Ordinary Differential Equations

Definition
An **nth order ODE** includes the nth derivate but no higher derivatives.

To solve an ODE we find the dependent variable as a function of the independent variables.
- In the real world: this is done numerically with computers.
- Here we do it analytically to demonstrate principles.

Some ODEs don't have analytical solutions.

First Order ODEs

Definition
Integrable ODEs have form

$$\frac{dy}{dx} = f(x)$$

where the RHS does not depend on y.

$$y = \int f(x) dx$$

If $F'(x) = f(x)$, then $y = F(x) + C$. y solves the ODE whatever C is - threr are infinitely many solutions.

$(*)$ is called the **general solution** if it contains all possible solutions.

If we choose any value for C, we get a particular solution.

Definition
Separable ODEs have form

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

$$g(y) \frac{dy}{dx} = f(x)$$

$$\int g(y) \frac{dy}{dx} dx = \int f(x) dx$$

$$\int g(y) dy = \int f(x) dx$$

$$G(y) = F(x) + C$$

Geometric Interpretation of First Order ODEs

Consider $\frac{dy}{dx} = F(x, y)$

For every point (x, y):
- There is a particular solution passing through the point
- Its gradient is $F(x, y)$

Note
2 particular solutions don't have to be the same shape (shifted versions) of each other.
