA, only the required minterms are generated using the **AND-plane** and **OR-plane**.



The PLA is programmed by selectively removing connections from the AND-plane and OR-plane.

Programmable Array Logic

To simplify design, the OR-plane is not programmable. There are instead multiple AND-planes each connected to an OR gate.

Memory Applications

Other memory devices includes:

- Non-volatile memory by ROMs and flash.
- Volatile memory offered by static RAM or dynamic RAM (much denser than SRAM, but has to be refreshed regularly).

Memory is connected to the CPU using **buses**.

Definition

Buses are a bunch of wires in parallel.

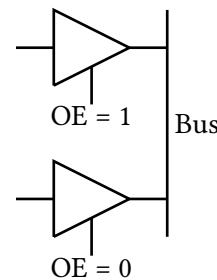
- The **address bus** specify the memory location being accessed.
- The **data bus** conveys data to and from that location.

Using Multiple Memory Devices

More than 1 memory device can be connected to the same bus wires. **Tristate buffers** controls which devices to enable by disconnecting the device from the bus when not selected.

The tristate buffers are controlled by **output enabled** (OE) control signals, the other control signals are:

- **Write enable** (WE) determines whether data is written or read.
- **Chip select** (CS) determines if the chip is activated, otherwise the chip is powered down to conserve power.



Sequential Logic

... is the end to combinational logic.

- **Combinational logic** depends only on the condition of the latest inputs.
- **Sequential logic** also depend on earlier inputs.

Definitions

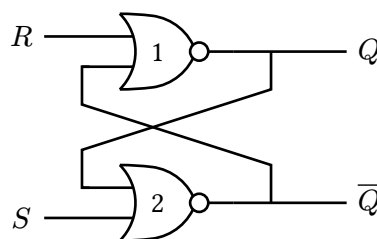
- **Memory** stores data from earlier.
- A snapstop of memory is called **state**.
- 1 bit memory is called a **bistable**.

Flip-flops and **latches** are are implementations of bistable.

RS-Latch

The RS-latch is a memory element with 2 inputs. Where Q is the current state, and Q' the next state.

S	R	Q'	\bar{Q}	Comment
0	0	Q	\bar{Q}	hold
0	1	0	1	reset
1	0	1	0	set
1	1	X	X	illegal



- Consider $R = 1$ and $S = 0$:
 - Gate 1 is 0, so Q is 0.
 - Gate 2 gives complement of gate 1, so 1.
- Consider $R = 0$ and $S = 0$:
 - Gate 1 gives the complement of gate 2.
 - Gate 2 gives the complement of gate 1, which is the hold condition.

State Transition Diagrams

1. Create a truth table of the RS-latch.

Q	S	R	Q'	Comment
0	0	0	0	hold
0	0	1	0	reset
0	1	0	1	set
0	1	1	0	illegal

Q	S	R	Q'	Comment
1	0	0	1	hold
1	0	1	0	reset
1	1	0	1	set
1	1	1	0	illegal

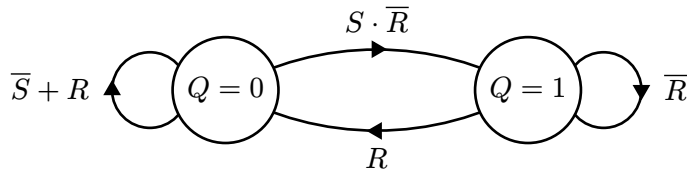
2. Consider all cases where $Q = 1$ and $Q' = 1$.

Q	S	R	Q'
1	0	0	1
1	1	0	1

So when $Q = 1$, the next state $Q' = 1$ if

$$\bar{S} \cdot \bar{R} + S \cdot \bar{R} = \bar{R}$$

3. Repeat this for all other state transitions, we get.



Transparent D-Latch

The output of a transparent D-latch will only change when there is a global **enable signal** called the system clock. This makes it easier to design large scale systems, as opposed to letting the circuits change state when ever the input changes.

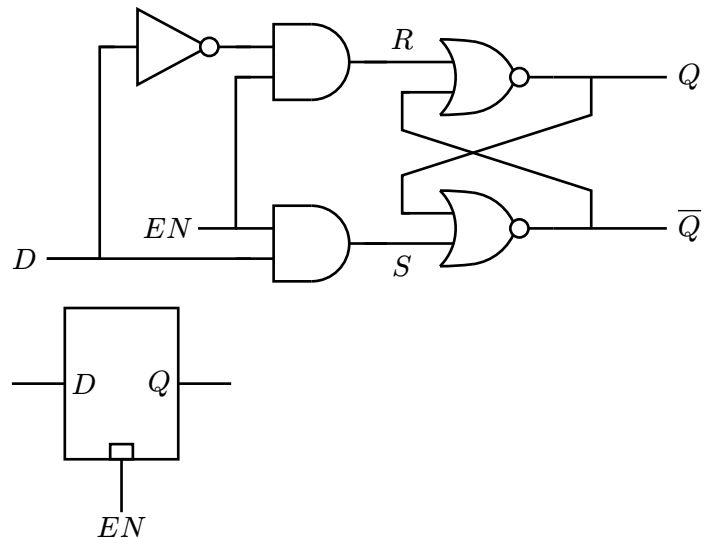
Definition

The **system clock** produces square wave signals at a frequency.

- Imposes order on state changes.
- Allows a lot of states to update simultaneously.

Modify the RS-Latch so the signal only change there is an enabling signal.

- There is only 1 input, R and S are complement of each other, so we can never set both of them to 1, avoiding the unwanted combination.



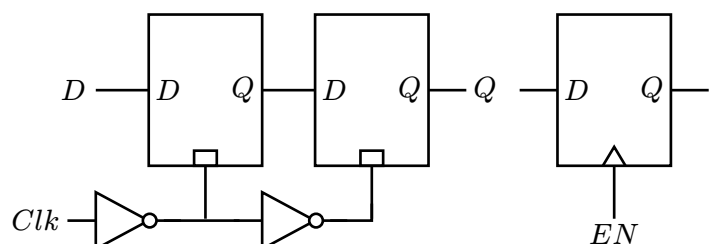
The square box means it is **level activated**. When it is enabled, it is in **transparent mode**.

- $Q = D$ if $EN = 1$
- Q remains in previous state if $EN = 0$

Master Slave Flip-Flops

The output changes only on a rising edge.

- It is much simpler to design sequential circuits if outputs only changes on rising or falling edges.
- In lab we use a truly F-F device (not affected by propagation delay).



Other Types of Flip-Flops

- The **JK F-F**: a clocked RS F-F that has 2 outputs Q and \bar{Q} . The illegal state now represents **toggle**.

- The **T F-F**: similar to the D F-F but there are 2 outputs.

Asynchronous Inputs

It is common for F-Fs to have additional asynchronous inputs which are independent of clock signals.

- Reset/clear: set Q to 0
- Preset/set: set Q to 1

Timing

The transparent D-latch requires

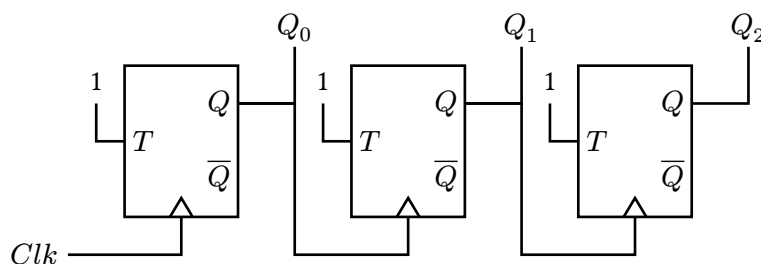
- D to change before a minimum **setup time** t_{su} before the clock signal.
- The data is held for **hold time** t_h after the clock edge.
- Then the output will change in **propagation delay** t_p after the clock edge.

Flip-Flops Applications

Definition

A **counter** is a clock sequential circuit that goes through a predetermined sequence of states.

A **ripple counter** can be made by cascading edge triggered T-type F-Fs operating in toggle mode.



A string of n F-Fs can have 2^n states.

Counters are used for

- Counting.
- Producing a delay of a particular duration.
- Generating sequences.
- Dividing frequencies by 2.

A ripple counter is not a synchronous device as the clock to the next stage comes from the previous stage, in a true synchronous device, all the clock input to the flip flops come from the clock.

- Output does not change synchronously - hard to know when the output is actually valid.
- Propagation delay builds up, limiting the maximum clock speed before **miscounting** occurs.

Where Q_0 changes to the next state before Q_2 manages to change to the current state.

BCD Counters

To count a number that is not a power of 2 (e.g. 10), we need

- An F-F with a clear/reset signal.
- An AND gate to detect the count of 10 and use its output to reset the F-F.

Synchronous Counters

We can identify a synchronous design if all F-F clock inputs are connected to the clock signal, so they all change at the same time.

The **excitation table** shows the input required for a particular transition. Note that D is the input we want to create, and Q' is caused by D .

$Q_2 Q_1 Q_0$	$Q_2' Q_1' Q_0'$	$D_2 D_1 D_0$
000	001	001
001	010	010
010	011	011
011	100	100
100	101	101
101	110	110
110	111	111
111	000	000

By inspection, we can see that $D_0 = \overline{Q_0}$ and $D_1 = Q_1 \oplus Q_0$.

To find out D_2 , we need to draw a K-map.

		$Q_1 Q_0$			
Q_2		00	01	11	10
	0			1	
	1	1	1		1

$$\begin{aligned}
 D_2 &= Q_2 \cdot \overline{Q_1} + Q_2 \cdot \overline{Q_0} + \overline{Q_2} \cdot Q_1 \cdot Q_0 \\
 &= Q_2 \cdot (\overline{Q_1} + \overline{Q_0}) + \overline{Q_2} \cdot Q_1 \cdot Q_0
 \end{aligned}$$