Computer Graphics

Computer graphics is part of a larger field that is visual computing, this includes

- Computer graphics and computer vision.
- Image capture and image display.

Computer graphics includes:

- User interface
- 3D renders
- AR/VR
- Special effects, including image editing

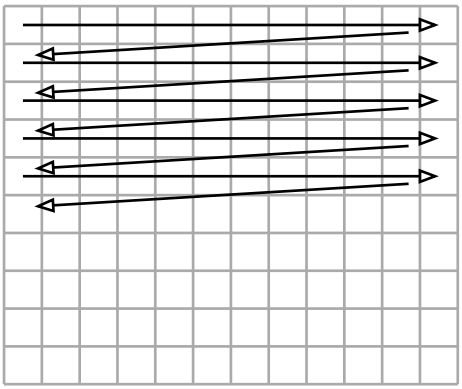
Image as a 2D Array

From a discrete perspecitive, an image is a **2D array of pixels**. The memory is not two-dimensional, how do we store it in memory? (how to linearise an image?)

Row Major

The first row is stored before the second row. The index of a pixel

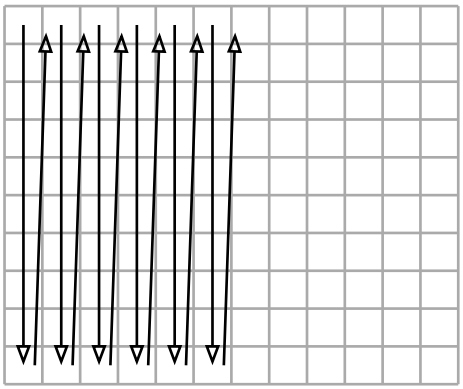
$$i(x, y) = x + y \cdot n_{\text{cols}}$$



Column Major

The first column is stored before the second column. The index of a pixel

$$i(x, y) = x \cdot n_{\text{rows}} + y$$



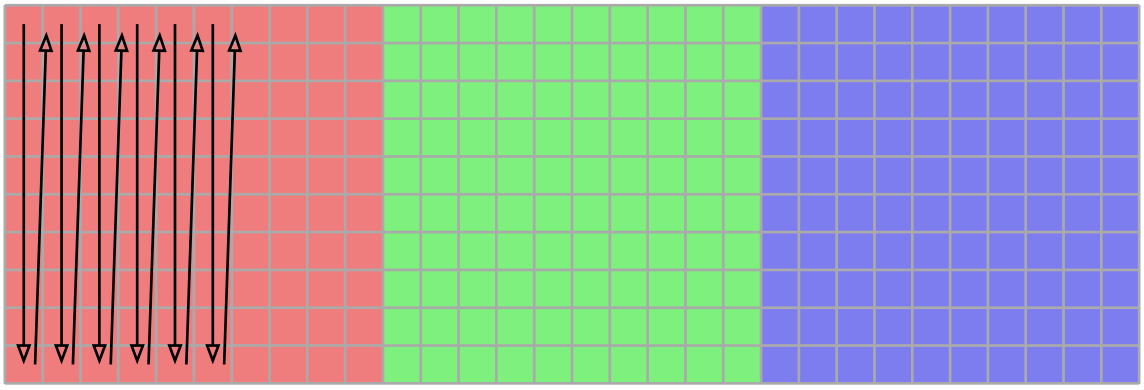
CRT TVs draw images from top to bottom, which is why most APIs represent images in row major.

RGB Representations

- **Interleaved, row major** stores all colours of a pixel next to each other, before moving on to the next pixel.

$$i(x, y, c) = 3x + 3y \cdot n_{\text{cols}} + c$$

- **Planar, column major** lays the R, G and B image next to each other, then use column major on the combined image.



$$i(x, y, c) = x \cdot n_{\text{rows}} + y + cxy$$

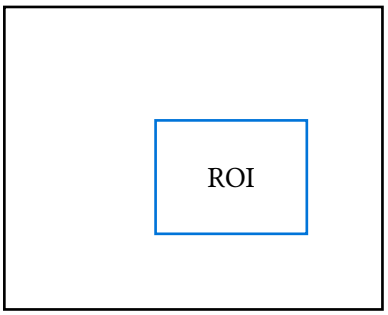
The general formula is $i(x, y, c) = xs_x + ys_y + cs_c$, where s_x is the number of pixels for each step in the x direction.

Padded Image
An algorithm that operates on neighbouring pixels need a padded image so every pixel in the image has neighbouring pixels.

The pixels in the region of interest are given by

$$i(x, y, c) = i_{\text{first pixel}} + xs_x + ys_y + cs_c$$

Where $i_{\text{first pixel}}$ is the index of the top left pixel of the ROI.



Pixels
Pixel is short for **picture element**.

Each pixel consist of 3 values R, G, B describing the colour.
- 0-255 for each colour, because it is convenient to use 1 byte per colour.
- It is also the number of colours we need to have no visible artifacts.

Colour Banding

Definition
Colour banding is when there are not enough bits to reprsent colour.

This is very visible to our eyes due to the mach band/chevreul illution where our eyes enhances the contrast of an edge, making the band more visible than it is.

Definition
Dithering adds noise to reduce banding: randomly add a value to the pixel value, the probability determined by the colour of the pixel.

Image as a Continuous Function

From a continuous perspective, an image is a continuous 2D function. This allows mathematical functions to run on the image.

Definition
A **pixel** is a point with no dimension.

We don't have continuous memory in computers, so we need to
- **Sample** the points, and
- **Quantise** the level of allowed values.

Rendering

Depth Perception

Our eyes will use anything to infer the depth of a scene, including:

- Occlusion: where one thing covers another because it is in front of the other thing.
- Relative size
- Distance to the horizon: the closer to the horizon the further away it is.
- Infer the shape of an object from shading.
- Red objects look closer than blue objects: chromatic abberation focus it further back in retina.
- Atmosphere/focus
- Perspective, this is easier when there are parallel lines.

In CG we want to use all the cues we can give.

Raytracing

We identify points on the image surface and calculate illumation for each pixel.

1. Shoot many rays from the camera.
2. Then calculate the colour of the pixel from where the ray hits, which is the closest intersection point (of the ray) to the camera.

Raytracing can easily handle reflection, refraction, shadows and motion blur, but is computationally expensive.

Finding Intersection

$$\text{ray} : \boldsymbol{r} = \boldsymbol{o} + s\boldsymbol{d}$$
$$\text{plane} : \boldsymbol{r} \cdot \boldsymbol{n} + a = 0$$

After solving

$$s = -\frac{a + \boldsymbol{n} \cdot \boldsymbol{o}}{\boldsymbol{n} \cdot \boldsymbol{d}}$$

To find an intersection with a polygon

1. Find intersections with the plane
2. Check whether the point is inside the polygon, which is just 2D geometry.

In the real word, the rays comes from the object

- It is computationally inefficient as a lot of the rays don't hit the eye.
- It can be mathematically proven that the result is the same regardless if you trace from the source to the eye or the other way round.

3D Object Intersection

Sphere

- Ray: $\boldsymbol{r} = \boldsymbol{o} + s\hat{\boldsymbol{d}}$
- Sphere: $(\boldsymbol{r} - \boldsymbol{c})^2 = r^2$

$$\left(\boldsymbol{o} - s\hat{\boldsymbol{d}} - \boldsymbol{c}\right)^2 - r^2 = 0$$
$$\hat{\boldsymbol{d}}^2 s^2 - 2s\hat{\boldsymbol{d}} \cdot (\boldsymbol{o} - \boldsymbol{c}) - r^2 = 0$$

This is the quadratic equation, solve for s.

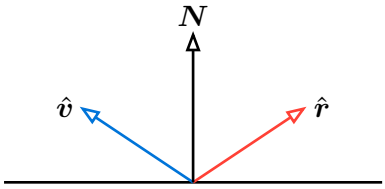
- If there are 2 real solutions, then cloose the closer intersection.

- If there are 0 real solutions, then there is no intersection.

You can also find intersections with a cylinder, cone and torus, it's easier if the objet is **axis aligned**.

Shading

1. Calculate the normal to the object at intersection.
2. Continue to look for a light source.
   - If the reflected ray intersect with another object, then the surface is not illuminated by the source. This is called a **shadow ray**.
3. If a surface is reflective, spawn a new ray to find the pixel's colour given by reflection.

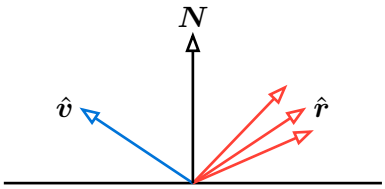


$$\hat{\boldsymbol{v}} \cdot \boldsymbol{N} = -\hat{\boldsymbol{r}} \cdot \boldsymbol{N}$$
$$\hat{\boldsymbol{r}} = -\hat{\boldsymbol{v}} + 2\boldsymbol{N}(\hat{\boldsymbol{v}} \cdot \boldsymbol{N})$$

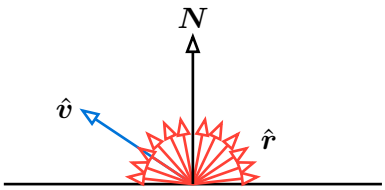
If we have a material that is both reflective and transparent, e.g. glass, then light will be refracted.
- 80% colour comes from reflection.
- 20% colour comes from refraction.

Types of Reflection

1. **Perfect reflection** as shown above.
2. **Imperfect specular reflection** (where the surface is not perfectly flat)



3. **Diffuse reflection** (where the structure is so complex light scatters in random direction)



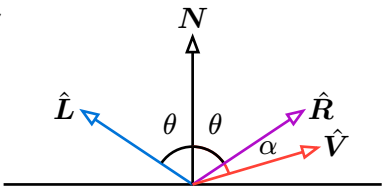
E.g. **plastic** has specular reflection on the light's colour, and diffuse reflection on the plastic's colour. Different wavelengths of light may reflect/scatter differently.

Phong's Imperfection Model

Definitions

Keyword	Definition
$\hat{\boldsymbol{L}}$	Normalised vector in direction of light source.
$\widehat{\boldsymbol{N}}$	Normal vector of the plane.
I_l	The intensity of light source.
k_d	The proportion of light diffusely reflected by the surface.
I	The intensity of the light being reflected.

$$I = I_l k_s \cos^n \alpha = I_l k_s \left(\hat{\boldsymbol{R}} \cdot \hat{\boldsymbol{L}}\right)^n$$



n determines how spread out the reflected light is - it is the **roughness factor**.

Overall Shading Equation

$$I = I_a k_d + \sum_i I_i k_d \hat{\boldsymbol{L}} \cdot \widehat{\boldsymbol{N}} + \sum_i I_i k_s \left(\hat{\boldsymbol{R}} \cdot \hat{\boldsymbol{V}}\right)^n$$

The $I_a k_d$ term gives the **ambient** shading. The next two terms gives the diffuse and specularity.

Sampling

So far we assume each ray passes through the centre of a pixel. This can lead to

- Jagged edges.
- Small objects being missed.
- Thin objects being split into pieces.

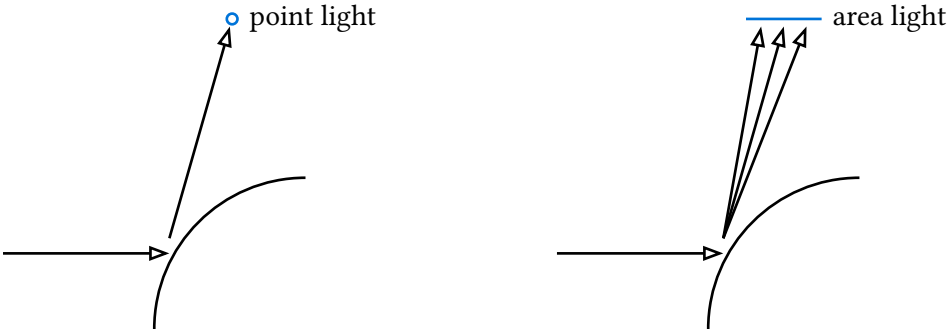
Antialiasing

Definition
Artifacts are also known as **aliasing**.

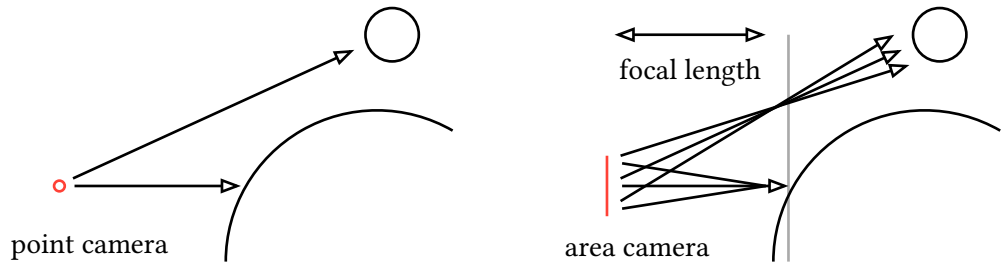
- **Single point sampling** just samples the point at the centre.
- **Super sampling** samples points in a regular grid/gaussian pattern.
  - **Random sampling** sample random points in the pixel, as our eyes are less sensitive to noise than artifact.
  - **Poisson disc sampling** reject rays less than distance ε from each other, but it takes n^2 comparisons to check for disc overlaps.
  - **Jitter sampling** is a type of stratified sampling: take a random grid, then random sample in each grid.
- **Adaptive super sampling** samples the four corners:
  - If the variance is high, do a super sample.
  - Otherwise, don't sample the pixel.

Distributed Sampling

- **Super sampling** - distributing samples in a pixel $\Longrightarrow$ **antialiasing**.
- **Area light** - distributing light on a plane $\Longrightarrow$ **soft shadows**.



- **Camera as an area** - distribute the camera position $\Longrightarrow$ **depth of field** effects.



And distributing sampling through time creates **motion blur**.

Rasterisation

Ray tracing gives very high quality results, but is computationally expensive.

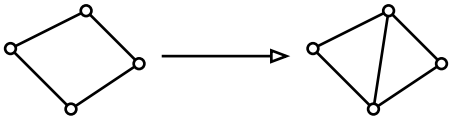
Real-time applications use rasterisation:
1. Model surfaces as polyhedrons.
2. Apply transformations to project the plane on screen.
3. Fill pixels with colours of the nearest visible polygon.

Most modern games use 90% rasterisation combined with 10% ray tracing.

Definition
Polyhedral surfaces are made of connected polygons surfaces.

We can approximate curved surfaces with polygons - the triangle is the simplest polygon as it vertices must be planar. GPUs are optimised to draw triangles.

We can split a polygon surface to triangles.



- **Postscript**: 2D transformations.
- **OpenGL**: 3D transformations.

Transformations as Matrices

Transformation	Matrix
Scale by factor m	$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$
Rotate by angle θ	$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
Shear parallel to x by factor a	$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$

Homogenous Coordinates

We could not represent transformation as matricies, unless we define the homogenous coordinates.

$$(homogenous) = (conventional)$$

$$(x, y, w) = \left(\frac{x}{w}, \frac{y}{w}\right)$$

There is an infinite number of homogenous coordinates that maps to the same point.

Transformation	Matrix
Scale by factor m	$\begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Translate by (x, y)	$\begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}$

Multiple transformations can be concatenated to make a more efficient transformation.

Note that in general, transformations are **not commutative**.

Scale/Rotation About a Point

To scale by factor of m about point (x_0, y_0)
1. Translate by $(-x_0, -y_0)$
2. Scale by factor of m
3. Translate by (x_0, y_0)

Similar for rotation.

3D Homogenous Coordinates

It is a simple extension of the 2D homogenous coordinates.

$$(x, y, z, w) \to \left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right)$$

Example: Placing a Cylinder in 3D Space

The program defines a cylinder as one with radius 1, height 2, oriented in direction of $(0, 0, 1)$, and centred at origin. We need to apply transformations to get it to the correct place.

We usually do in order
1. Scale
2. Rotate
3. Translate

So each step does not interfere with the previous steps. (what's the word for that?)

Rotation is the only nontrivial step, it is easier to do it in reverse order
1. Find the rotation R_1 about the y axis so the desired shape is oriented in direction $(0, y, z)$
2. Find the rotation R_2 about the x axis so the desired shape is oriented in direction $(0, 0, 1)$

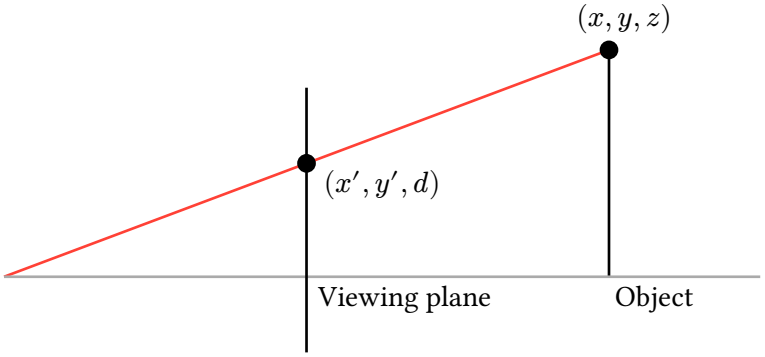
$$\text{The combined transformation} = T \times (R_1)^{-1} \times (R_2)^{-1} \times S$$

With transformations an object can be modelled once, and multiple instances can be placed.

2D Projection

Parallel projection	Perspective projection
Used in CAD.	Thing's get smaller as they get further away.
Less realistic.	More realistic.

Project to Viewing Plane



By similar triangles.

$$x' = x\frac{d}{z}$$
$$y' = y\frac{d}{z}$$

The furthere things are the smaller they look, because it is divided by larger z.

We also want $z' = 1/z$ to use in the z-buffer algorithm.

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 1/d \\ z/d \end{pmatrix}$$

Which gives conventional coordinates $(x \cdot d/z, y \cdot d/z, 1/z)$.

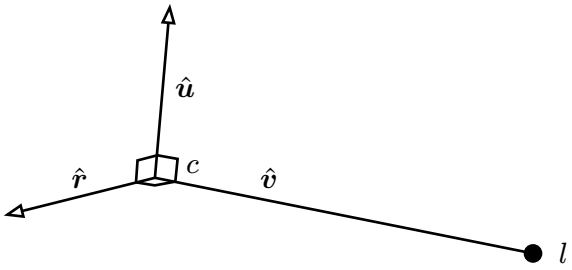
Viewing Coordinates

Instead of projecting all objects to an arbitrary plane, it is easier to transform all aobjects to a viewing coordinates system.

> **Note**
> OpenGL uses **right-handed coordinates**, which we will be using. (also note y is up)

We want to place the camera
- Centred at c
- Directed at l
- Up in direction of $\boldsymbol{u}$

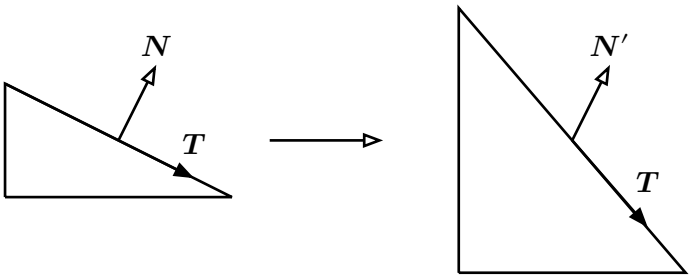
$$\hat{\boldsymbol{v}} = \frac{l - c}{|l - c|}$$
$$\hat{\boldsymbol{r}} = \frac{\boldsymbol{u} \times \hat{\boldsymbol{v}}}{|\boldsymbol{u} \times \hat{\boldsymbol{v}}|}$$
$$\hat{\boldsymbol{u}} = \hat{\boldsymbol{v}} \times \hat{\boldsymbol{r}}$$



We need to use this formula as $\boldsymbol{u}$ is given by the user, we cannot be sure that $\boldsymbol{u} \perp \hat{\boldsymbol{v}}$.

Transformaing Normal Vectors

Transformation by non-orthogonal matrix does not preserve angle, this breaks normals.



We want $\boldsymbol{N} \cdot \boldsymbol{T} = 0$ after transformation M. Then we need to transform $\boldsymbol{N}$ by matrix G.

$$T' = MT$$
$$N' = GN$$

For two vectors A and B, $A \cdot B = A^T B$ in matrix multiplication.

Note
Let A and B be matrices, and M^T the transpose operator.

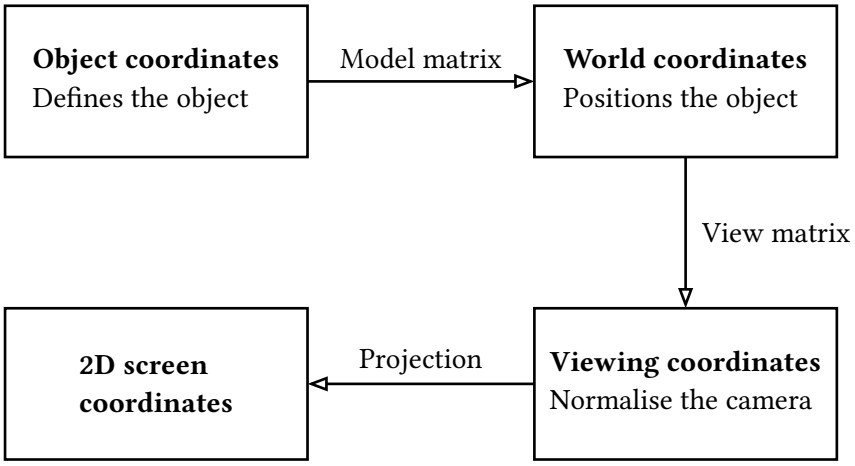
$$(AB)^T = B^T A^T$$

(Why?)

$$(GN) \cdot (MT) = (GN)^T (MT)$$
$$N^T G^T MT = 0$$

Because $N^T T = 0$, then $G^T M = I$, and $G = \left(M^{-1}\right)^T$.

The overall process to display an object on screen.



Scene Graph
To attach object B to object A.
1. Apply scale to A.
2. Apply scale, rotation and translation to move B to where it will attach to A.
3. Apply rotation and translation to both A and B.

To attach object C to B, add extra steps between 1 and 2 to attach it to B, all other transformations applied to B should also be applied to C.

We can build a **scene graph** by attachments, traversing the scene graph draws the scene.

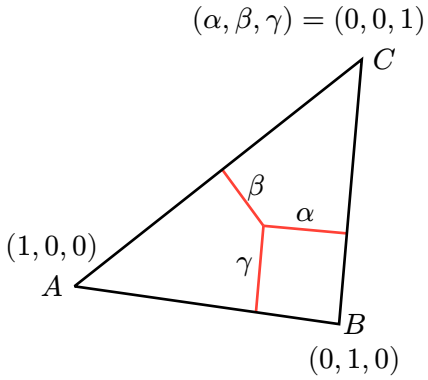
Rasterisation Algorithm

The algorithm goes as:

1. Set the model, view, projection (MVP) transforms.
2. For all triangles, transform the vertices with MVP.
3. If the triangle is in **view frustum**, clip triangle to screen boarder.
4. For each **fragment** (pixel in the triangle), interpolate the attributes of the fragment between pixels (e.g. colour and normal).
5. If the fragment is closer to the camera than pixels drawn so far, update the screen pixels with fragment colour.

Interpolation

Homogenous barycentric coordinates are used for interpolating attributes.



Where α, β and γ are the distances of point P from the line $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$.

We want this distance to be normalised between 0 and 1, so

$$\alpha = \frac{(P - A) \cdot \overrightarrow{AB}}{(C - A) \cdot \overrightarrow{AB}}$$

Similar for β and γ. If α, β and γ are all between 0 and 1, then the point is in the triangle.

> **Note**
> We don't need to write this algorithm because it is already included in the GPU.

Occlusion

The **z-buffer** algorithm initialises a **colour buffer** and a **depth buffer** (which describes how far objects are away from the camera).

1. Initialise the colour buffer to background colour, the depth buffer to as far as possible.
2. For every fragment in every triangle, calculate z for current fragment (pixel in triangle).
3. If $z < \text{depth}(x, y)$ and $z > z_{\text{min}}$, then set $\text{depth}(x, y) = z$ and $\text{colour}(x, y) = \text{fragment colour}$.

The z-buffer stores distances with finite precision, we store $1/z$ instead of z so infinite distance can be easily represented.

> **Note**
> **Z-fighting** happens when two planes have the same depth.

The GPU

> **Definition**
> The **GPU** is optimised for floating point operations on large arrays of data.

It is optimised for **parallel operations** such as

- Ray tracing
- Rasterisation
- Shading
- Video encoding

GPUs have thousands of **SIMD processors**, and a much higher memory access speed than the CPU.

GPU APIs

OpenGL	**Vulkan**
An older API.	Has lower overhead than OpenGL.
A good starting point for doing grapahics.	Reduces CPU load.
	Allows for finer controls.
	But very verbose, it is used in performance critical code like game engines.

There are **general purpose GPU** APIs, as OpenGL and Vulkan are only for graphics.
- CUDA is only supported by Nvidia.
- OpenCL is cross platform.

Then there are graphics API that run on the browser:
- WebGL is a wrapper for OpenGL.
- WebGPU is a wrapper for Vulkan.

OpenGL Programming

We need to write both CPU and GPU code:
1. GL functions create OpenGL objects.
2. Data is copied between CPU and GPU

Definition
GPU code are called **shaders**, and are written in **GLSL**.
