Deep_Learning_Personal_Project

July 12, 2019

1 Deep Learning Personal Project: SGA implementation, Comparison and Evaluation

- Code implemented by Sirius See, SDCS, SYSU
- Student Name: *** **
- Student ID: *******

1.1 Acknowledgement

- The Symplectic Gradient Adjustment (SGA) algorithm is first introduced in The Mechanics of n-Player Differentiable Games by **David Balduzzi** in ICML 2018.
- The code is inspired by **Deep Mind**'s SGA algorithm in 2018.
- The Optimistic Mirror Descent (OMD) is first introduced in Optimistic mirror descent in saddle-point problems: Going the extra (gradient) mile by **Panayotis Mertikopoulos** in ICLR 2019
- **A series of experiments is added by Sirius See (* * * * *)** as the **solo project of Deep Learning course** in Sun Yat-sen University. Further detailed description of my work will be listed in the following section.
- **Google Colab** and **SYSU InPlusLab** provide the devices (mainly GPUs) for me to conduct the following experiments.

1.2 Table of Content

1.3 Tensorflow Environment Preparation

In [0]:
```
!pip install dm-sonnet
!pip install 'kfac[tensorflow]'
```

```
Requirement already satisfied: dm-sonnet in /usr/local/lib/python3.6/dist-packages (1.33)
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Collecting kfac[tensorflow]
  Downloading https://files.pythonhosted.org/packages/83/6f/937896d2d4156db31bca44f9137badee33
     || 143kB 4.6MB/s
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Requirement already satisfied: keras-preprocessing>=1.0.5 in /usr/local/lib/python3.6/dist-pac
Requirement already satisfied: setuptools>=41.0.0 in /usr/local/lib/python3.6/dist-packages (f
Requirement already satisfied: werkzeug>=0.11.15 in /usr/local/lib/python3.6/dist-packages (fr
```

```
Requirement already satisfied: markdown>=2.6.8 in /usr/local/lib/python3.6/dist-packages (from
Requirement already satisfied: h5py in /usr/local/lib/python3.6/dist-packages (from keras-appli
Installing collected packages: kfac
Successfully installed kfac-0.1.4
```

```
In [0]: from __future__ import absolute_import
        from __future__ import division
        from __future__ import print_function

        import math
        import os
        import numpy as np
        import sonnet as snt
        import tensorflow as tf
        import kfac

        import matplotlib.pyplot as plt
        import scipy as sp
```

```
W0711 01:50:12.525845 140060646823808 deprecation_wrapper.py:119] From /usr/local/lib/python3.6

W0711 01:50:12.527219 140060646823808 deprecation_wrapper.py:119] From /usr/local/lib/python3.6
```

1.4 Experiment 1: Symplectic Gradient Adjustment (SGA) algorithm implementation

```
In [0]: # help function of SymplecticOptimizer

        def list_divide_scalar(xs, y):
          return [x / y for x in xs]


        def list_subtract(xs, ys):
          return [x - y for (x, y) in zip(xs, ys)]


        def jacobian_vec(ys, xs, vs):
          return kfac.utils.fwd_gradients(
              ys, xs, grad_xs=vs, stop_gradients=xs)


        def jacobian_transpose_vec(ys, xs, vs):
          dydxs = tf.gradients(ys, xs, grad_ys=vs, stop_gradients=xs)
          dydxs = [
              tf.zeros_like(x) if dydx is None else dydx for x, dydx in zip(xs, dydxs)
          ]
```

```
    return dydxs


  def _dot(x, y):
    dot_list = []
    for xx, yy in zip(x, y):
      dot_list.append(tf.reduce_sum(xx * yy))
    return tf.add_n(dot_list)
```

In [0]:
```
#@title Default title text
# impl of SGA optimizer

class SymplecticOptimizer(tf.train.Optimizer):
  """Optimizer that corrects for rotational components in gradients."""

  def __init__(self,
               learning_rate,
               reg_params=1.,
               use_signs=True,
               use_locking=False,
               name='symplectic_optimizer'):
    super(SymplecticOptimizer, self).__init__(
        use_locking=use_locking, name=name)
    self._gd = tf.train.RMSPropOptimizer(learning_rate)
    self._reg_params = reg_params
    self._use_signs = use_signs

  def compute_gradients(self,
                        loss,
                        var_list=None,
                        gate_gradients=tf.train.Optimizer.GATE_OP,
                        aggregation_method=None,
                        colocate_gradients_with_ops=False,
                        grad_loss=None):
    return self._gd.compute_gradients(loss, var_list, gate_gradients,
                                      aggregation_method,
                                      colocate_gradients_with_ops, grad_loss)

  def apply_gradients(self, grads_and_vars, global_step=None, name=None):
    grads, vars_ = zip(*grads_and_vars)
    n = len(vars_)
    h_v = jacobian_vec(grads, vars_, grads)
    ht_v = jacobian_transpose_vec(grads, vars_, grads)
    at_v = list_divide_scalar(list_subtract(ht_v, h_v), 2.)
    if self._use_signs:
      grad_dot_h = _dot(grads, ht_v)
      at_v_dot_h = _dot(at_v, ht_v)
      mult = grad_dot_h * at_v_dot_h
```

```
      lambda_ = tf.sign(mult / n + 0.1) * self._reg_params
    else:
      lambda_ = self._reg_params
    apply_vec = [(g + lambda_ * ag, x)
                 for (g, ag, x) in zip(grads, at_v, vars_)
                 if at_v is not None]
    return self._gd.apply_gradients(apply_vec, global_step, name)
```

1.5 Experiment 2: Basic multilayer perceptron (MLP) model implementation

In [0]:
```
class MLP(snt.AbstractModule):

  def __init__(self, depth, hidden_size, out_dim, name='SimpleNet'):
    super(MLP, self).__init__(name=name)
    self._depth = depth
    self._hidden_size = hidden_size
    self._out_dim = out_dim

  def _build(self, input):
    h = input
    for i in range(self._depth):
      h = tf.nn.relu(snt.Linear(self._hidden_size)(h))
    return snt.Linear(self._out_dim)(h)
```

In [0]:
```
def reset_and_build_graph(depth, width, x_real_builder, z_dim, batch_size, learning_ra
  tf.reset_default_graph()

  x_real = x_real_builder(batch_size)
  x_dim = x_real.get_shape().as_list()[1]
  generator = MLP(depth, width, x_dim, 'generator')
  discriminator = MLP(depth, width, 1, 'discriminator')
  z = tf.random_normal([batch_size, z_dim])
  x_fake = generator(z)
  disc_out_real = discriminator(x_real)
  disc_out_fake = discriminator(x_fake)

  # Loss
  disc_loss_real = tf.reduce_mean(
      tf.nn.sigmoid_cross_entropy_with_logits(
          logits=disc_out_real, labels=tf.ones_like(disc_out_real)))
  disc_loss_fake = tf.reduce_mean(
      tf.nn.sigmoid_cross_entropy_with_logits(
          logits=disc_out_fake, labels=tf.zeros_like(disc_out_fake)))
  disc_loss = disc_loss_real + disc_loss_fake

  gen_loss = tf.reduce_mean(
      tf.nn.sigmoid_cross_entropy_with_logits(
          logits=disc_out_fake, labels=tf.ones_like(disc_out_fake)))
```

```
    gen_vars = generator.variable_scope.trainable_variables()
    disc_vars = discriminator.variable_scope.trainable_variables()
    # Compute gradients
    xs = disc_vars + gen_vars
    disc_grads = tf.gradients(disc_loss, disc_vars)
    gen_grads = tf.gradients(gen_loss, gen_vars)
    Xi = disc_grads + gen_grads
    apply_vec = list(zip(Xi, xs))

    if mode == 'RMS':
      optimizer = tf.train.RMSPropOptimizer(learning_rate)
    elif mode == 'SGD':
      optimizer = tf.train.GradientDescentOptimizer(learning_rate)
    elif mode == 'ADG':
      optimizer = tf.train.AdagradOptimizer(learning_rate)
    elif mode == 'ADA':
      optimizer = tf.train.AdamOptimizer(learning_rate)
    elif mode == 'SGA':
      optimizer = SymplecticOptimizer(learning_rate)
    else:
      raise ValueError('Mode %s not recognised' % mode)

    with tf.control_dependencies([g for (g, v) in apply_vec]):
      train_op = optimizer.apply_gradients(apply_vec)

    init = tf.global_variables_initializer()

    return train_op, x_fake, z, init, disc_loss, gen_loss
```

```
In [0]: # visualization

        def kde(mu, tau, bbox=None, xlabel="", ylabel="", cmap='Blues'):
            values = np.vstack([mu, tau])
            kernel = sp.stats.gaussian_kde(values)

            fig, ax = plt.subplots()
            ax.axis(bbox)
            ax.set_aspect(abs(bbox[1]-bbox[0])/abs(bbox[3]-bbox[2]))
            ax.set_xlabel(xlabel)
            ax.set_ylabel(ylabel)

            xx, yy = np.mgrid[bbox[0]:bbox[1]:300j, bbox[2]:bbox[3]:300j]
            positions = np.vstack([xx.ravel(), yy.ravel()])
            f = np.reshape(kernel(positions).T, xx.shape)
            cfset = ax.contourf(xx, yy, f, cmap=cmap)
            plt.show()
```

1.6 Experiment 4: Comparison of optimizers when learning a 4 by 4 mixture of Gaussian in 2D

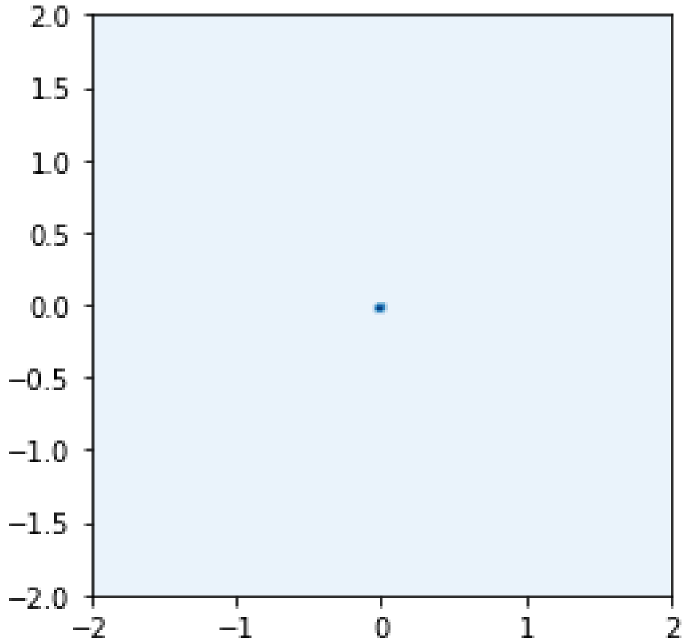
```
In [0]: def train(train_op, x_fake, z, init, disc_loss, gen_loss, z_dim,
                  n_iter=10001, n_save=2000):
          bbox = [-2, 2, -2, 2]
          batch_size = x_fake.get_shape()[0].value
          ztest = [np.random.randn(batch_size, z_dim) for i in range(10)]

          with tf.Session() as sess:
            sess.run(init)

            for i in range(n_iter):
              disc_loss_out, gen_loss_out, _ = sess.run(
                  [disc_loss, gen_loss, train_op])
              if i % n_save == 0:
                print('i = %d, discriminant loss = %.4f, generator loss =%.4f' %
                      (i, disc_loss_out, gen_loss_out))
                x_out = np.concatenate(
                    [sess.run(x_fake, feed_dict={z: zt}) for zt in ztest], axis=0)
                kde(x_out[:, 0], x_out[:, 1], bbox=bbox)

        def learn_mixture_of_gaussians(mode):
          print(mode)
          def x_real_builder(batch_size):
            sigma = 0.1
            skel = np.array([
                [ 1.50,  1.50],
                [ 1.50,  0.50],
                [ 1.50, -0.50],
                [ 1.50, -1.50],
                [ 0.50,  1.50],
                [ 0.50,  0.50],
                [ 0.50, -0.50],
                [ 0.50, -1.50],
                [-1.50,  1.50],
                [-1.50,  0.50],
                [-1.50, -0.50],
                [-1.50, -1.50],
                [-0.50,  1.50],
                [-0.50,  0.50],
                [-0.50, -0.50],
                [-0.50, -1.50],
            ])
            temp = np.tile(skel, (batch_size // 16 + 1,1))
            mus = temp[0:batch_size,:]
            return mus + sigma*tf.random_normal([batch_size, 2])*.2
```

```
z_dim = 64
train_op, x_fake, z, init, disc_loss, gen_loss = reset_and_build_graph(
    depth=6, width=384, x_real_builder=x_real_builder, z_dim=z_dim,
    batch_size=256, learning_rate=1e-4, mode=mode)

train(train_op, x_fake, z, init, disc_loss, gen_loss, z_dim)
```

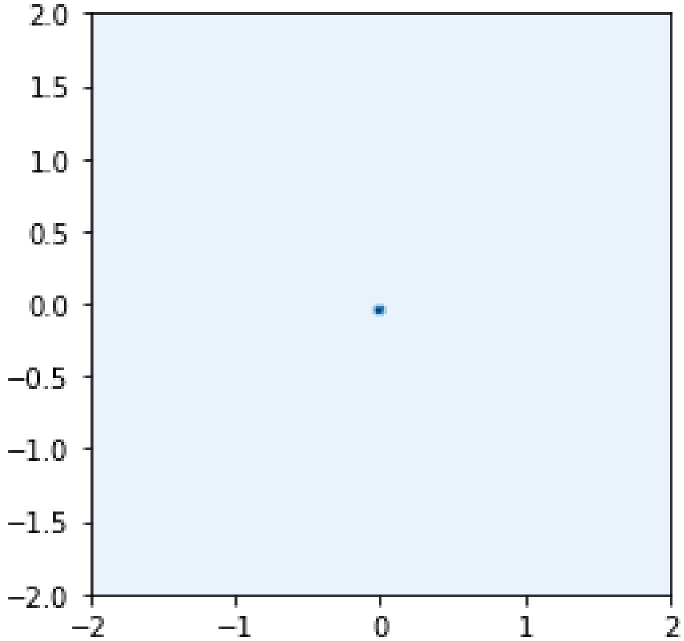
1.6.1 Using SGD

```
In [0]: learn_mixture_of_gaussians('SGD')
```

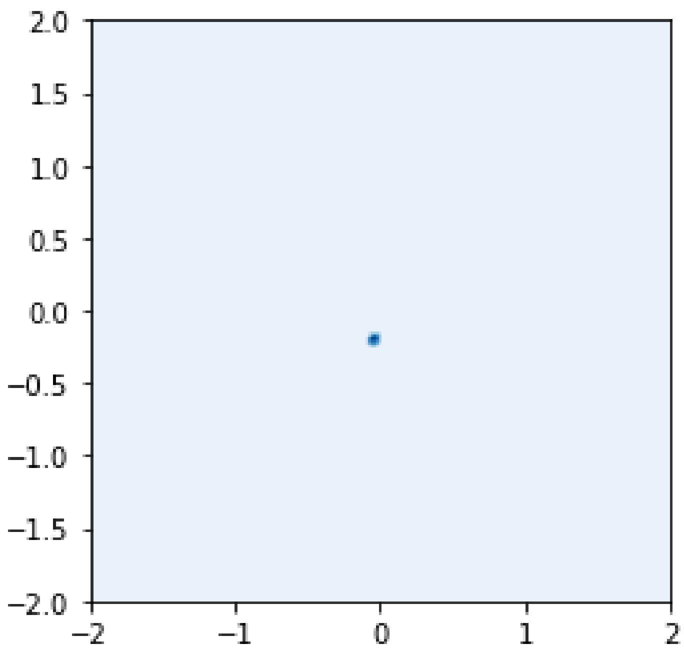
```
SGD
i = 0, discriminant loss = 1.4003, generator loss =0.6933
```



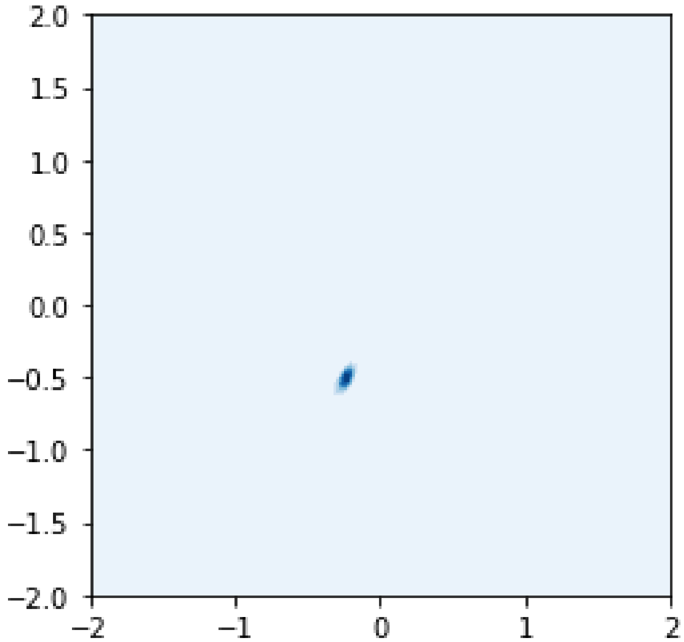

```
i = 2000, discriminant loss = 1.2123, generator loss =0.6958
```



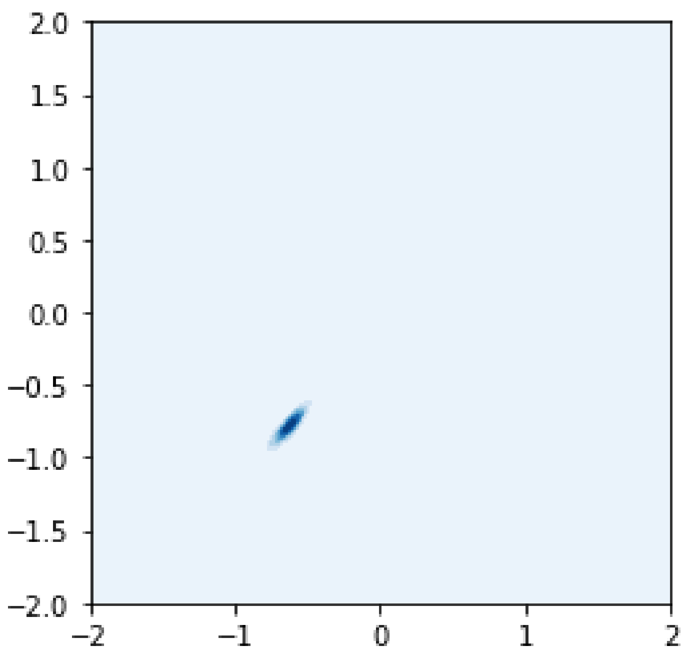

```
i = 4000, discriminant loss = 1.0371, generator loss =0.6724
```



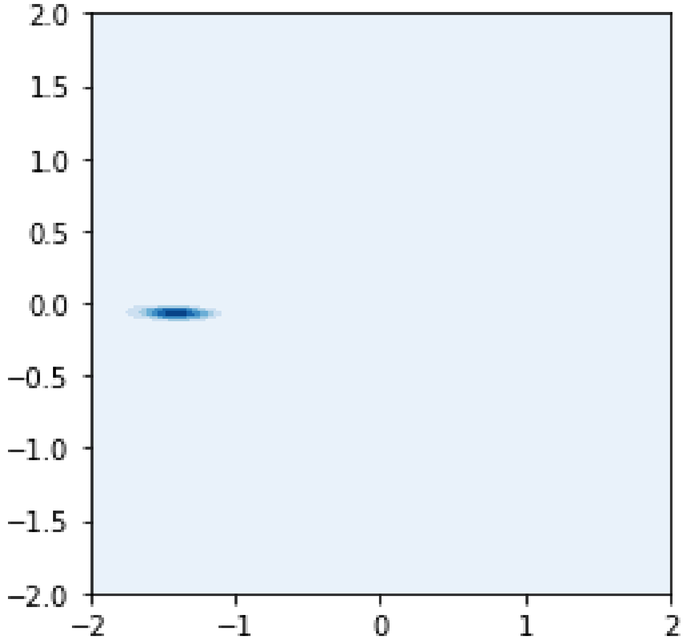
```
i = 6000, discriminant loss = 1.0122, generator loss =0.6308
```




```
i = 8000, discriminant loss = 0.9194, generator loss =0.7399
```



```
i = 10000, discriminant loss = 0.9195, generator loss =0.8402
```



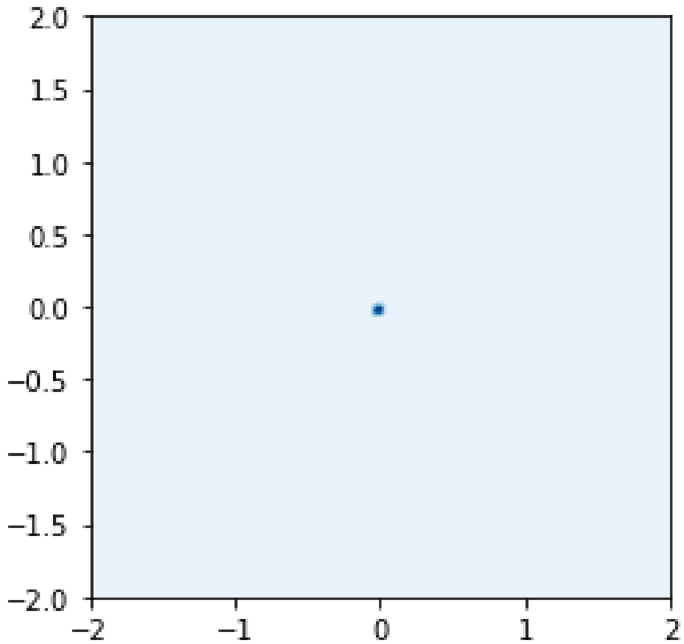
1.6.2 Using AdaGrad

```
In [0]: learn_mixture_of_gaussians('ADG')
```

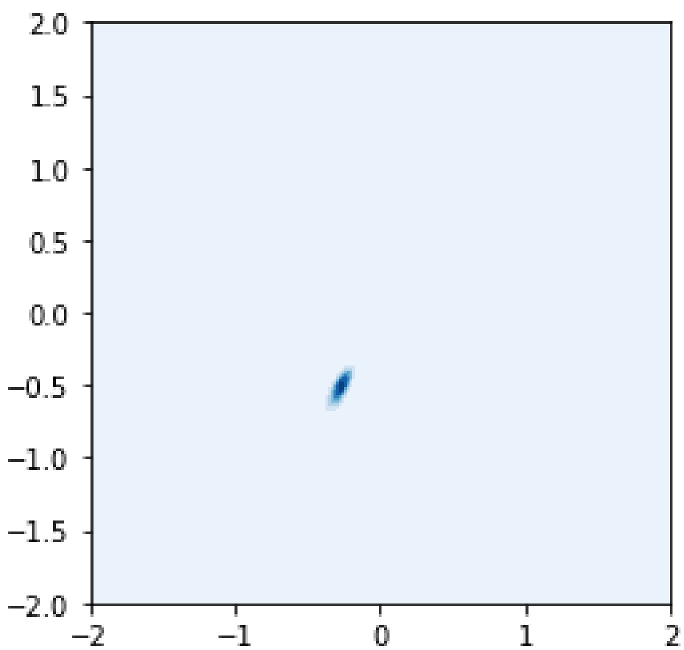
```
ADG
```

```
W0710 13:55:39.310465 140150887180160 deprecation.py:506] From /usr/local/lib/python3.6/dist-pa
Instructions for updating:
Call initializer instance with the dtype argument instead of passing it to the constructor
```

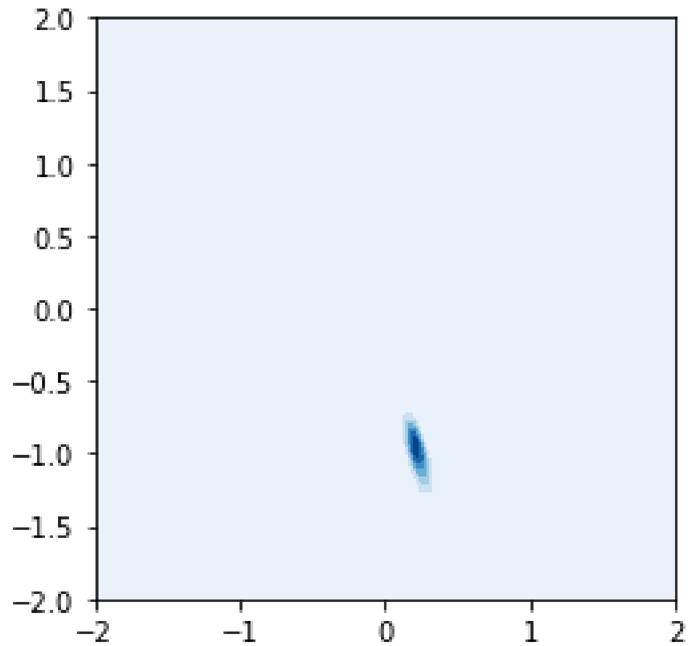
```
i = 0, discriminant loss = 1.3714, generator loss =0.6926
```



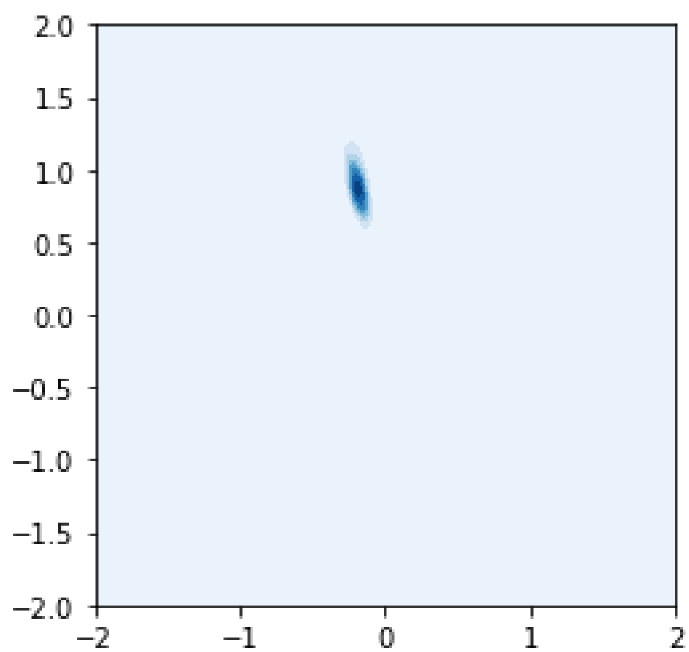

```
i = 2000, discriminant loss = 1.0970, generator loss =0.6096
```



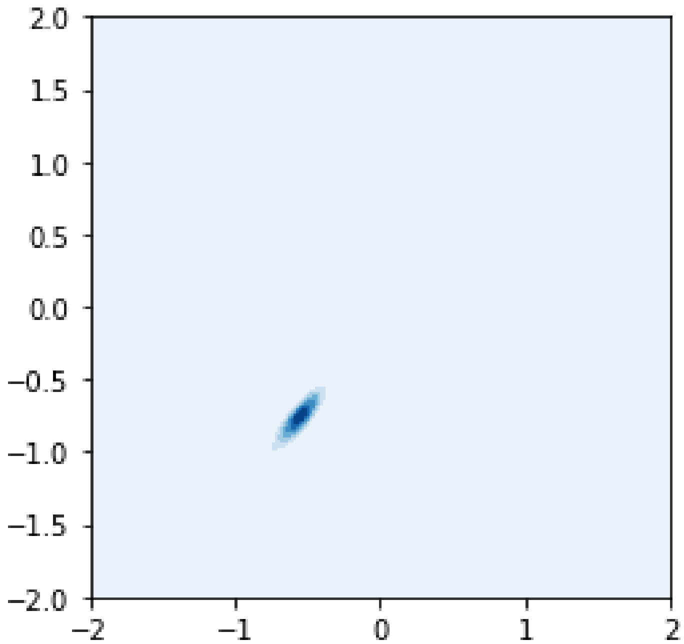
```
i = 4000, discriminant loss = 0.9673, generator loss =0.6934
```



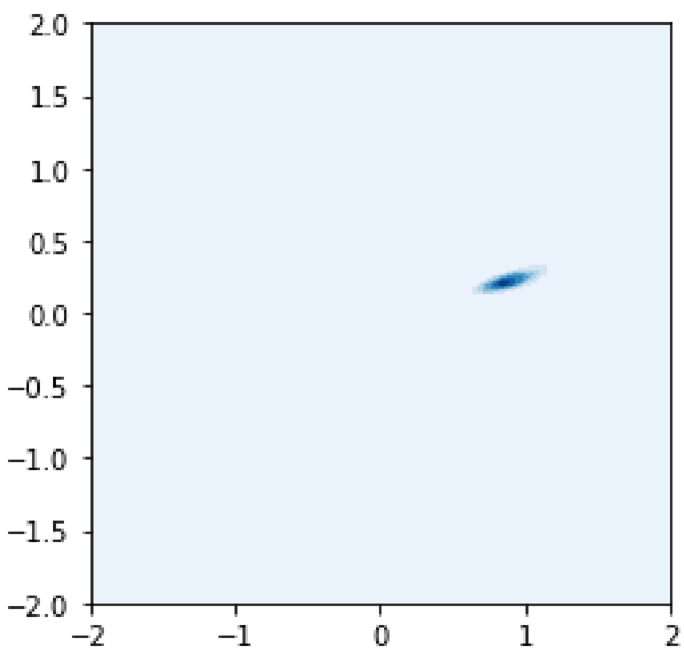

```
i = 6000, discriminant loss = 1.1352, generator loss =0.8187
```



```
i = 8000, discriminant loss = 1.0060, generator loss =0.7855
```



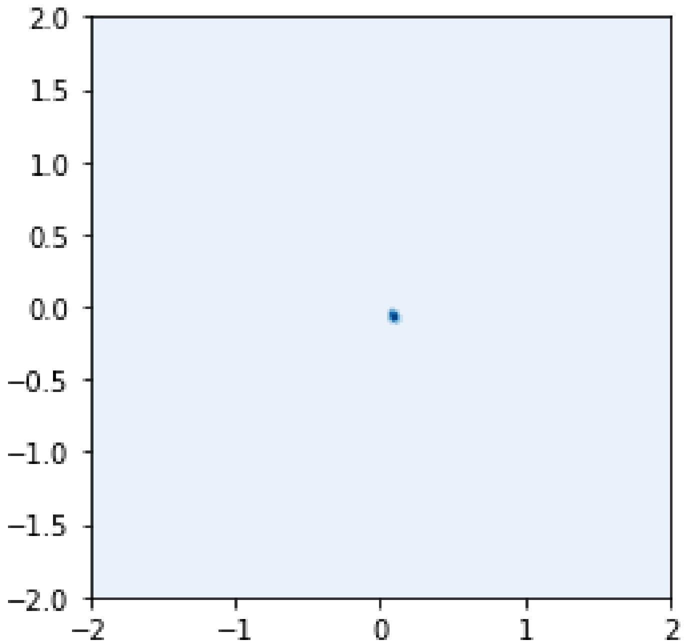
```
i = 10000, discriminant loss = 0.9412, generator loss =0.8080
```



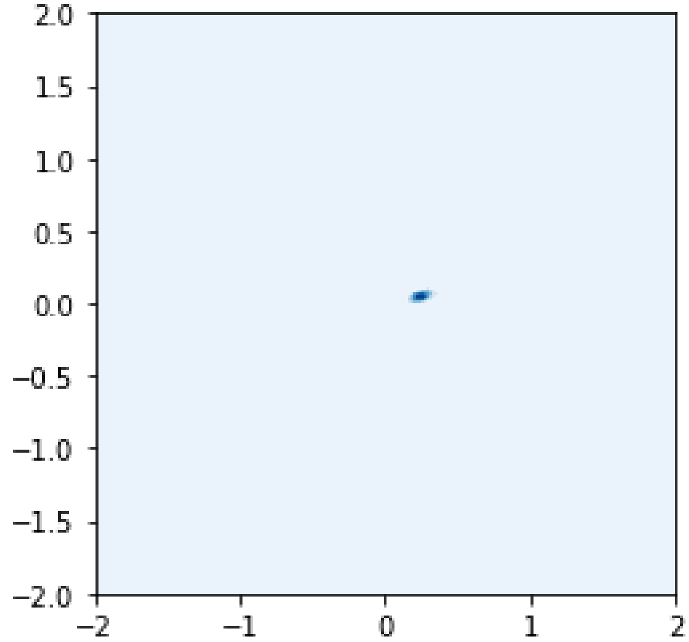
1.6.3 Using Adam

```
In [0]: learn_mixture_of_gaussians('ADA')
```

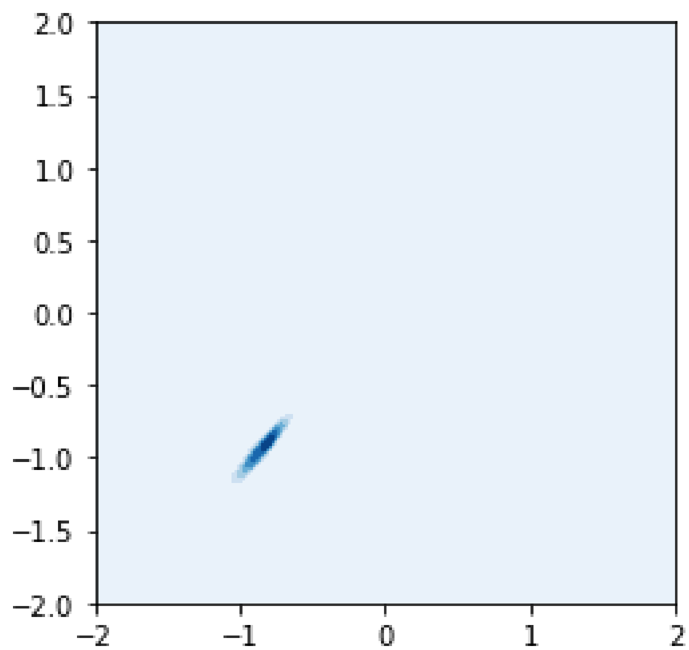
```
ADA
i = 0, discriminant loss = 1.3380, generator loss =0.6903
```



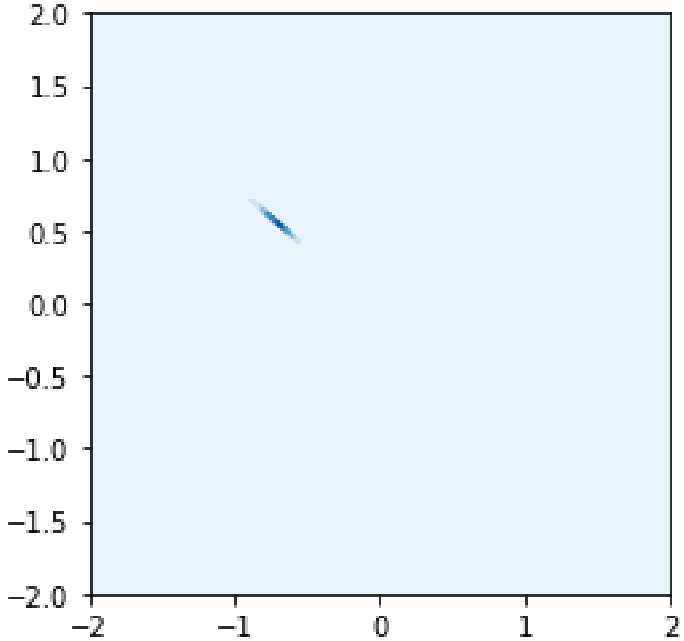
```
i = 2000, discriminant loss = 1.1390, generator loss =0.8283
```



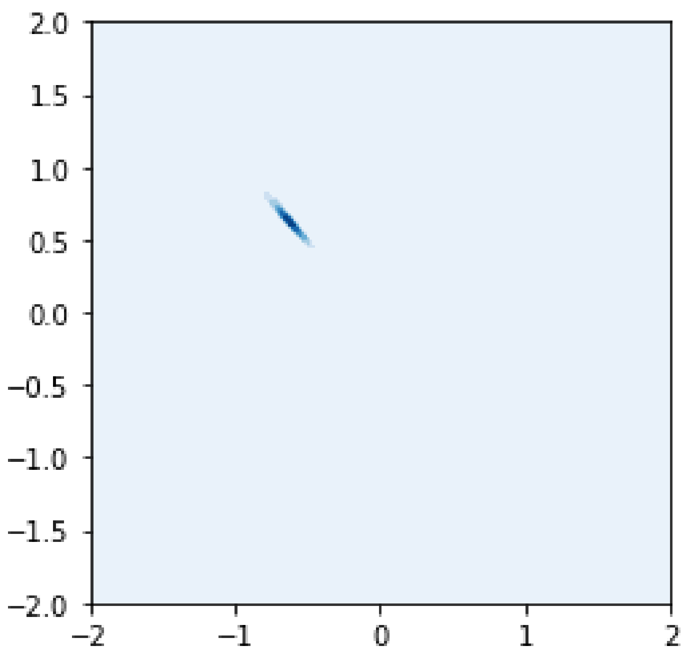

```
i = 4000, discriminant loss = 0.8575, generator loss =1.1069
```



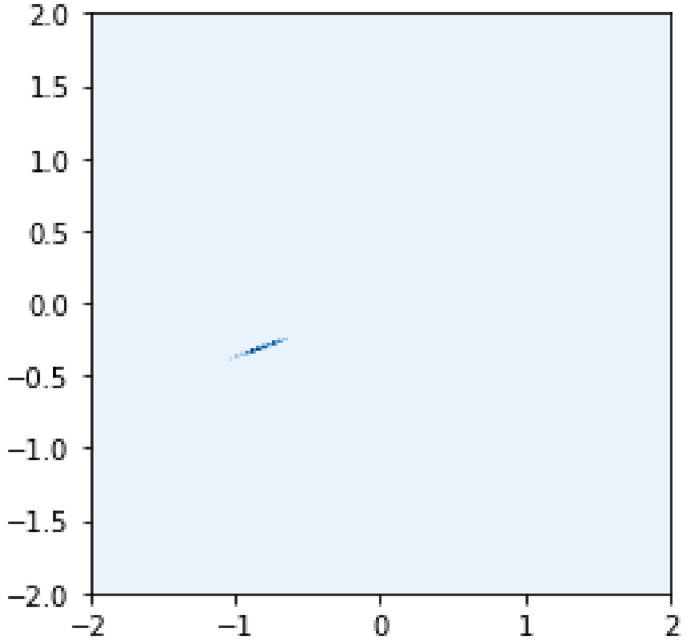
```
i = 6000, discriminant loss = 1.4832, generator loss =0.9082
```




```
i = 8000, discriminant loss = 1.3655, generator loss =0.9416
```



```
i = 10000, discriminant loss = 0.7635, generator loss =0.9369
```



1.6.4 Using RMSProp

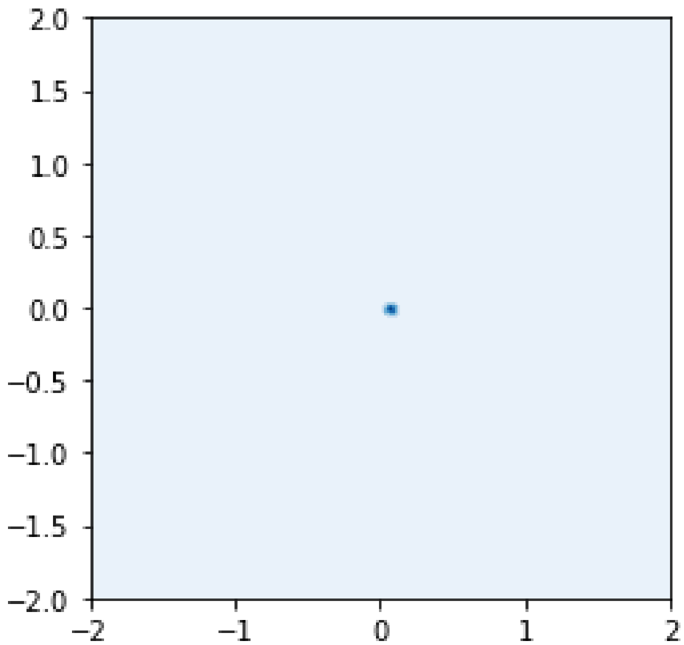
```
In [0]: learn_mixture_of_gaussians('RMS')
```

```
W0710 11:44:01.528981 140150887180160 deprecation_wrapper.py:119] From /usr/local/lib/python3.6

W0710 11:44:01.554638 140150887180160 deprecation_wrapper.py:119] From /usr/local/lib/python3.6

W0710 11:44:01.561922 140150887180160 deprecation.py:506] From /usr/local/lib/python3.6/dist-pa
Instructions for updating:
Call initializer instance with the dtype argument instead of passing it to the constructor
W0710 11:44:01.562939 140150887180160 deprecation.py:506] From /usr/local/lib/python3.6/dist-pa
Instructions for updating:
Call initializer instance with the dtype argument instead of passing it to the constructor
W0710 11:44:01.564574 140150887180160 deprecation_wrapper.py:119] From /usr/local/lib/python3.6
```

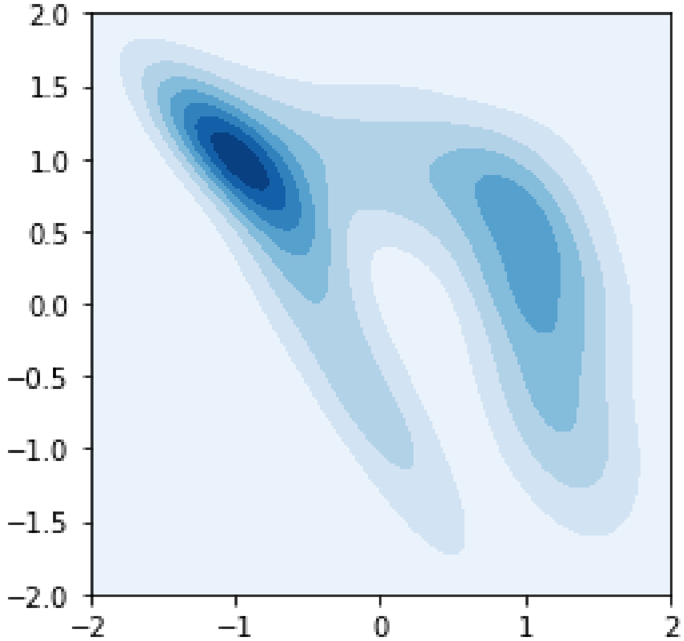
```
RMS
```

```
W0710 11:44:01.825356 140150887180160 deprecation.py:323] From /usr/local/lib/python3.6/dist-pa
Instructions for updating:
Use tf.where in 2.0, which has the same broadcast rule as np.where
W0710 11:44:02.175057 140150887180160 deprecation.py:506] From /usr/local/lib/python3.6/dist-pa
Instructions for updating:
Call initializer instance with the dtype argument instead of passing it to the constructor
```

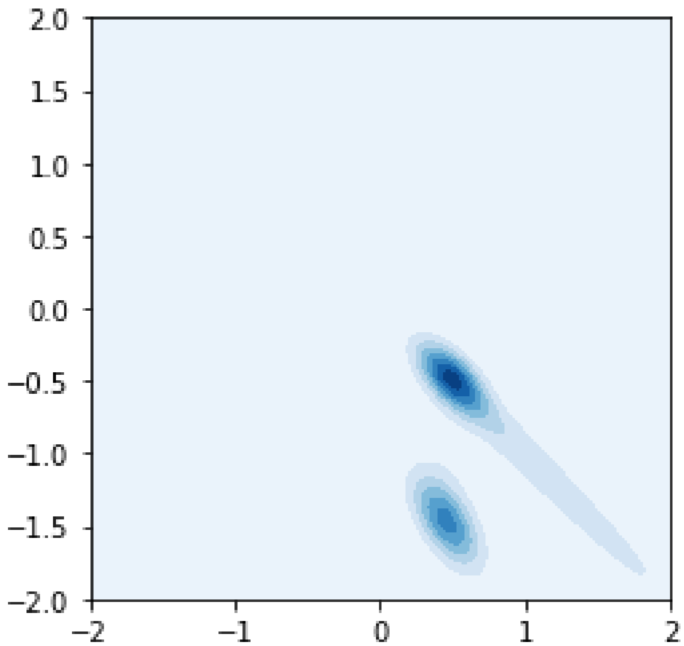
```
i = 0, discriminant loss = 1.4136, generator loss =0.6941
```



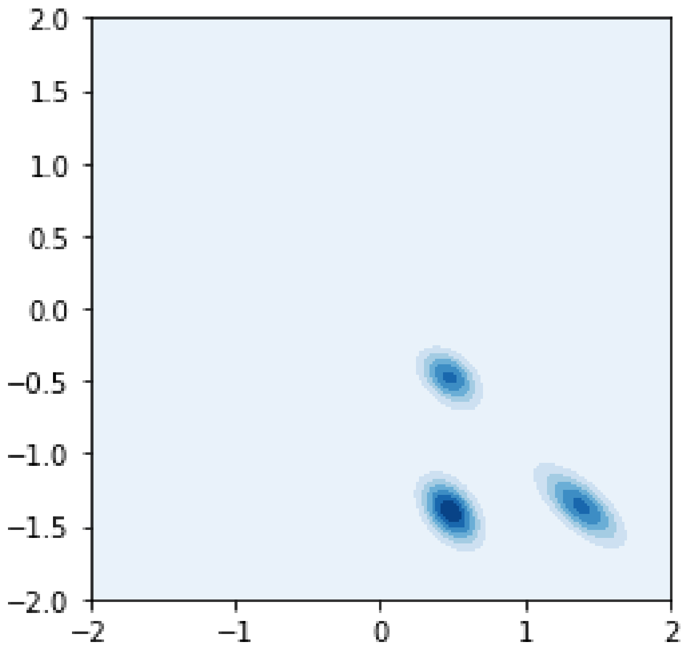

```
i = 2000, discriminant loss = 1.0527, generator loss =1.5996
```



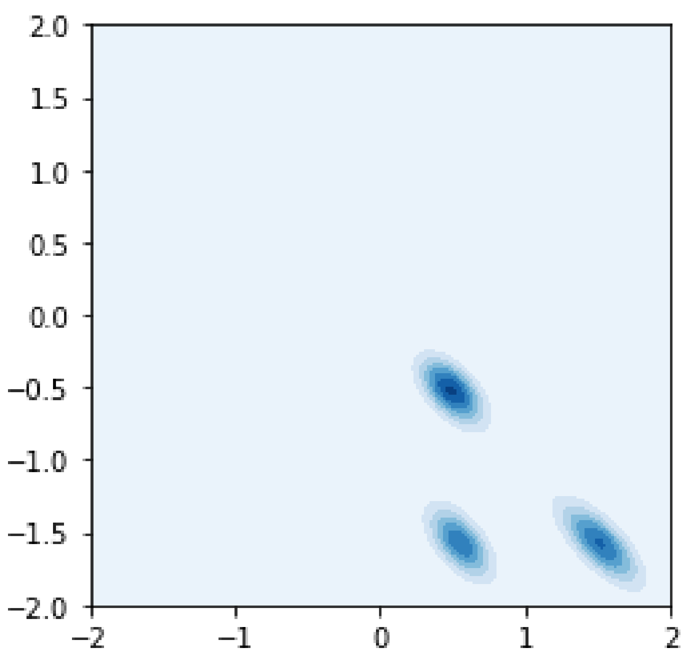

```
i = 4000, discriminant loss = 0.4317, generator loss =2.0905
```



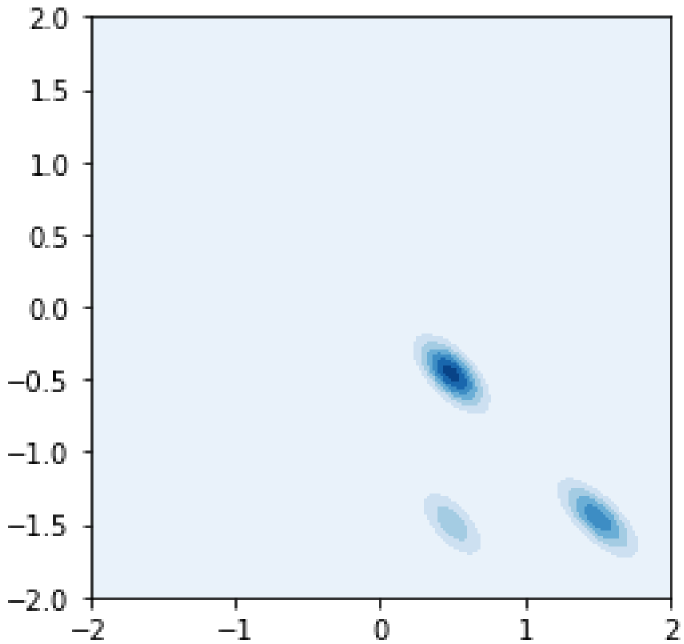
```
i = 6000, discriminant loss = 0.4968, generator loss =2.1840
```




```
i = 8000, discriminant loss = 0.4876, generator loss =1.7091
```

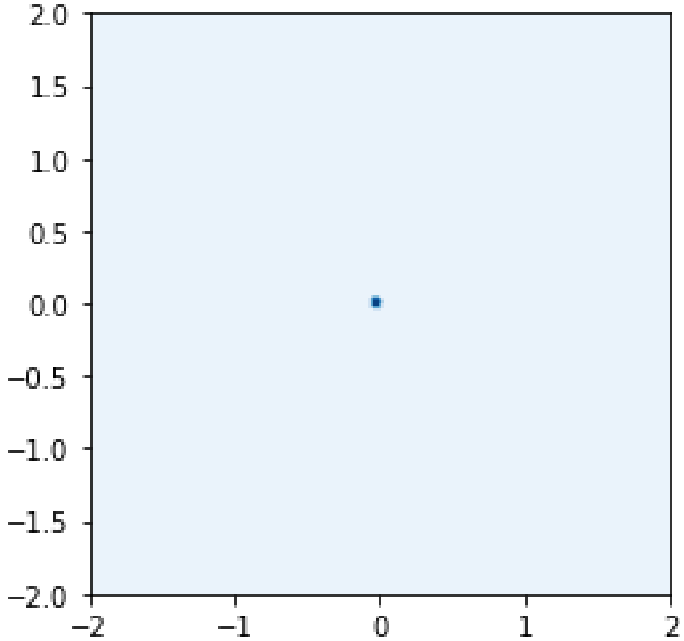


```
i = 10000, discriminant loss = 0.5137, generator loss =1.7146
```

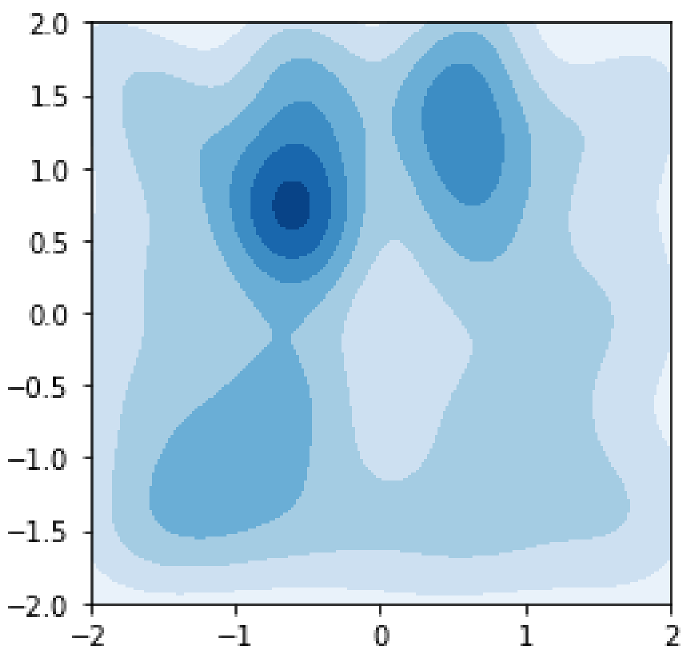



```
In [0]: # Use Symplectic Gradient Adjustment to optimise the GAN parameters.
        # With SGA, all modes are produced by the trained GAN.
        learn_mixture_of_gaussians('SGA')
```

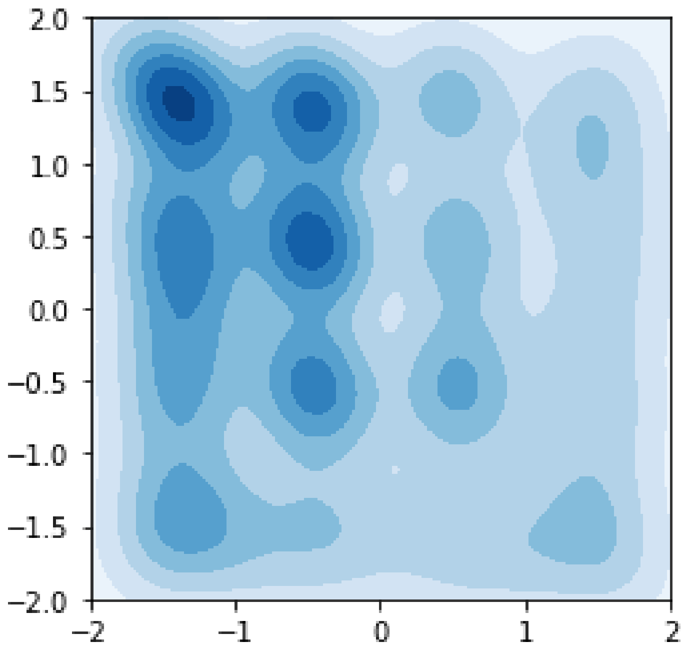
```
SGA
i = 0, discriminant loss = 1.4156, generator loss =0.6938
```



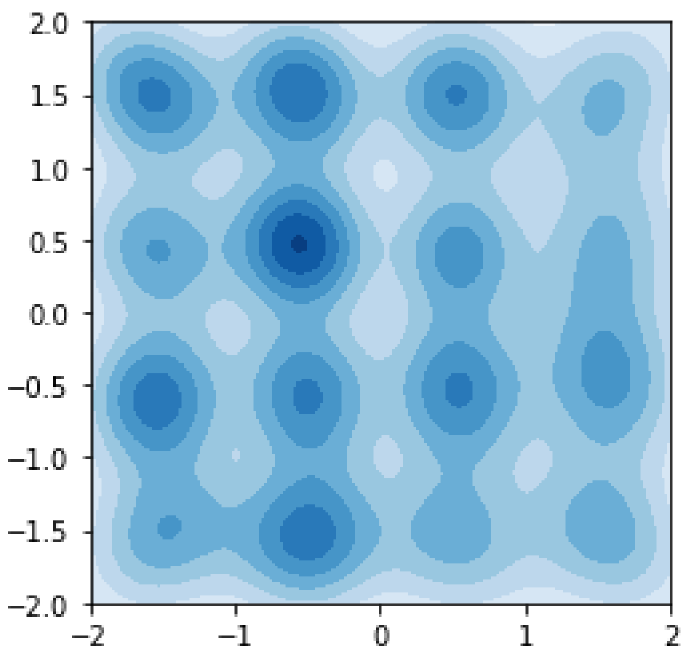

```
i = 2000, discriminant loss = 1.3443, generator loss =0.7208
```



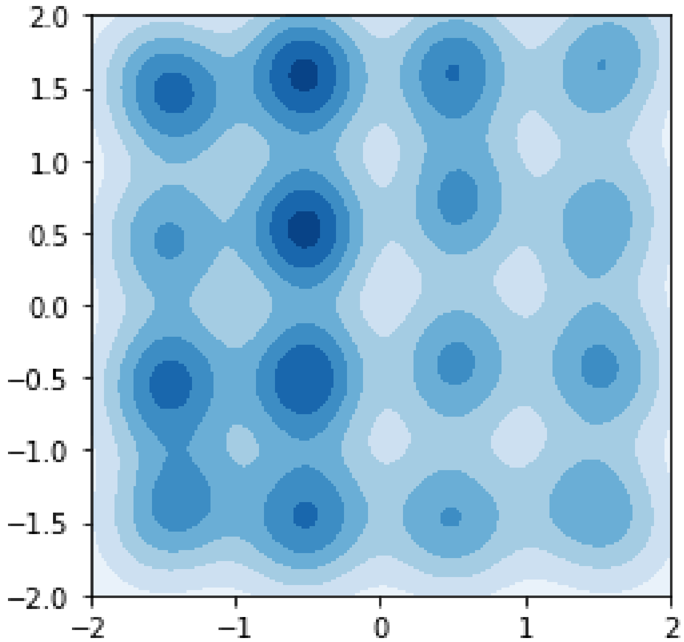
```
i = 4000, discriminant loss = 1.3732, generator loss =0.7006
```



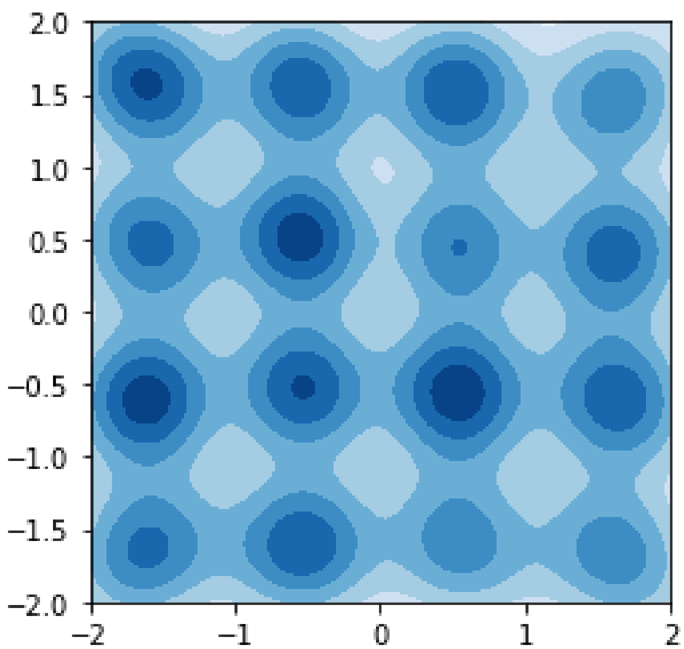

```
i = 6000, discriminant loss = 1.3827, generator loss =0.6935
```



```
i = 8000, discriminant loss = 1.3852, generator loss =0.6940
```




```
i = 10000, discriminant loss = 1.3853, generator loss =0.6939
```



1.7 Experiment 5: Optimistic Mirror Descent (OMD) algorithm implementation

In [0]:
```
from tensorflow.python.ops import control_flow_ops
from tensorflow.python.ops import math_ops
from tensorflow.python.ops import state_ops
from tensorflow.python.framework import ops
from tensorflow.python.training import optimizer


class AdamirrorOptimizer(optimizer.Optimizer):


  def __init__(self, learning_rate=0.001, beta1=0.9, beta2=0.999, epsilon=1e-8,
               use_locking=False, name="Adamirror"):


    super(AdamirrorOptimizer, self).__init__(use_locking, name)
    self._lr = learning_rate
    self._beta1 = beta1
    self._beta2 = beta2

    self._lr_t = None
    self._beta1_t = None
    self._beta2_t = None
    self._t = None

  def _prepare(self):
    self._lr_t = ops.convert_to_tensor(self._lr, name="learning_rate")
    self._beta1_t = ops.convert_to_tensor(self._beta1, name="beta1")
    self._beta2_t = ops.convert_to_tensor(self._beta2, name="beta2")
    self._t = ops.convert_to_tensor(0, name="t")


  def _create_slots(self, var_list):
    for v in var_list:
      self._zeros_slot(v, "m", self._name)
      self._zeros_slot(v, "v", self._name)
      self._zeros_slot(v, "g", self._name)

  def _apply_dense(self, grad, var):
    lr_t = math_ops.cast(self._lr_t, var.dtype.base_dtype)
    beta1_t = math_ops.cast(self._beta1_t, var.dtype.base_dtype)
    beta2_t = math_ops.cast(self._beta2_t, var.dtype.base_dtype)
    if var.dtype.base_dtype == tf.float16:
        eps = 1e-7
```

```
        else:
            eps = 1e-8
        t = self._t

        t = t+1
        v = self.get_slot(var, "v")
        v_t = v.assign(beta2_t * v + (1. - beta2_t) * tf.square(grad))
        m = self.get_slot(var, "m")
        m_t = m.assign( beta1_t * m + (1. - beta1_t) * grad )
        v_t_hat = tf.div(v_t, 1. - tf.pow(beta2_t, t))
        m_t_hat = tf.div(m_t, 1. - tf.pow(beta1_t, t))

        g_t = tf.div( m_t_hat, tf.sqrt(v_t_hat)+eps )
        g_t_1 = self.get_slot(var, "g")
        g_t = g_t_1.assign( g_t )

        t_t = self._t.assign(t)

        var_update = state_ops.assign_sub(var, 2. * lr_t * g_t - lr_t * g_t_1) #Adam would
        return control_flow_ops.group(*[var_update, m_t, v_t, g_t, t_t])

      def _apply_sparse(self, grad, var):
        raise NotImplementedError("Sparse gradient updates are not supported.")
```

In [0]:
```
x = tf.constant([[2, 2.0], [3, 3]])
y = tf.constant([[8, 16], [2, 3]])
print(tf.pow(x, y))  # [[256, 65536], [9, 27]]
```

```
---------------------------------------------------------------------------

ValueError                                Traceback (most recent call last)

/usr/local/lib/python3.6/dist-packages/tensorflow/python/framework/op_def_library.py i
    526                 as_ref=input_arg.is_ref,
--> 527                 preferred_dtype=default_dtype)
    528           except TypeError as err:


/usr/local/lib/python3.6/dist-packages/tensorflow/python/framework/ops.py in internal_
   1223     if ret is None:
-> 1224       ret = conversion_func(value, dtype=dtype, name=name, as_ref=as_ref)
   1225


/usr/local/lib/python3.6/dist-packages/tensorflow/python/framework/ops.py in _TensorTe
   1017         "Tensor conversion requested dtype %s for Tensor with dtype %s: %r" %
-> 1018         (dtype.name, t.dtype.name, str(t)))
```

```
   1019   return t

    ValueError: Tensor conversion requested dtype float32 for Tensor with dtype int32: 'Te

During handling of the above exception, another exception occurred:

    TypeError                                 Traceback (most recent call last)

    <ipython-input-29-dce0577ce282> in <module>()
      1 x = tf.constant([[2, 2.0], [3, 3]])
      2 y = tf.constant([[8, 16], [2, 3]])
----> 3 print(tf.pow(x, y))  # [[256, 65536], [9, 27]]

    /usr/local/lib/python3.6/dist-packages/tensorflow/python/util/dispatch.py in wrapper(*a
    178     """Call target, and fall back on dispatchers if there is a TypeError."""
    179     try:
--> 180       return target(*args, **kwargs)
    181     except (TypeError, ValueError):
    182       # Note: convert_to_eager_tensor currently raises a ValueError, not a

    /usr/local/lib/python3.6/dist-packages/tensorflow/python/ops/math_ops.py in pow(x, y, n
    448   """
    449   with ops.name_scope(name, "Pow", [x]) as name:
--> 450     return gen_math_ops._pow(x, y, name=name)
    451
    452

    /usr/local/lib/python3.6/dist-packages/tensorflow/python/ops/gen_math_ops.py in _pow(x
   6970   # Add nodes to the TensorFlow graph.
   6971   _, _, _op = _op_def_lib._apply_op_helper(
-> 6972         "Pow", x=x, y=y, name=name)
   6973   _result = _op.outputs[:]
   6974   _inputs_flat = _op.inputs

    /usr/local/lib/python3.6/dist-packages/tensorflow/python/framework/op_def_library.py in
    561                   "%s type %s of argument '%s'." %
    562                   (prefix, dtypes.as_dtype(attrs[input_arg.type_attr]).name,
--> 563                    inferred_from[input_arg.type_attr]))
    564
    565           types = [values.dtype]
```

```
TypeError: Input 'y' of 'Pow' Op has type int32 that does not match type float32 of arg
```

1.8 Experiment 6: Comparison of learning rate threshold of OMD and SGA in Gaussian mixture model

In [0]:
```
def reset_and_build_graph_with_lr(depth, width, x_real_builder, z_dim, batch_size, lea
    tf.reset_default_graph()

    x_real = x_real_builder(batch_size)
    x_dim = x_real.get_shape().as_list()[1]
    generator = MLP(depth, width, x_dim, 'generator')
    discriminator = MLP(depth, width, 1, 'discriminator')
    z = tf.random_normal([batch_size, z_dim])
    x_fake = generator(z)
    disc_out_real = discriminator(x_real)
    disc_out_fake = discriminator(x_fake)

    # Loss
    disc_loss_real = tf.reduce_mean(
        tf.nn.sigmoid_cross_entropy_with_logits(
            logits=disc_out_real, labels=tf.ones_like(disc_out_real)))
    disc_loss_fake = tf.reduce_mean(
        tf.nn.sigmoid_cross_entropy_with_logits(
            logits=disc_out_fake, labels=tf.zeros_like(disc_out_fake)))
    disc_loss = disc_loss_real + disc_loss_fake

    gen_loss = tf.reduce_mean(
        tf.nn.sigmoid_cross_entropy_with_logits(
            logits=disc_out_fake, labels=tf.ones_like(disc_out_fake)))
    gen_vars = generator.variable_scope.trainable_variables()
    disc_vars = discriminator.variable_scope.trainable_variables()
    # Compute gradients
    xs = disc_vars + gen_vars
    disc_grads = tf.gradients(disc_loss, disc_vars)
    gen_grads = tf.gradients(gen_loss, gen_vars)
    Xi = disc_grads + gen_grads
    apply_vec = list(zip(Xi, xs))

    if mode == 'RMS':
      optimizer = tf.train.RMSPropOptimizer(learning_rate)
    elif mode == 'OMD':
      optimizer = AdamirrorOptimizer(learning_rate)
    elif mode == 'SGA':
      optimizer = SymplecticOptimizer(learning_rate)
    else:
      raise ValueError('Mode %s not recognised' % mode)
```

```
        with tf.control_dependencies([g for (g, v) in apply_vec]):
          train_op = optimizer.apply_gradients(apply_vec)

        init = tf.global_variables_initializer()

        return train_op, x_fake, z, init, disc_loss, gen_loss
```

```
In [0]: def learn_mixture_of_gaussians_with_lr(mode, learning_rate):
          print(mode)
          def x_real_builder(batch_size):
            sigma = 0.1
            skel = np.array([
                [ 1.50,  1.50],
                [ 1.50,  0.50],
                [ 1.50, -0.50],
                [ 1.50, -1.50],
                [ 0.50,  1.50],
                [ 0.50,  0.50],
                [ 0.50, -0.50],
                [ 0.50, -1.50],
                [-1.50,  1.50],
                [-1.50,  0.50],
                [-1.50, -0.50],
                [-1.50, -1.50],
                [-0.50,  1.50],
                [-0.50,  0.50],
                [-0.50, -0.50],
                [-0.50, -1.50],
            ])
            temp = np.tile(skel, (batch_size // 16 + 1,1))
            mus = temp[0:batch_size,:]
            return mus + sigma*tf.random_normal([batch_size, 2])*.2

          z_dim = 64
          train_op, x_fake, z, init, disc_loss, gen_loss = reset_and_build_graph_with_lr(
              depth=6, width=384, x_real_builder=x_real_builder, z_dim=z_dim,
              batch_size=256, learning_rate=learning_rate, mode=mode)

          train(train_op, x_fake, z, init, disc_loss, gen_loss, z_dim)
```

```
In [0]: lrs = [1e-5, 5e-5, 1e-4, 5e-4, 1e-3]
```

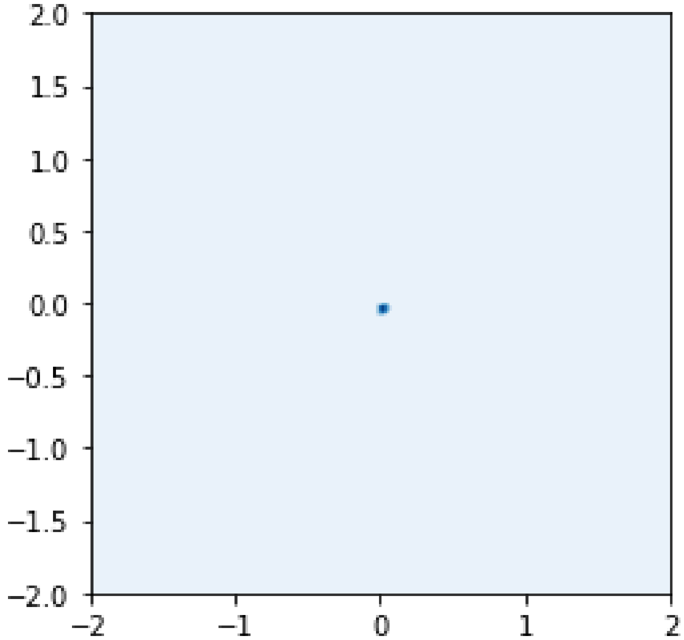
1.8.1 Using OMD

```
In [0]: for lr in lrs:
          learn_mixture_of_gaussians_with_lr('OMD', lr)
```

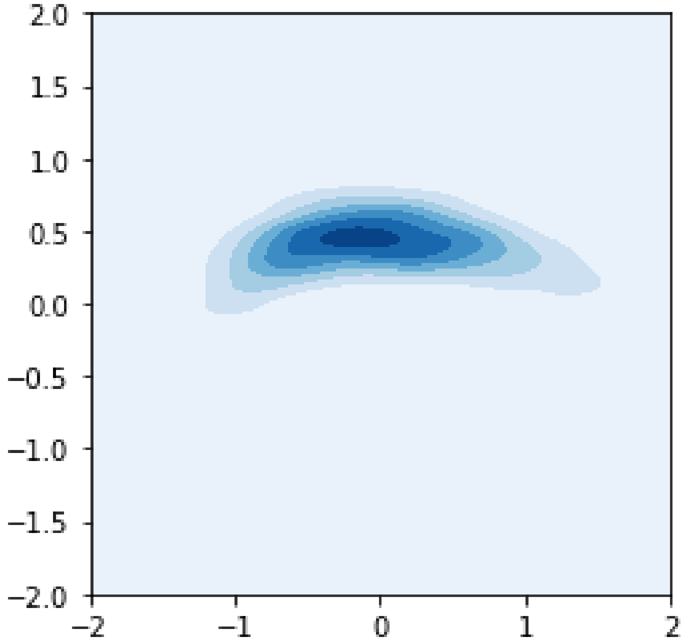
1.8.2 Using SGA

```
In [0]: for lr in lrs:
            learn_mixture_of_gaussians_with_lr('SGA', lr)
```

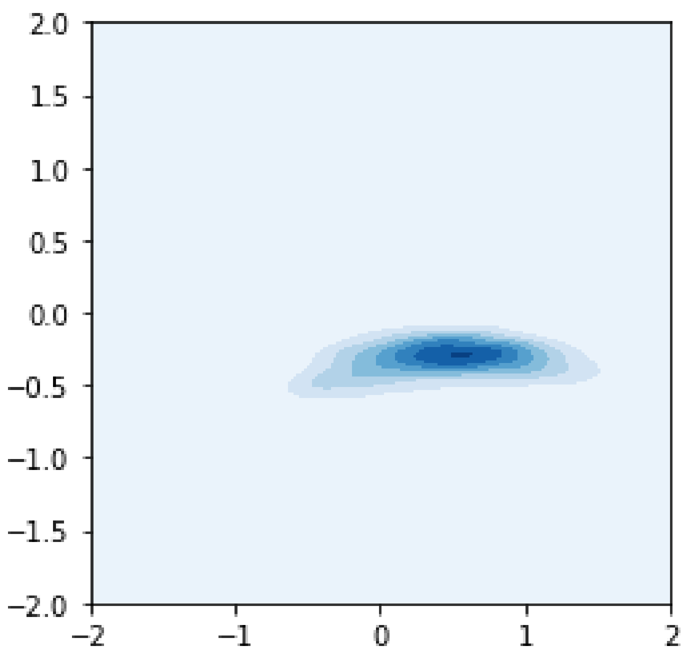
```
SGA
i = 0, discriminant loss = 1.3660, generator loss =0.6924
```



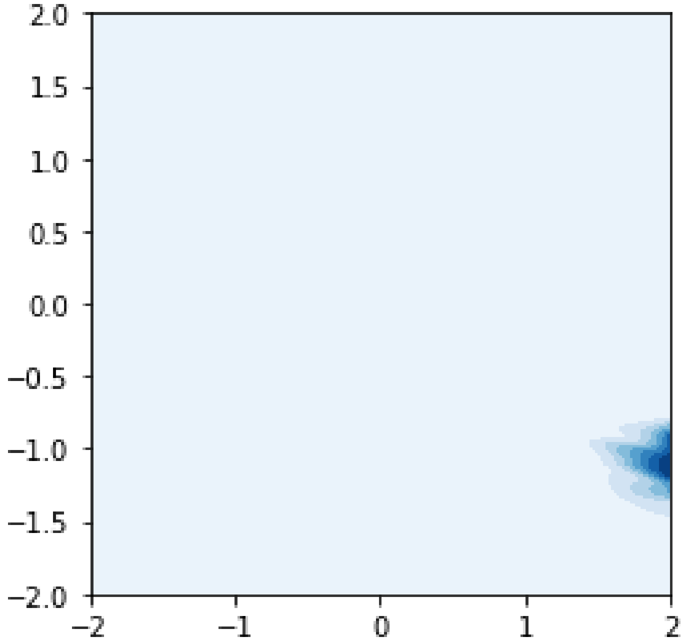

```
i = 2000, discriminant loss = 0.9693, generator loss =0.7837
```



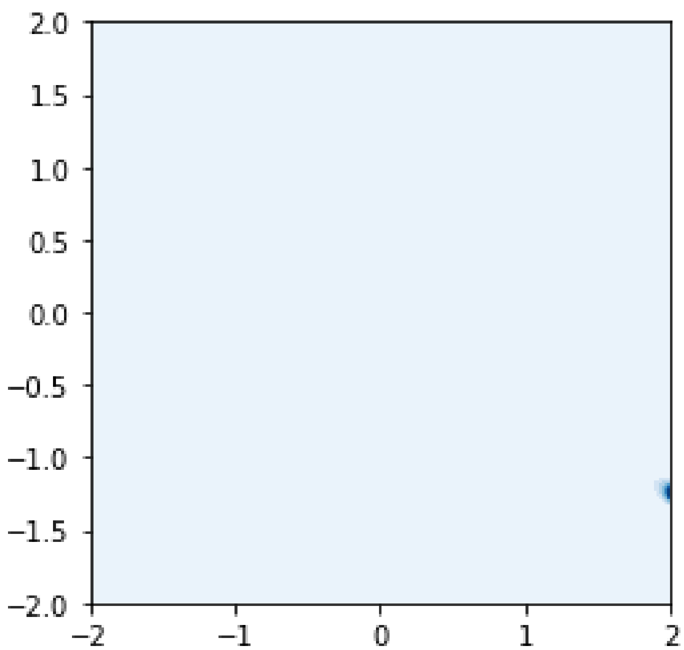

```
i = 4000, discriminant loss = 5.6121, generator loss =0.0429
```



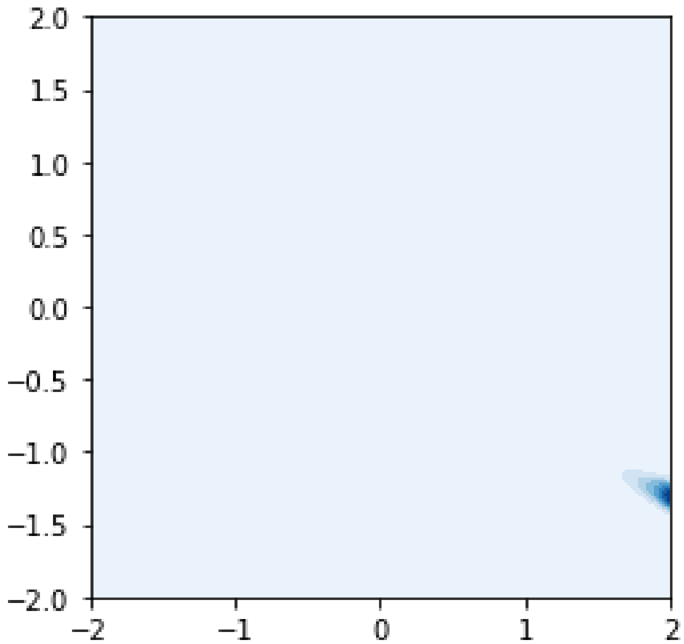
```
i = 6000, discriminant loss = 5.6527, generator loss =0.3095
```



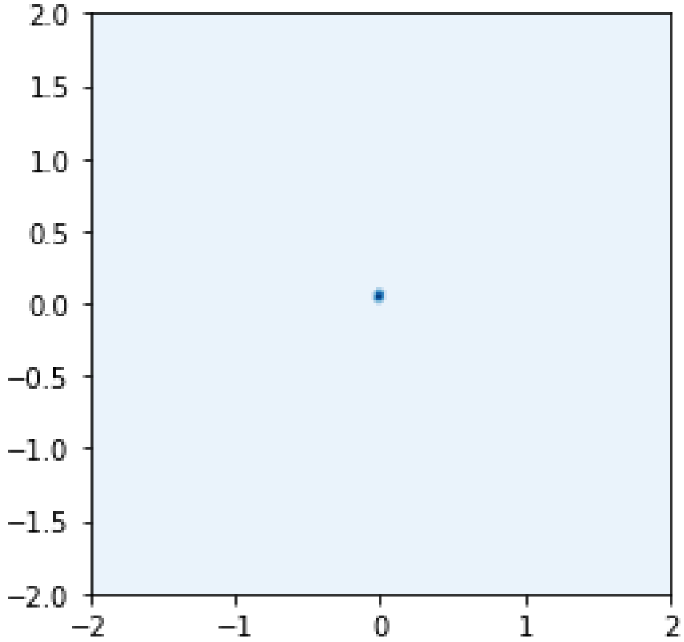

```
i = 8000, discriminant loss = 49.1593, generator loss =1.2838
```



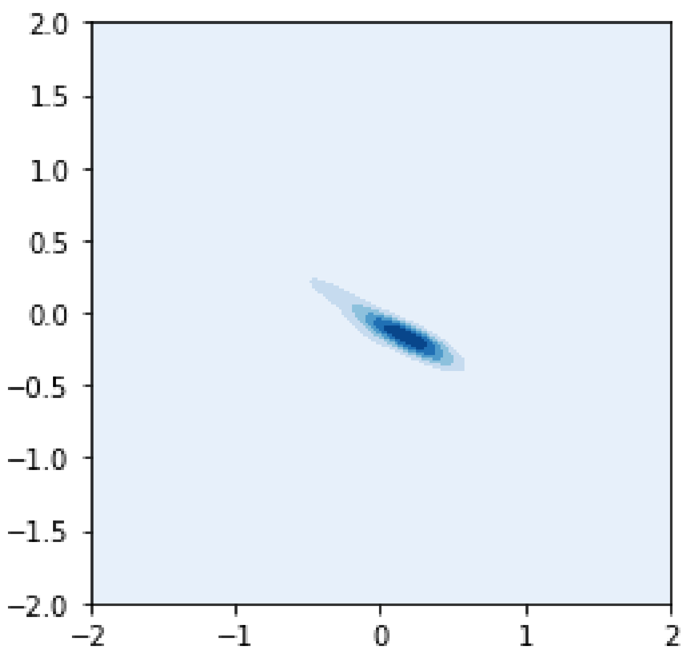
```
i = 10000, discriminant loss = 294.4630, generator loss =12.8131
```



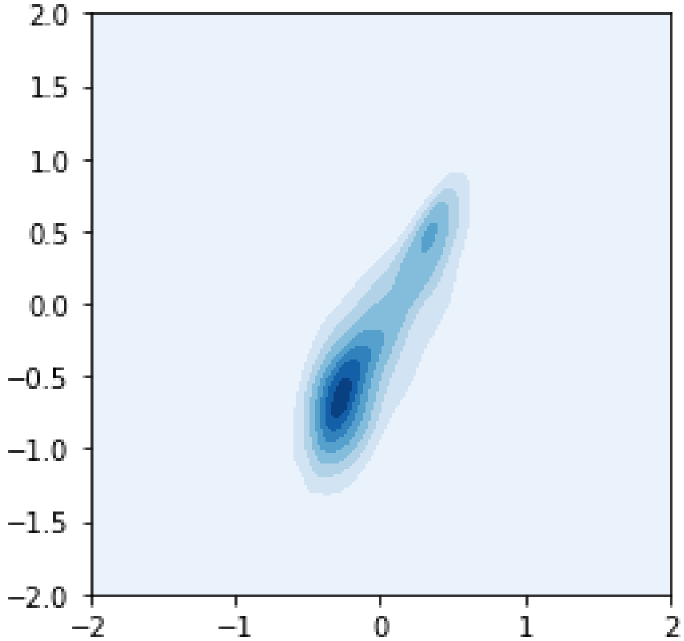

```
SGA
i = 0, discriminant loss = 1.3483, generator loss =0.6920
```



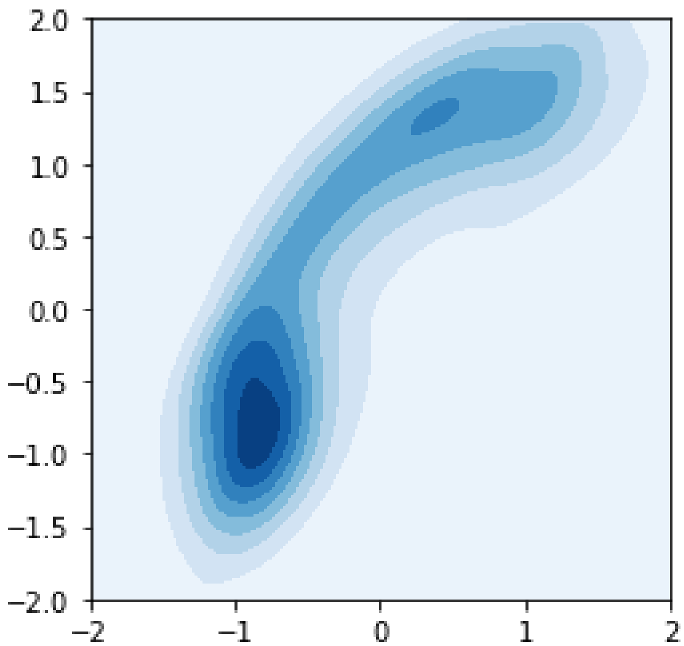

```
i = 2000, discriminant loss = 0.8996, generator loss =0.7104
```



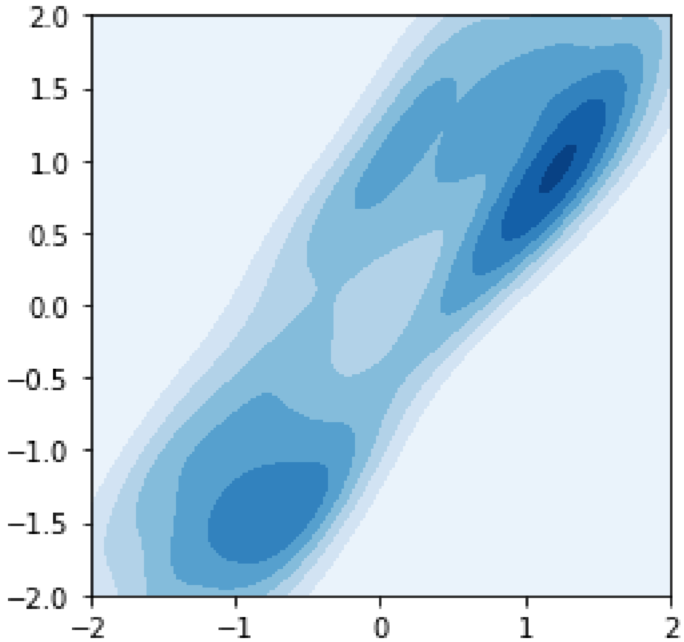
```
i = 4000, discriminant loss = 0.6597, generator loss =1.3435
```



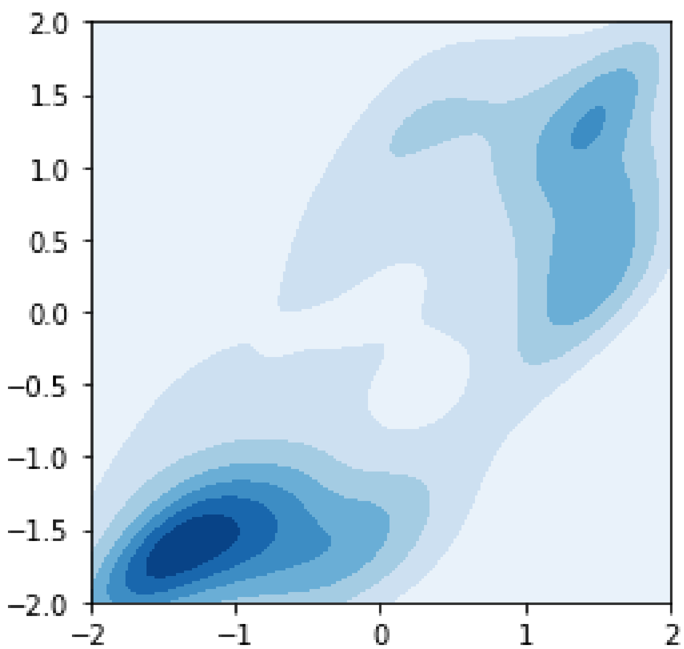

```
i = 6000, discriminant loss = 1.1008, generator loss =0.9895
```



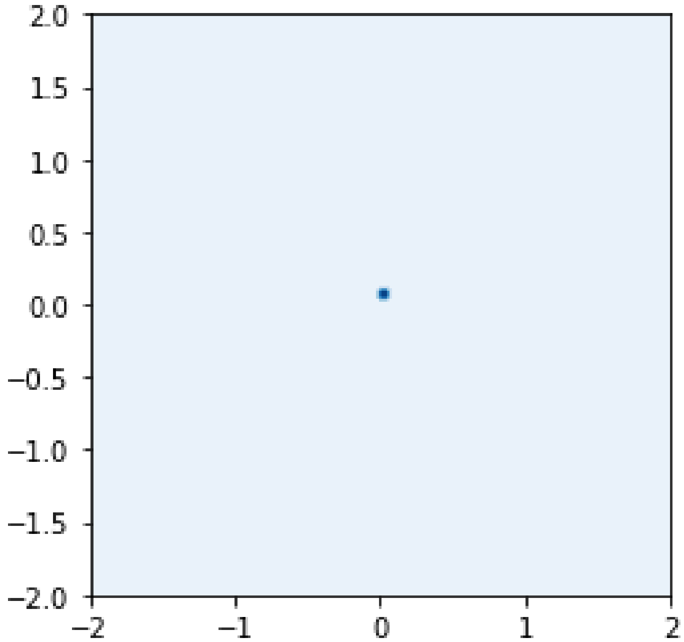
```
i = 8000, discriminant loss = 0.9388, generator loss =1.0736
```



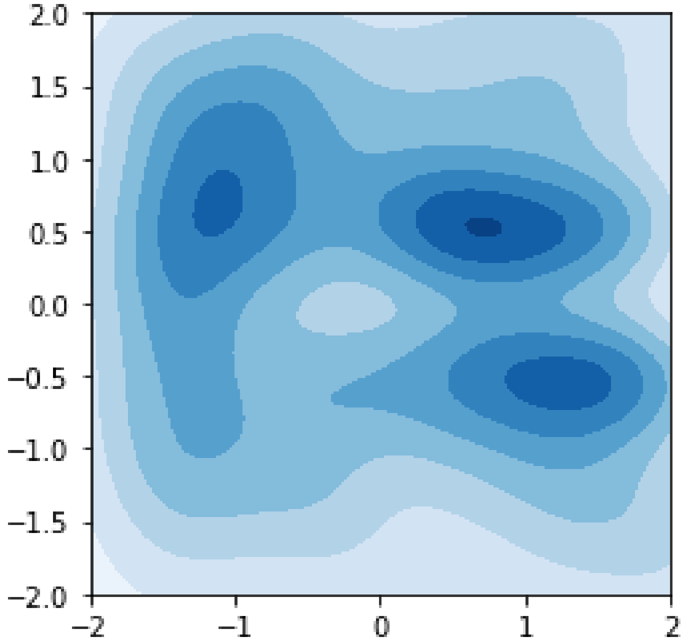
```
i = 10000, discriminant loss = 0.9927, generator loss =1.1249
```



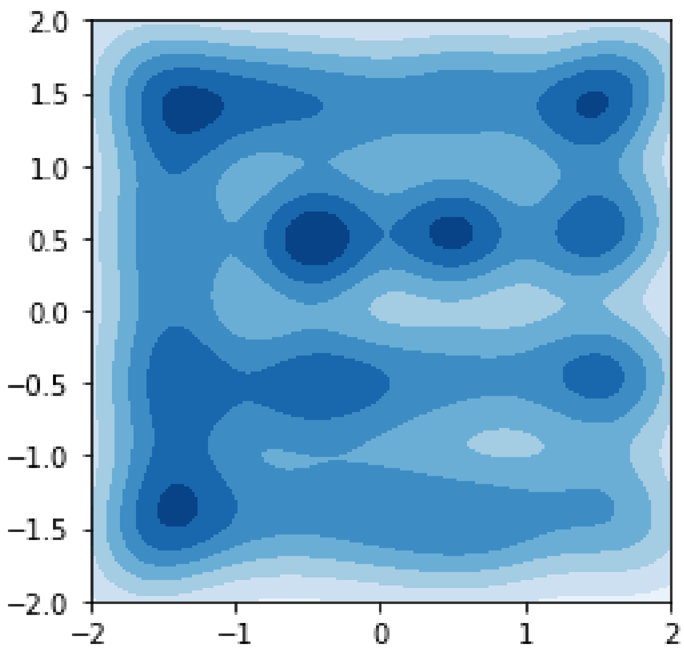
```
SGA
i = 0, discriminant loss = 1.3930, generator loss =0.6940
```



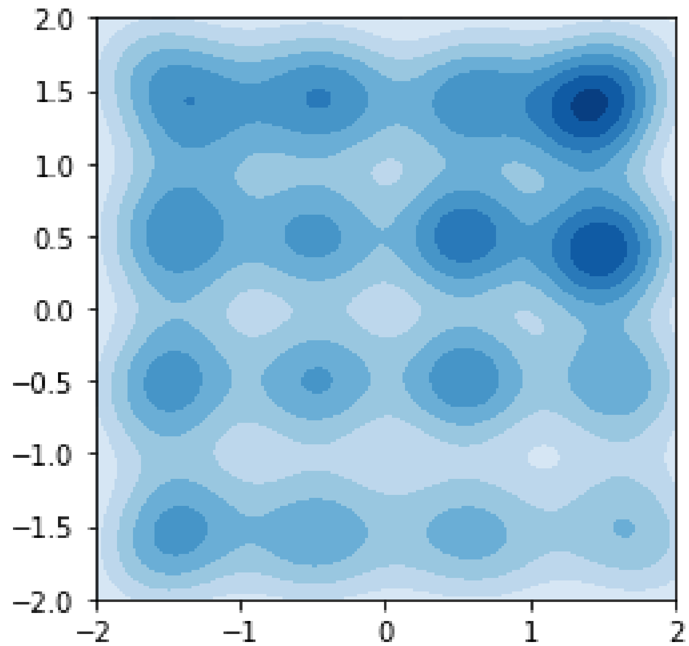

```
i = 2000, discriminant loss = 1.3426, generator loss =0.7182
```



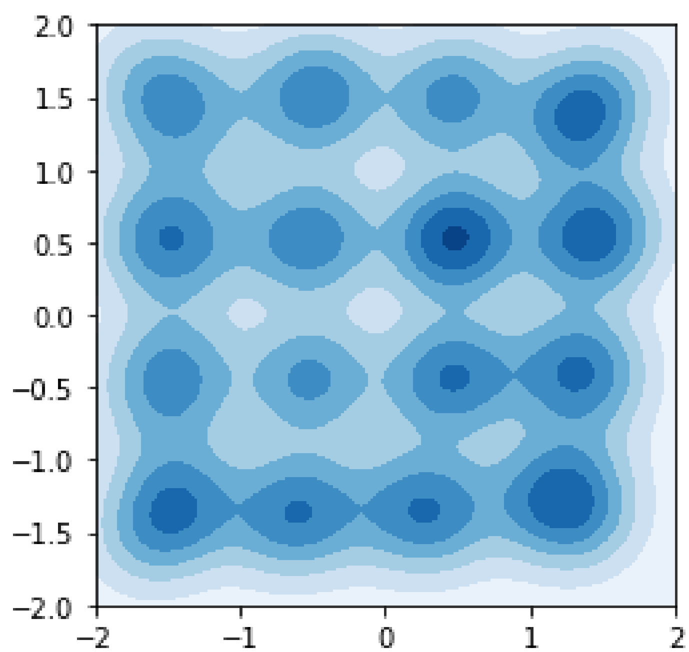

```
i = 4000, discriminant loss = 1.3656, generator loss =0.7059
```



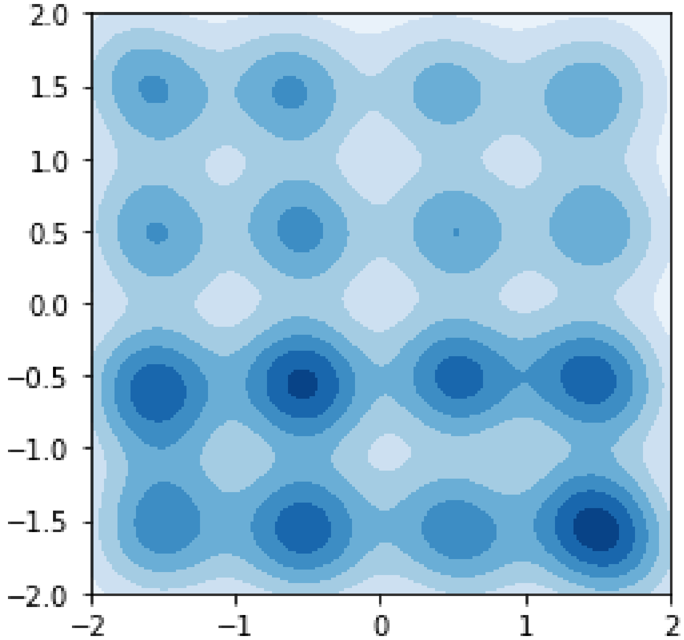
```
i = 6000, discriminant loss = 1.3823, generator loss =0.6944
```



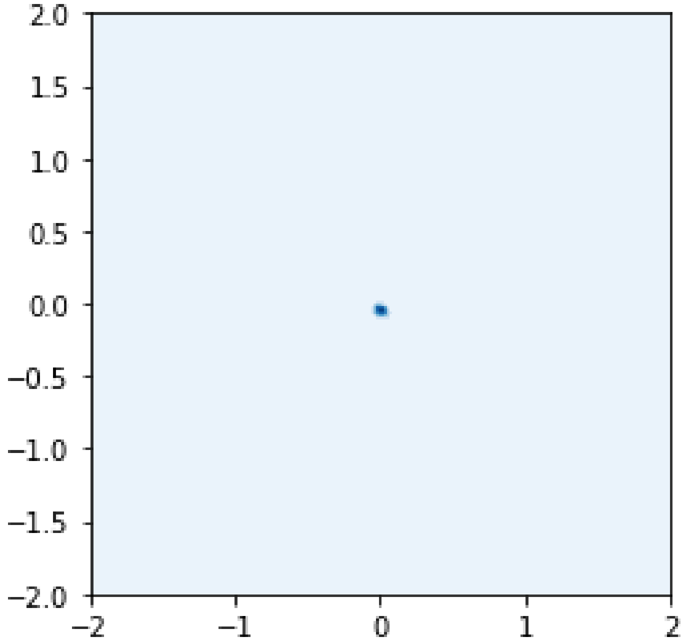

```
i = 8000, discriminant loss = 1.3850, generator loss =0.6939
```



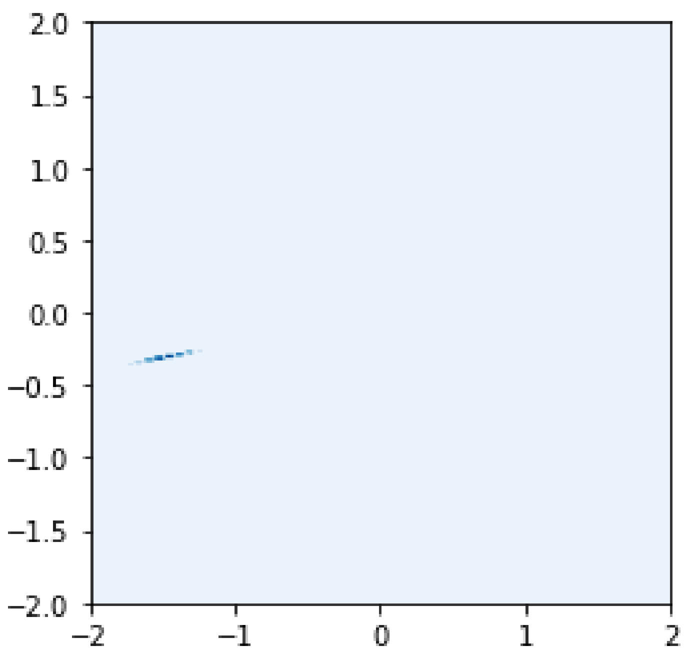
```
i = 10000, discriminant loss = 1.3854, generator loss =0.6939
```



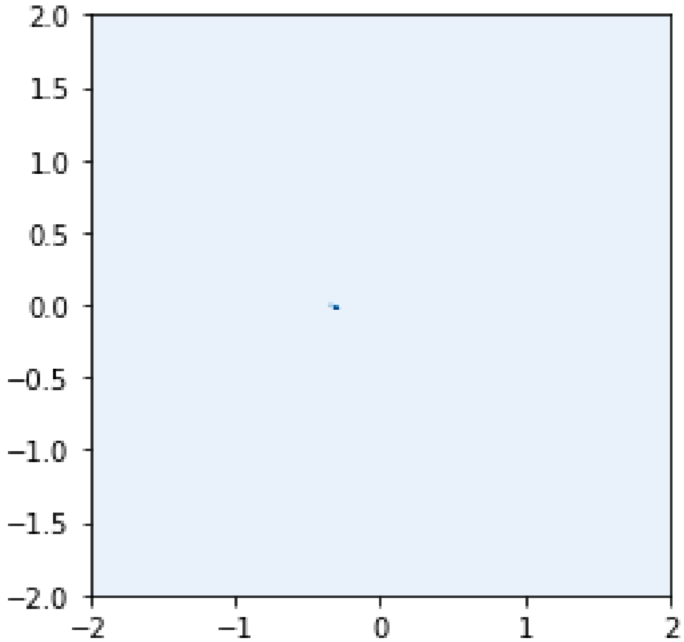

```
SGA
i = 0, discriminant loss = 1.3845, generator loss =0.6932
```



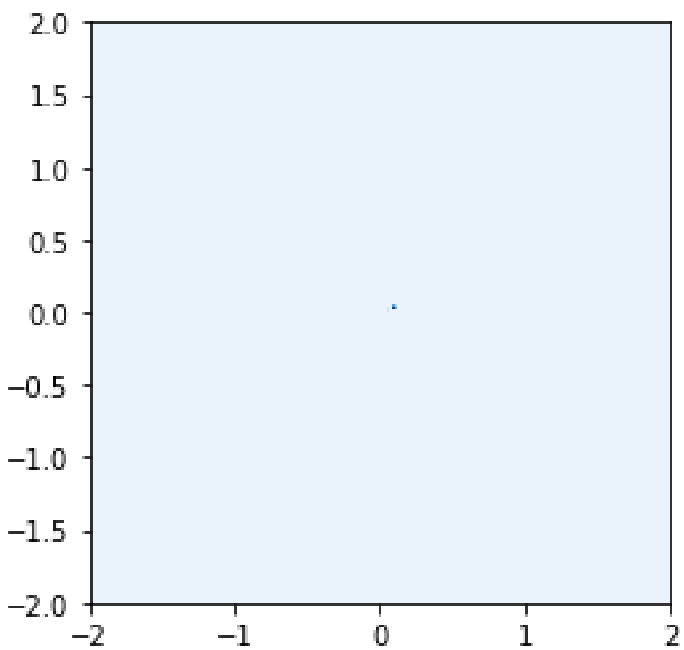
i = 2000, discriminant loss = 0.2664, generator loss =1.9103



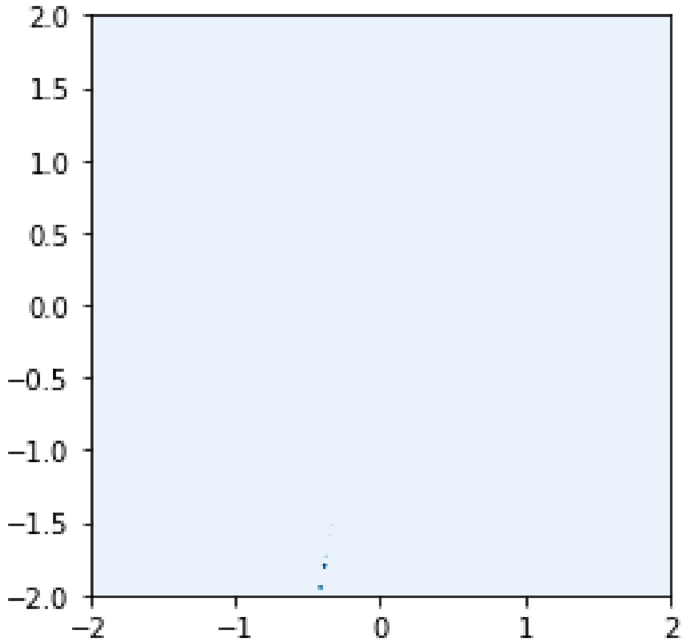
```
i = 4000, discriminant loss = 0.1573, generator loss =2.3711
```



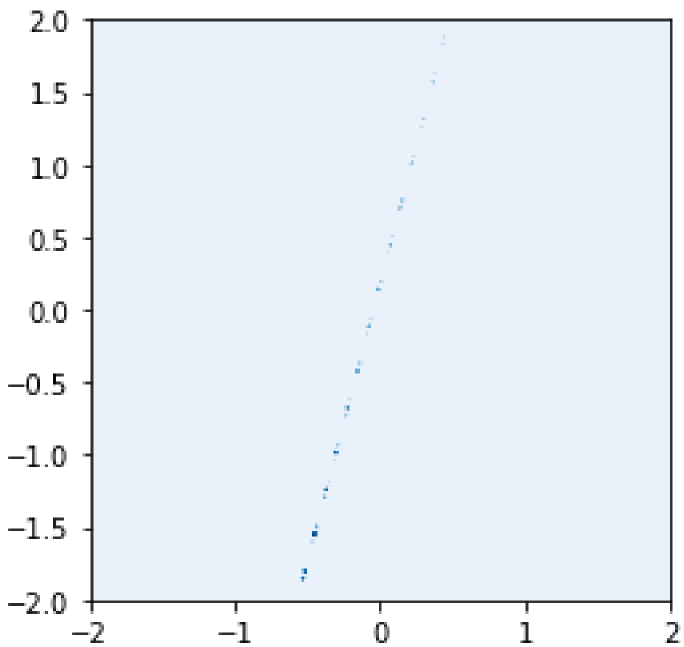

```
i = 6000, discriminant loss = 0.1241, generator loss =2.2053
```



i = 8000, discriminant loss = 0.0150, generator loss =4.2232

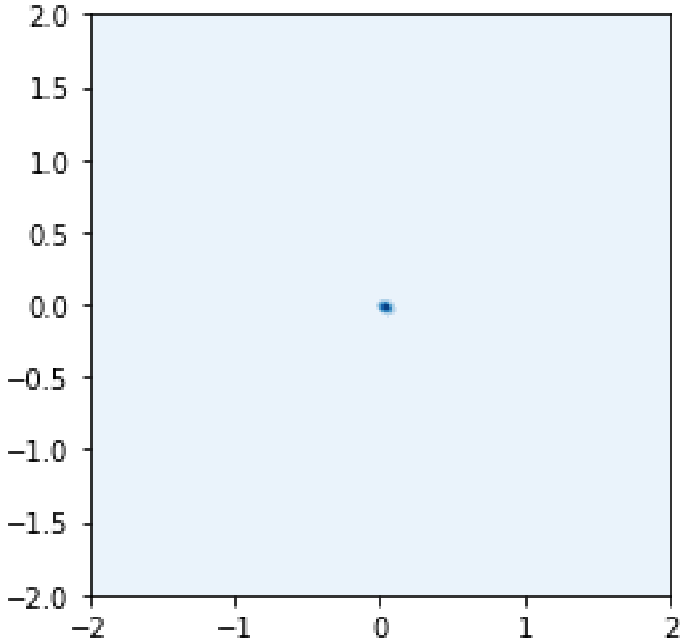


i = 10000, discriminant loss = 0.0006, generator loss =7.6488

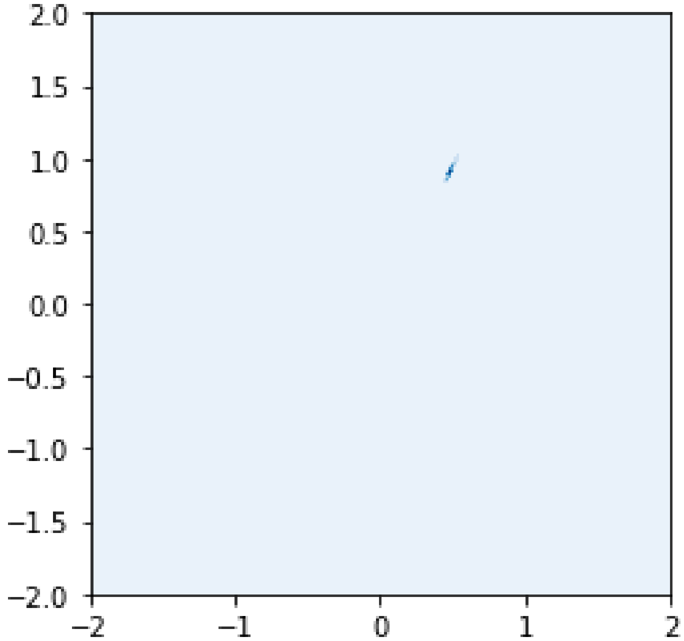


SGA

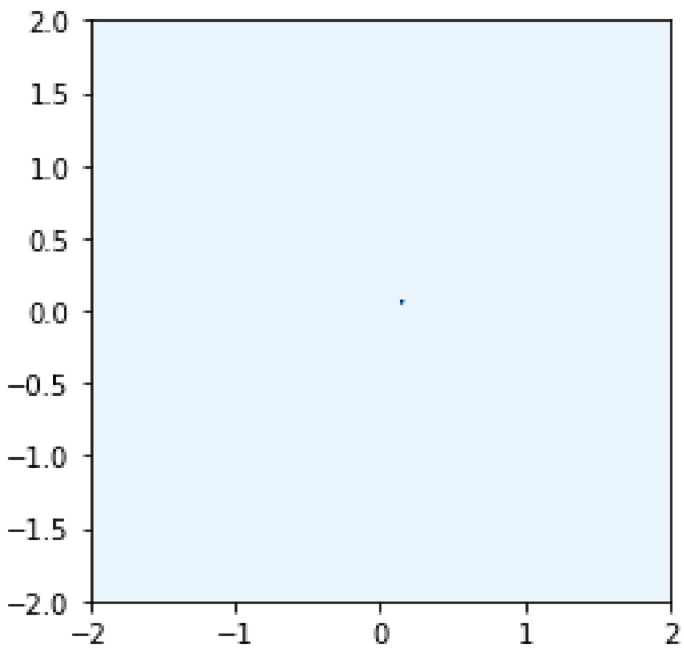
i = 0, discriminant loss = 1.3746, generator loss =0.6926



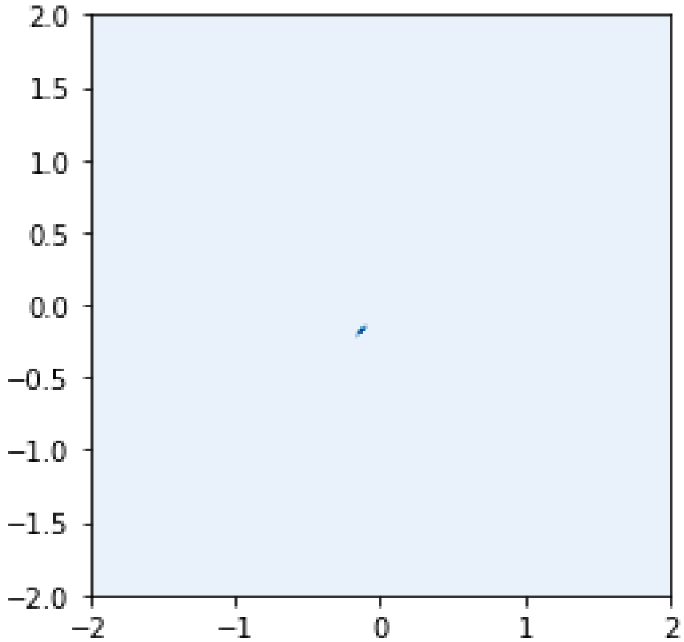
i = 2000, discriminant loss = 0.1350, generator loss =2.0975



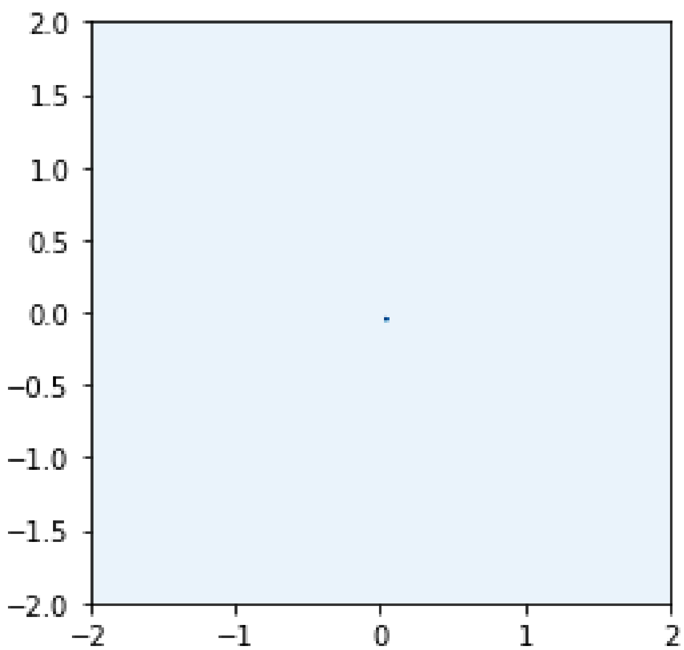
`i = 4000, discriminant loss = 0.0353, generator loss =3.3729`



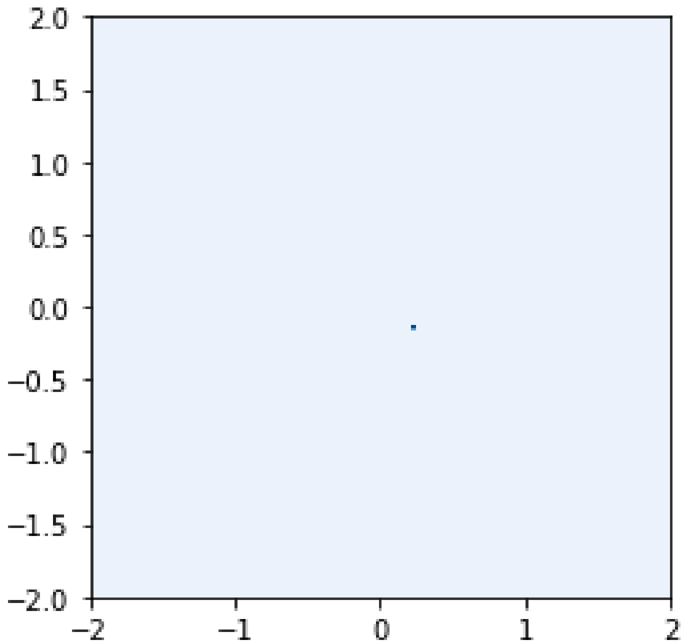
```
i = 6000, discriminant loss = 0.4416, generator loss =2.4975
```




```
i = 8000, discriminant loss = 0.0646, generator loss =2.8549
```

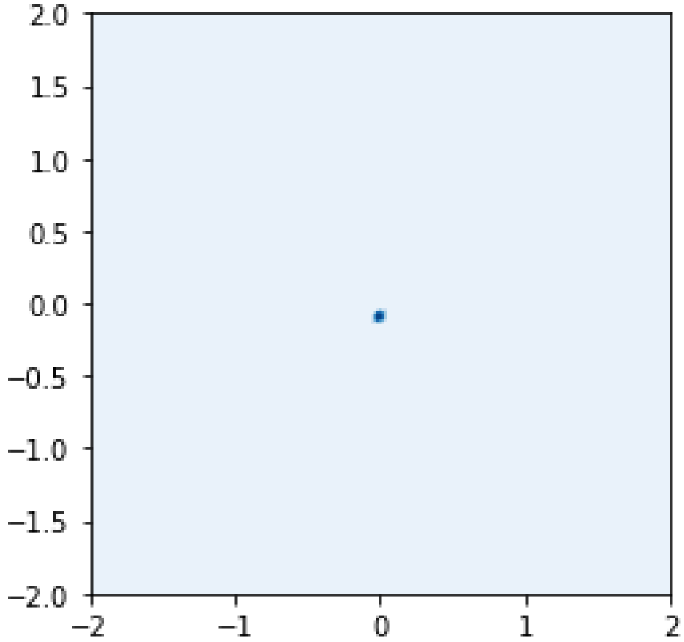


```
i = 10000, discriminant loss = 67803.4141, generator loss =0.0000
```

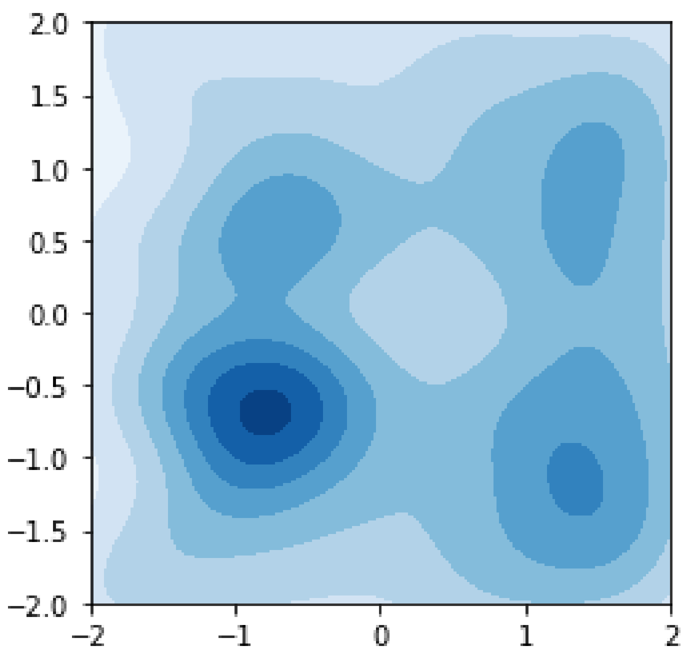



```
In [0]: lrs = [8e-5, 1e-4, 1.2e-4, 1.4e-4, 1.6e-4]
        for lr in lrs:
          learn_mixture_of_gaussians_with_lr('SGA', lr)
```

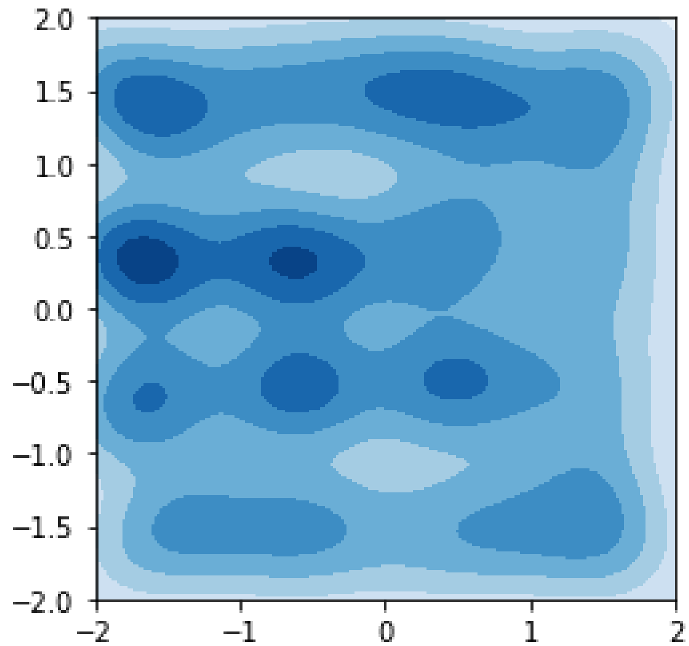
```
SGA
i = 0, discriminant loss = 1.3353, generator loss =0.6883
```



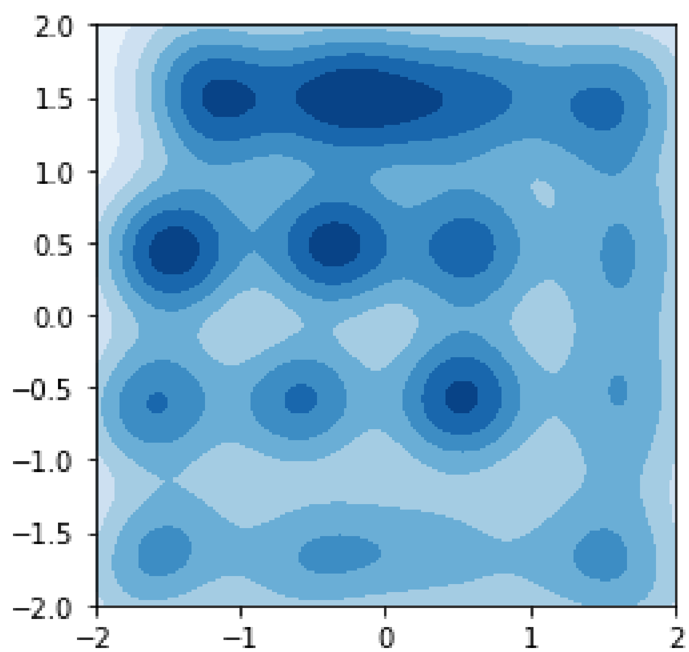
`i = 2000, discriminant loss = 1.3481, generator loss =0.7221`



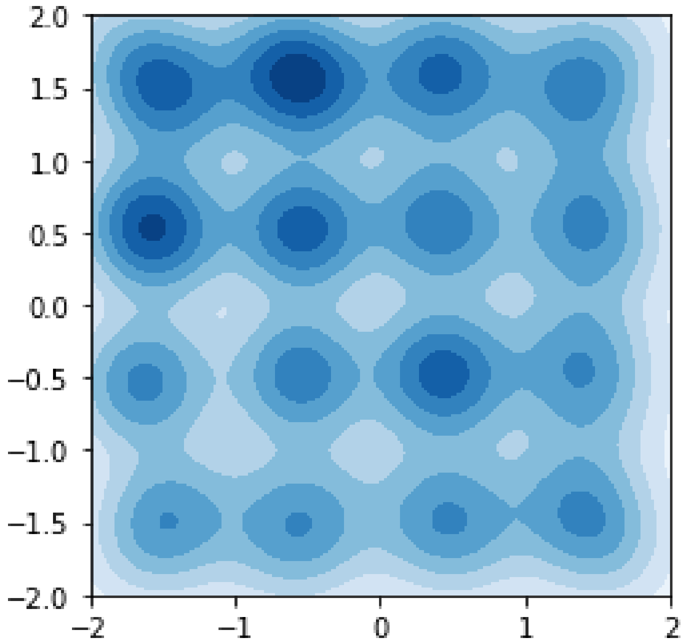
```
i = 4000, discriminant loss = 1.3642, generator loss =0.7061
```



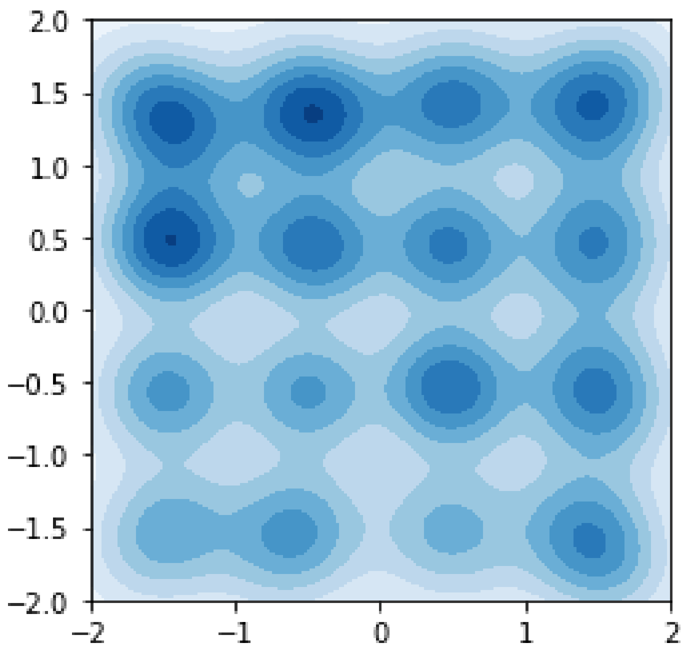

```
i = 6000, discriminant loss = 1.3772, generator loss =0.6963
```



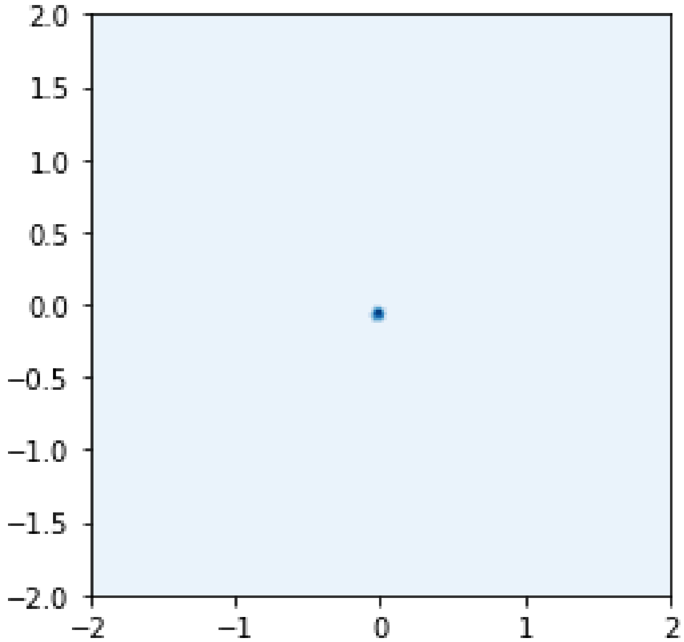
i = 8000, discriminant loss = 1.3843, generator loss =0.6940



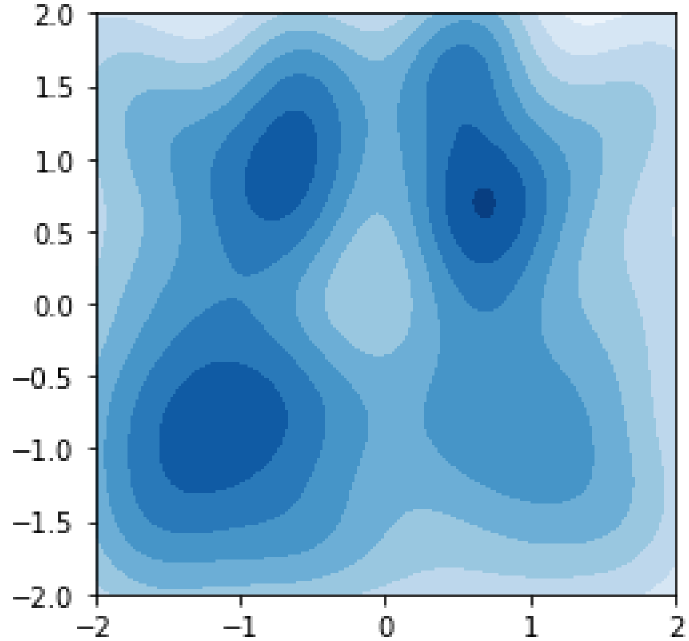
i = 10000, discriminant loss = 1.3854, generator loss =0.6938



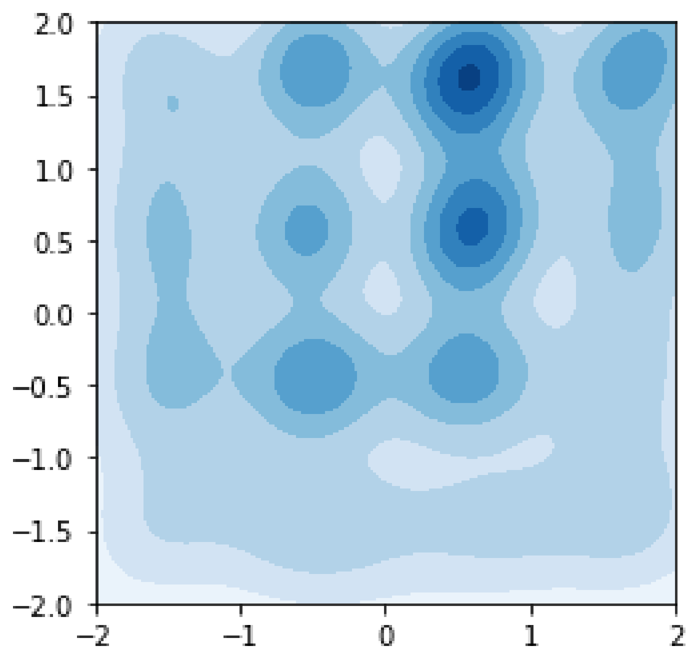
```
SGA
i = 0, discriminant loss = 1.3819, generator loss =0.6926
```



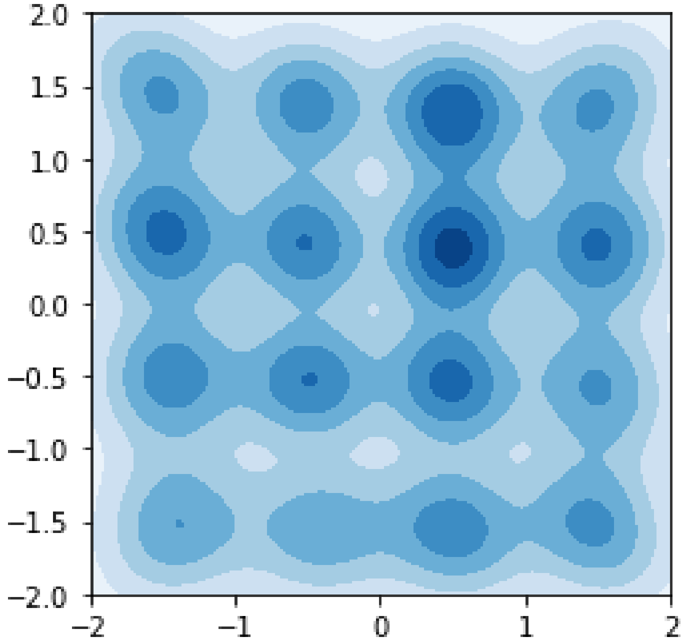

```
i = 2000, discriminant loss = 1.3461, generator loss =0.7409
```



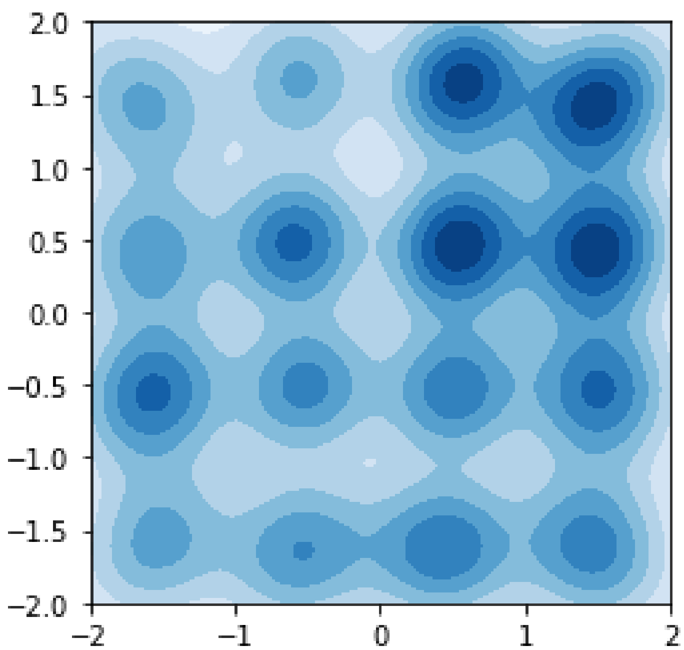

```
i = 4000, discriminant loss = 1.3761, generator loss =0.6976
```



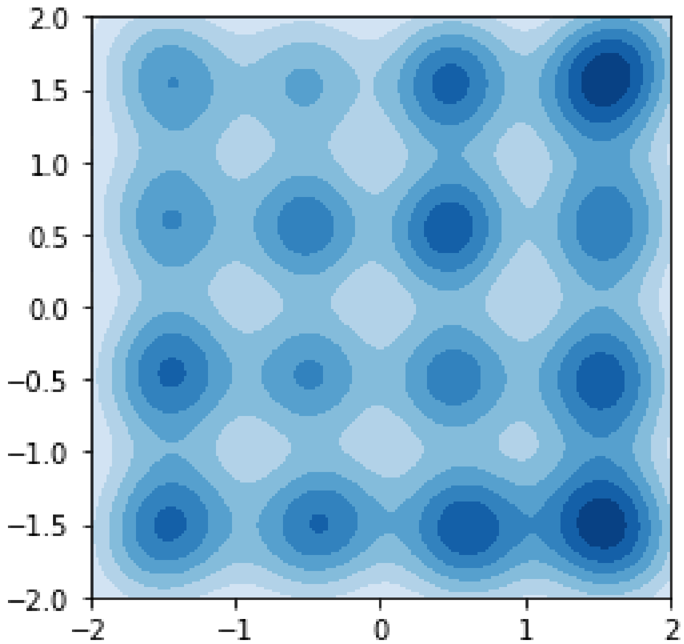
```
i = 6000, discriminant loss = 1.3846, generator loss =0.6938
```



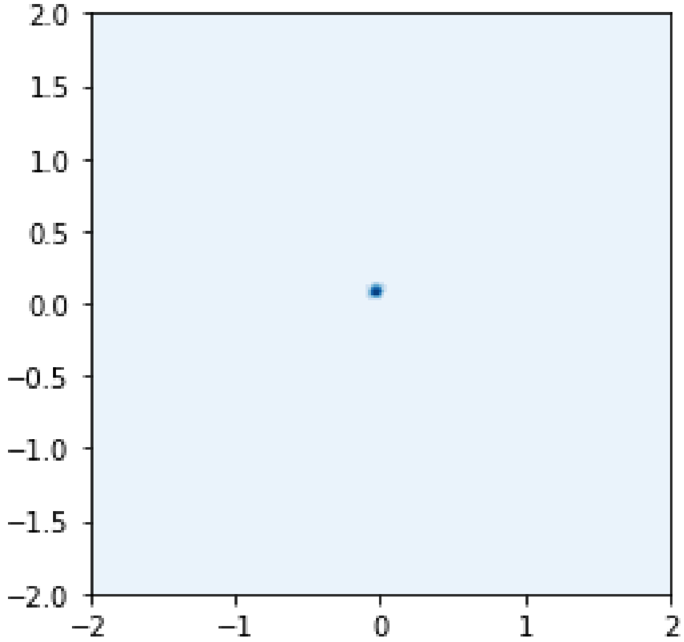

```
i = 8000, discriminant loss = 1.3851, generator loss =0.6940
```



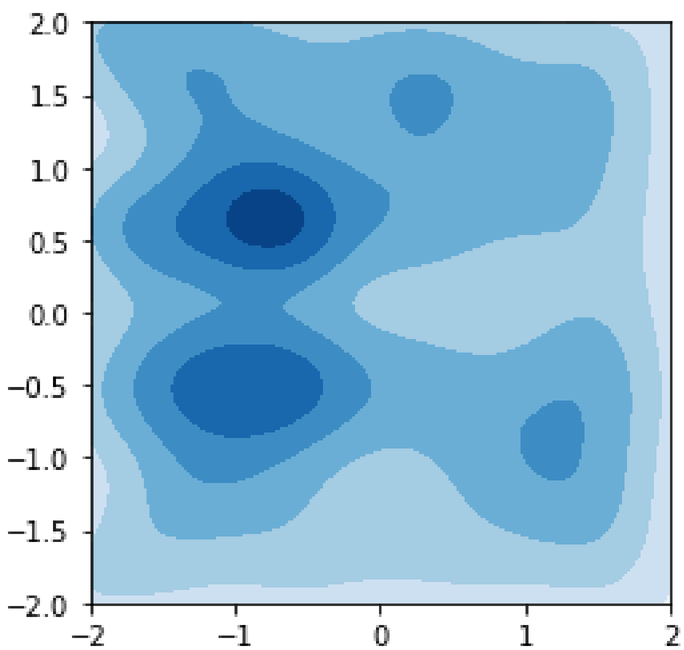
```
i = 10000, discriminant loss = 1.3854, generator loss =0.6940
```



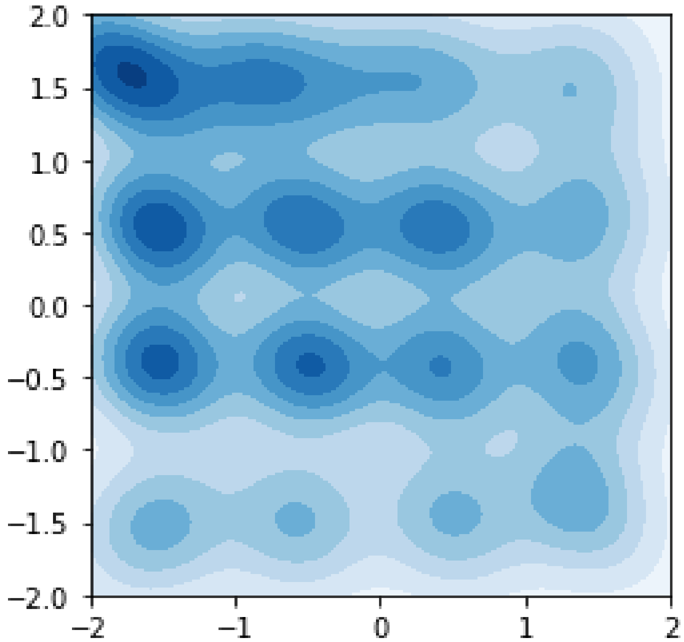

```
SGA
i = 0, discriminant loss = 1.3610, generator loss =0.6920
```



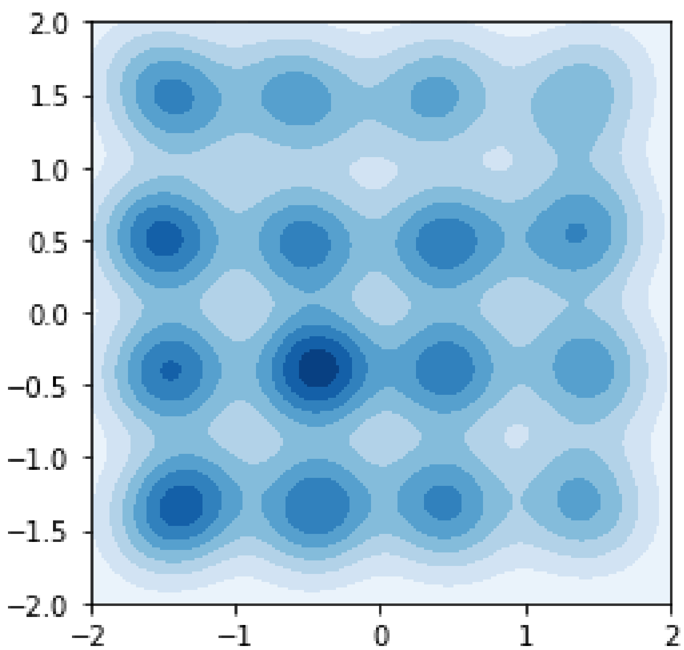

```
i = 2000, discriminant loss = 1.3472, generator loss =0.7152
```



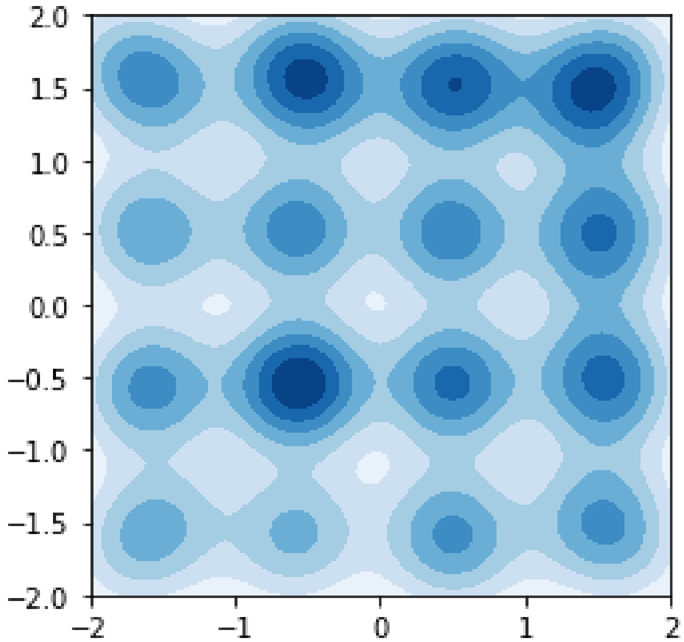
```
i = 4000, discriminant loss = 1.3788, generator loss =0.6979
```



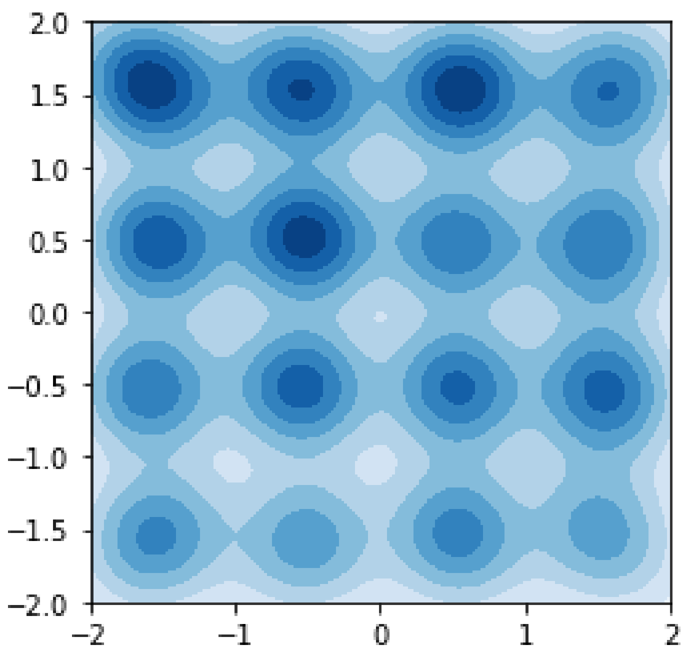

```
i = 6000, discriminant loss = 1.3850, generator loss =0.6937
```



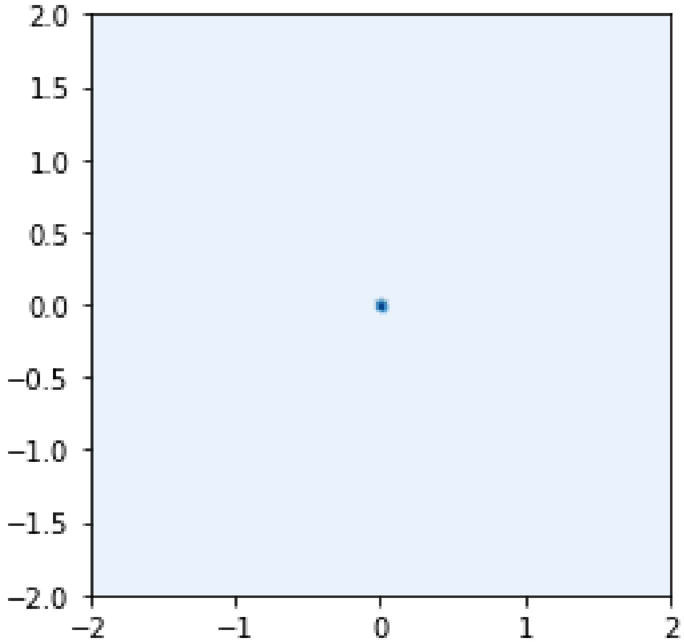
```
i = 8000, discriminant loss = 1.3852, generator loss =0.6936
```



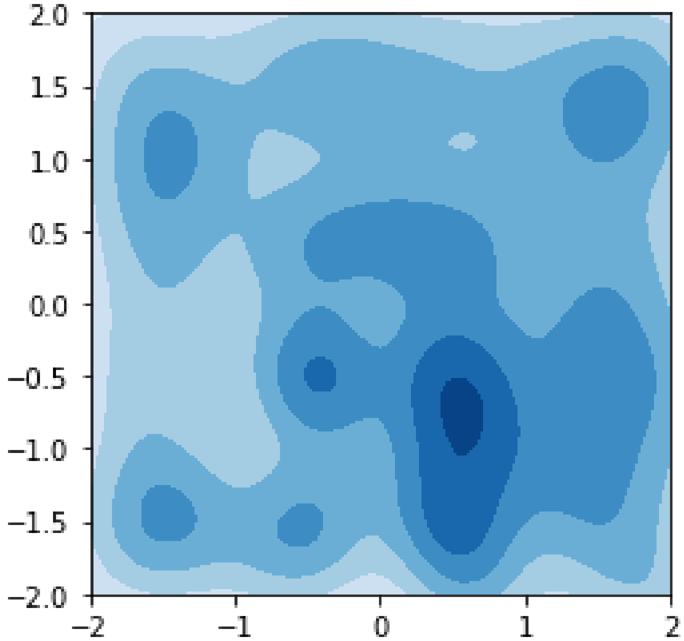

```
i = 10000, discriminant loss = 1.3857, generator loss =0.6931
```



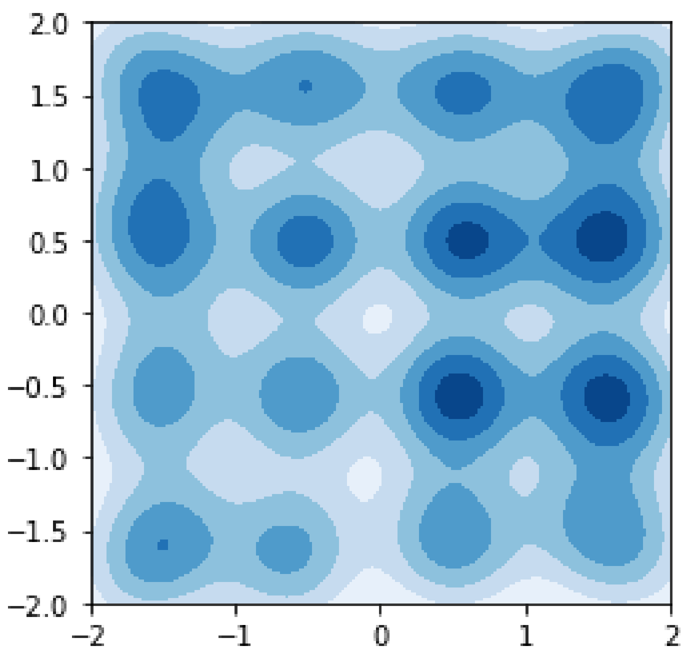
SGA
i = 0, discriminant loss = 1.3879, generator loss =0.6932



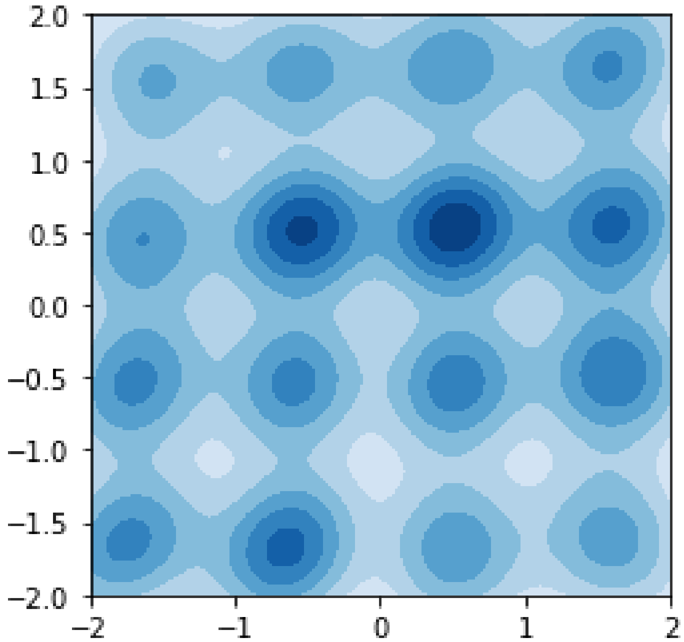
i = 2000, discriminant loss = 1.3501, generator loss =0.7214



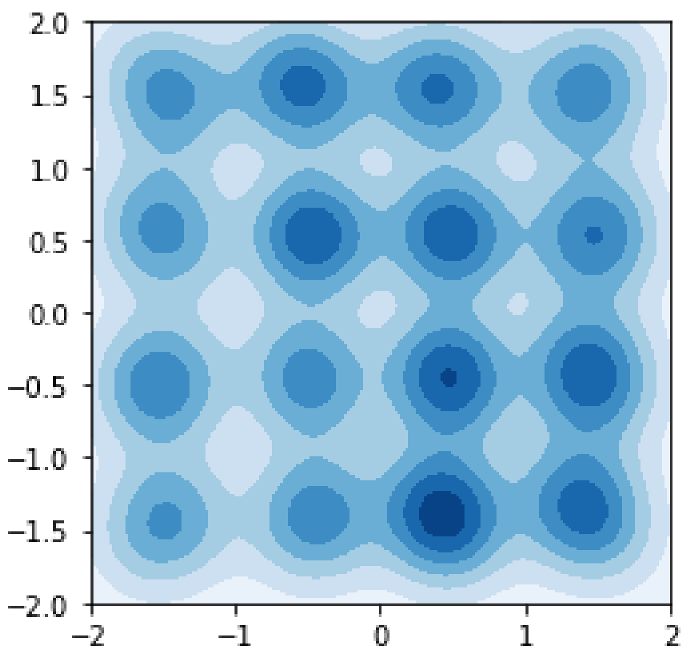

```
i = 4000, discriminant loss = 1.3834, generator loss =0.6938
```



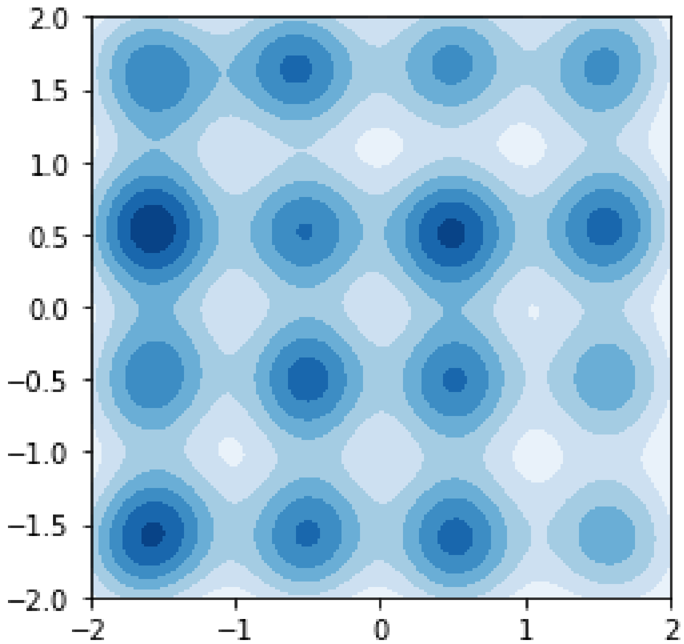
```
i = 6000, discriminant loss = 1.3856, generator loss =0.6934
```



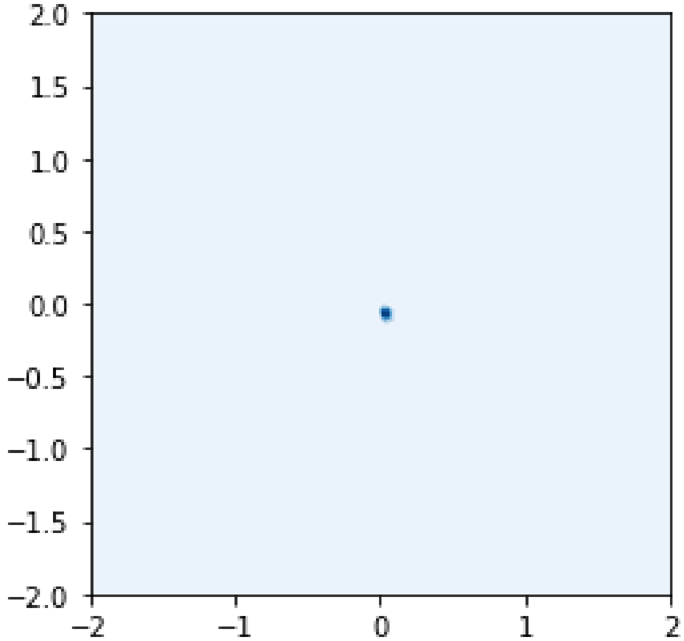

```
i = 8000, discriminant loss = 1.3858, generator loss =0.6933
```



```
i = 10000, discriminant loss = 1.3855, generator loss =0.6939
```




```
SGA
i = 0, discriminant loss = 1.3549, generator loss =0.6912
```



1.9 Experiment 7: Comparison of optimizers when learning a high dimensional Gaussian

Learning a high dimensional Gaussian (dimension 75 in the experiment below) using a GAN is an experiment that was proposed in A Classification–Based Study of Covariate Shift in GAN Distributions by Santurkar in ICML 2018 .

In the experiments below, the graphs plot all 75 singular values, in decreasing order, of the covariance matrix of the data generated by the GAN.

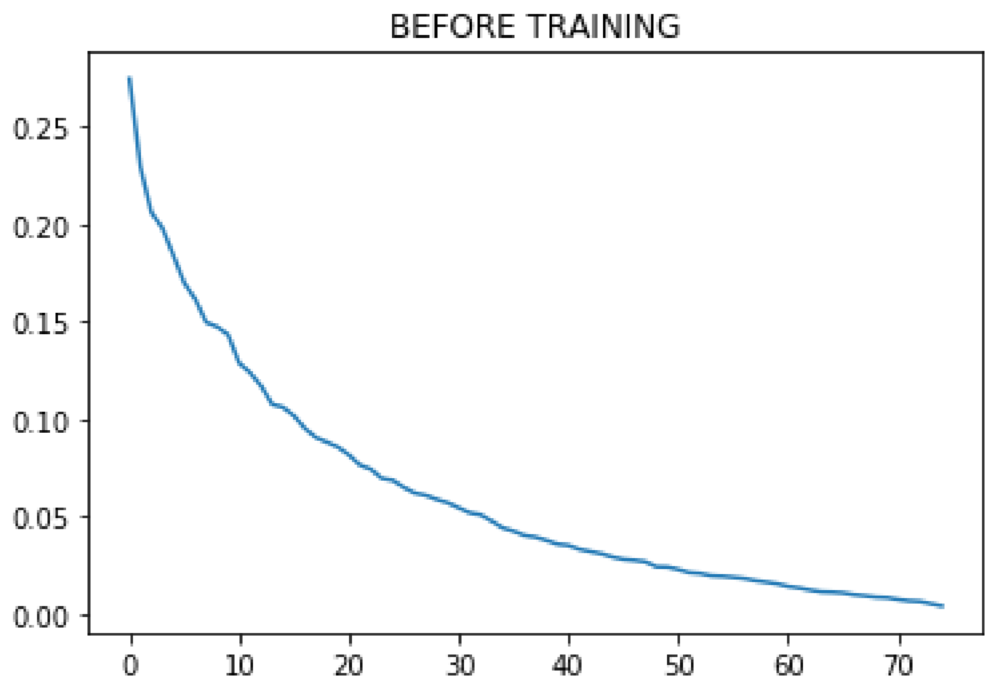
```
In [0]: def compute_eigenvalue(sess, x, n_pts, title):
          """Computes the singular values of the covariance matrix of x.

          The singular values are displayed in decreasing order in a plot.

          Args:
            sess: a Session object.
            x: a Tensor of shape ```(batch_size, x_dim)```
            n_pts: an int; the number of points used to compute the covariance matrix
            title: a string; the title of the displayed plot
          """
          batch_size, x_dim = x.get_shape().as_list()
          # Round n_pts to the next multiple of batch_size
          n_runs = (n_pts + batch_size - 1) // batch_size
          n_pts = n_runs * batch_size
          mean = np.zeros([x_dim])
          moment = np.zeros([x_dim, x_dim])
```

```
  for _ in range(n_runs):
    x_out = sess.run(x)
    mean += np.sum(x_out, axis=0)
    moment += np.matmul(x_out.transpose(), x_out)
  mean /= n_pts
  moment /= n_pts
  mean_2 = np.expand_dims(mean, 0)
  cov = moment - np.matmul(mean_2.transpose(), mean_2)
  u, s, vh = np.linalg.svd(cov)
  plt.plot(s)
  plt.title(title)
  plt.show()


def train(train_op, x_fake, init, disc_loss, gen_loss):
  n_iter = 20001
  n_save = 2000

  with tf.Session() as sess:
    sess.run(init)

    compute_eigenvalue(sess, x_fake, 2**20, 'BEFORE TRAINING')

    for i in range(n_iter):
      sess.run(train_op)
      disc_loss_out, gen_loss_out = sess.run([disc_loss, gen_loss])
      if i % n_save == 0:
        print('i = %d, discriminant loss = %.4f, generator loss =%.4f' %
              (i, disc_loss_out, gen_loss_out))
        compute_eigenvalue(sess, x_fake, 2**15, 'iter %d' % i)

    compute_eigenvalue(sess, x_fake, 2**20, 'AFTER TRAINING')


def high_dim_gaussian_experiment(mode):
  print(mode)

  x_dim = 75
  def x_real_builder(batch_size):
    return tf.random_normal([batch_size, x_dim])

  train_op, x_fake, unused_z, init, disc_loss, gen_loss = reset_and_build_graph(
      depth=2, width=200, x_real_builder=x_real_builder, z_dim=200,
      batch_size=64, learning_rate=2e-4, mode=mode)

  train(train_op, x_fake, init, disc_loss, gen_loss)
```

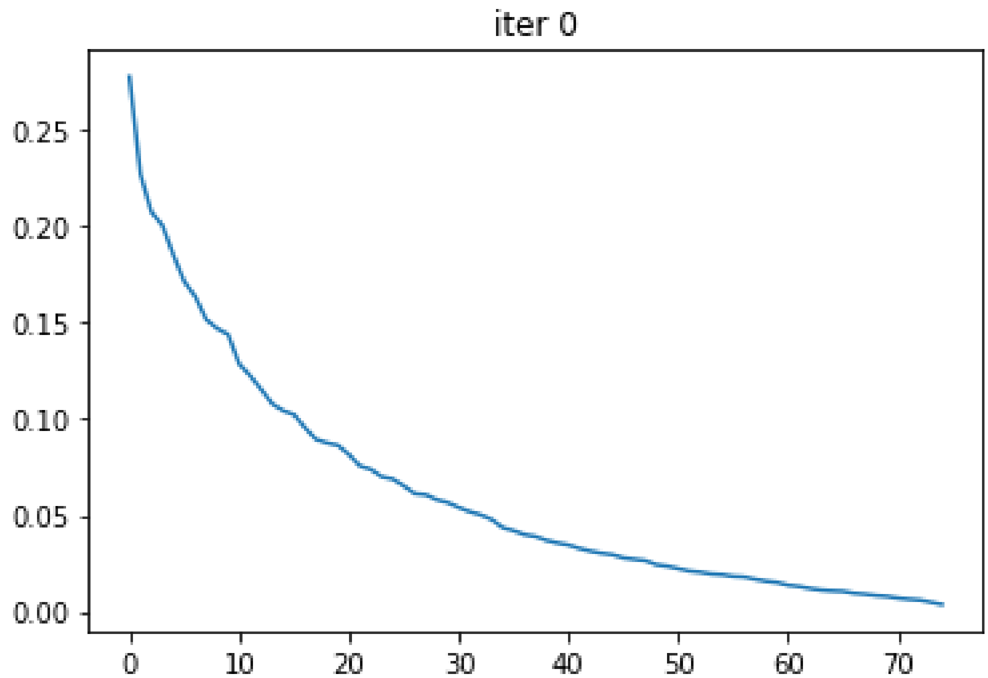
1.9.1 Using RMS

```
In [0]: # Use plain RmsProp to optimise the GAN parameters.
        # This experiment demonstrate how traditional GAN training fails to learn all
        # the directions of a high dimensional Gaussian.
        high_dim_gaussian_experiment('RMS')
```

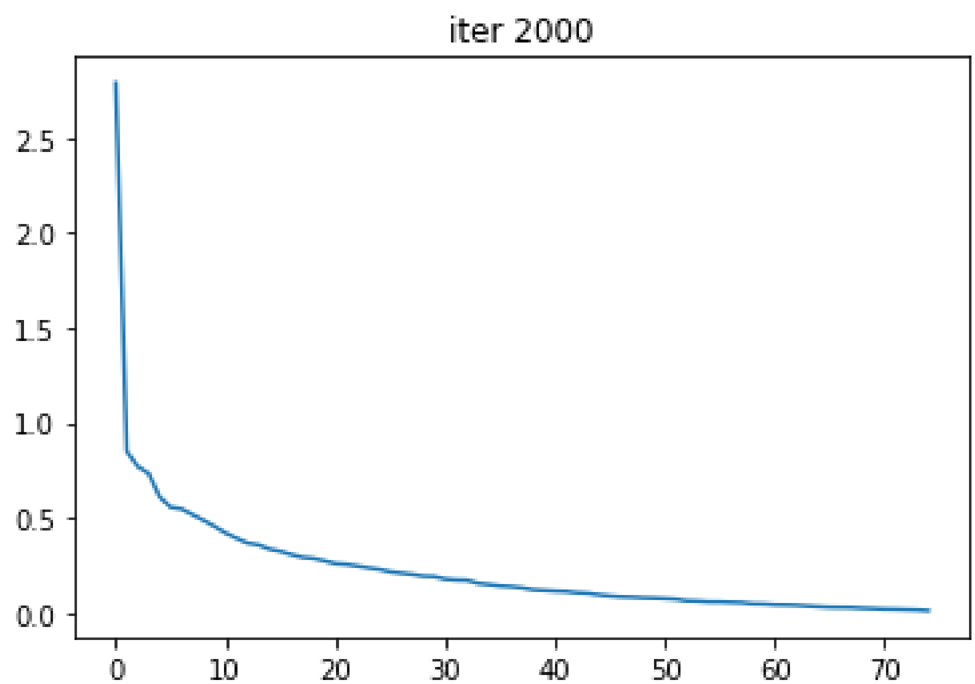
```
RMS
```



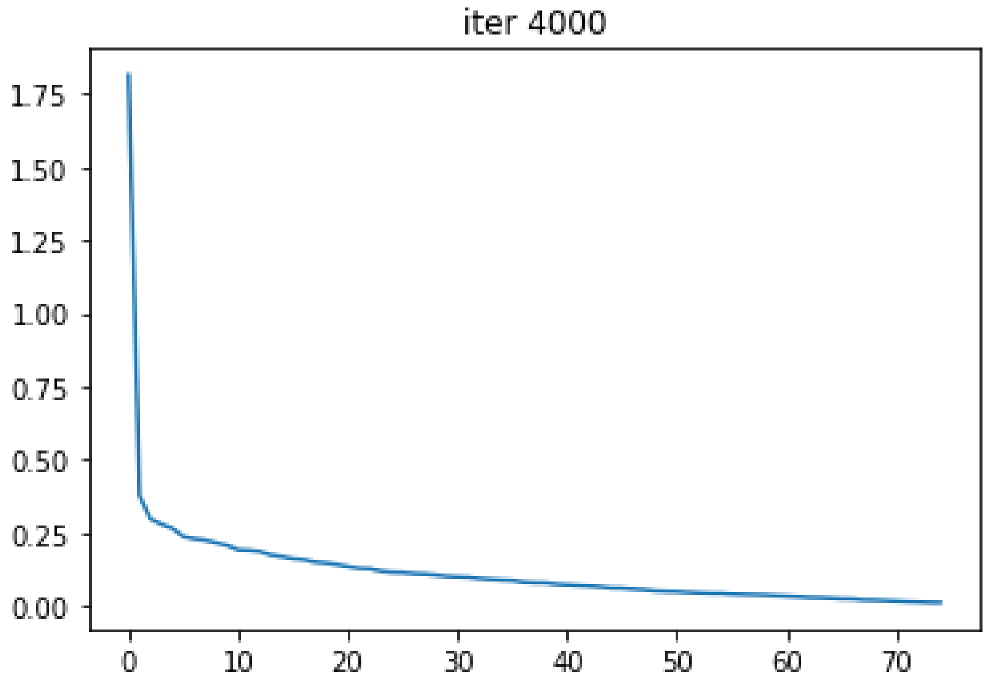

```
i = 0, discriminant loss = 1.3822, generator loss =0.6249
```



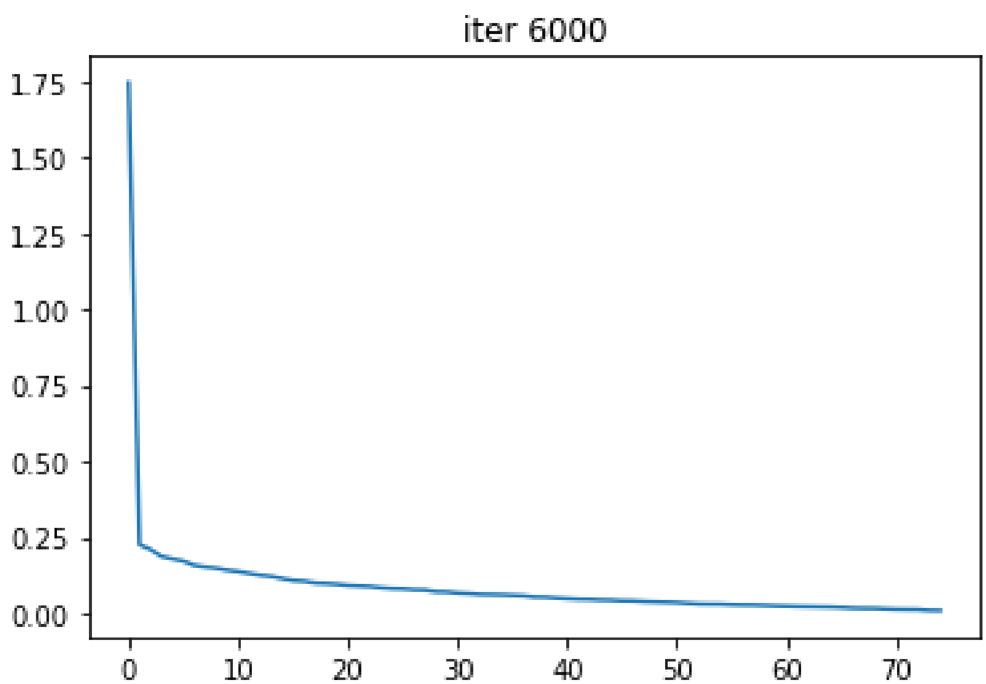

```
i = 2000, discriminant loss = 1.4030, generator loss =0.6816
```



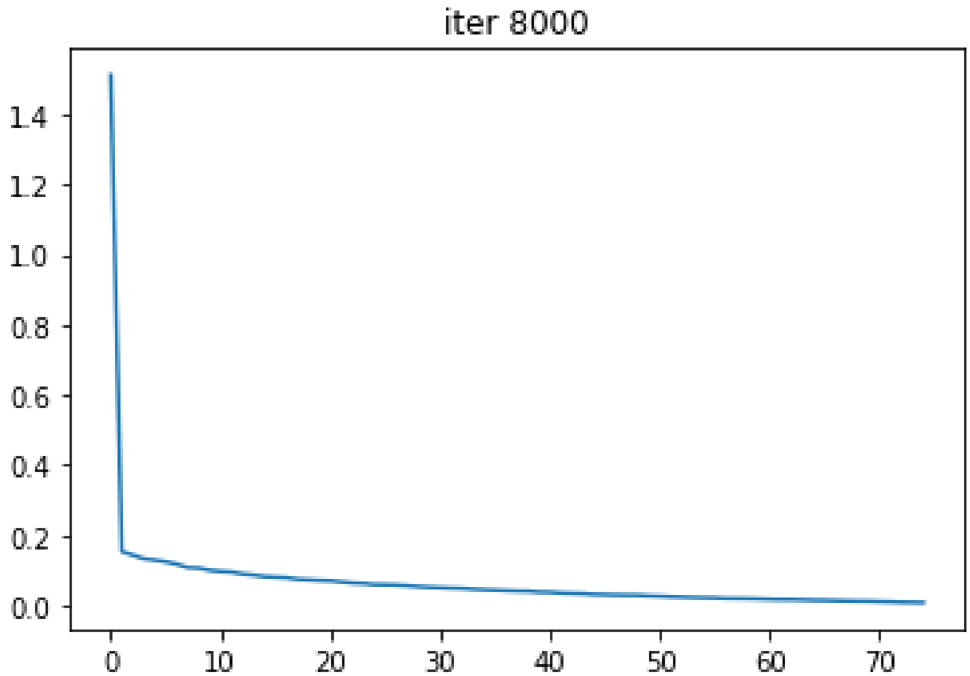
```
i = 4000, discriminant loss = 1.4097, generator loss =0.6523
```



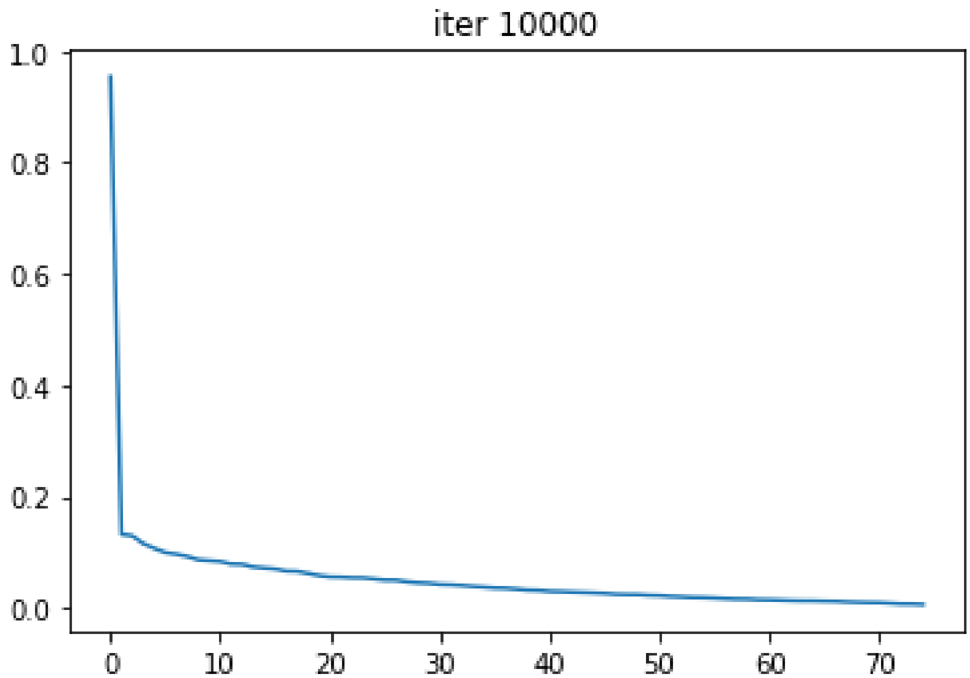

```
i = 6000, discriminant loss = 1.4174, generator loss =0.6912
```



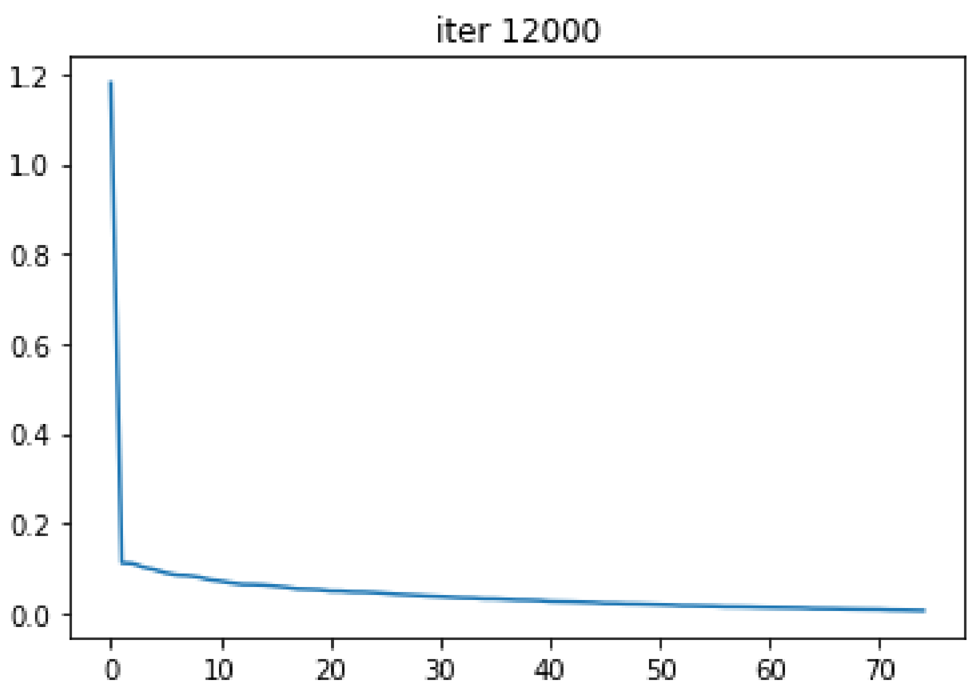
```
i = 8000, discriminant loss = 1.4118, generator loss =0.6706
```



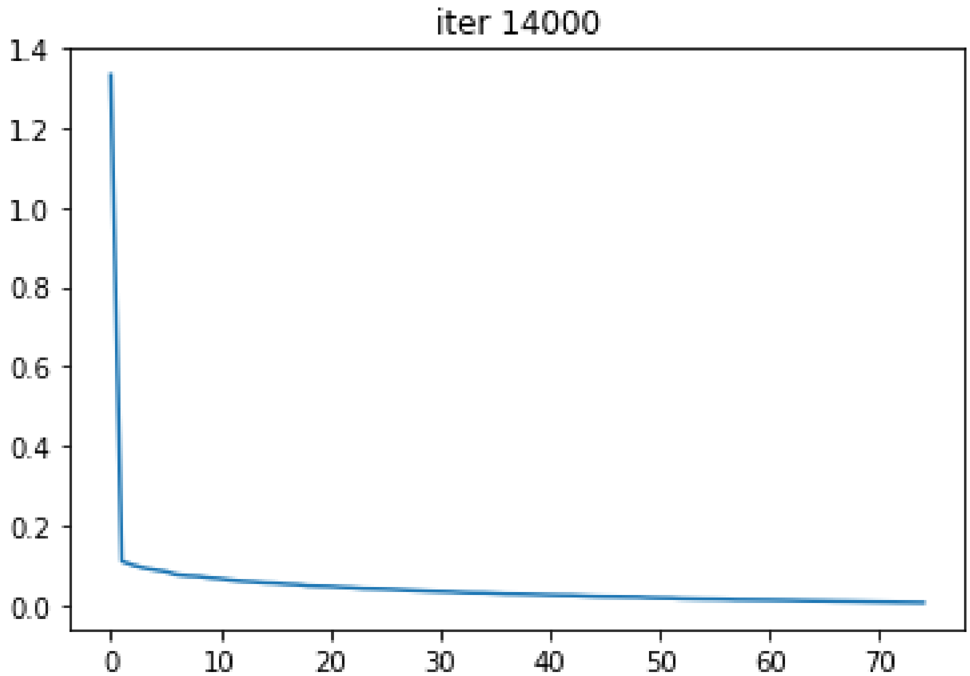

```
i = 10000, discriminant loss = 1.3706, generator loss =0.7215
```



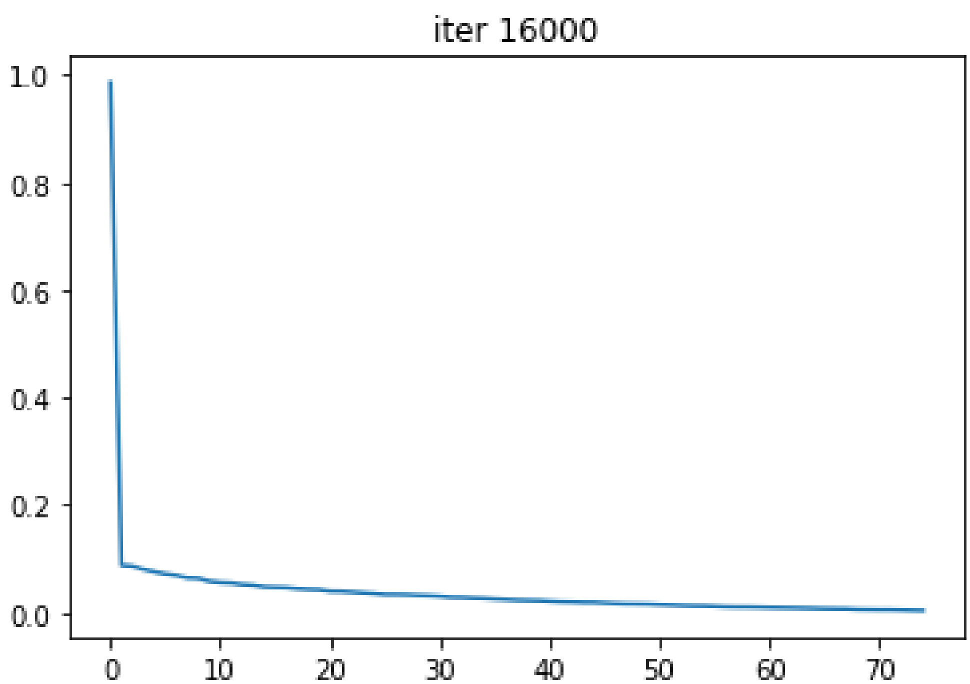
i = 12000, discriminant loss = 1.3736, generator loss =0.7113



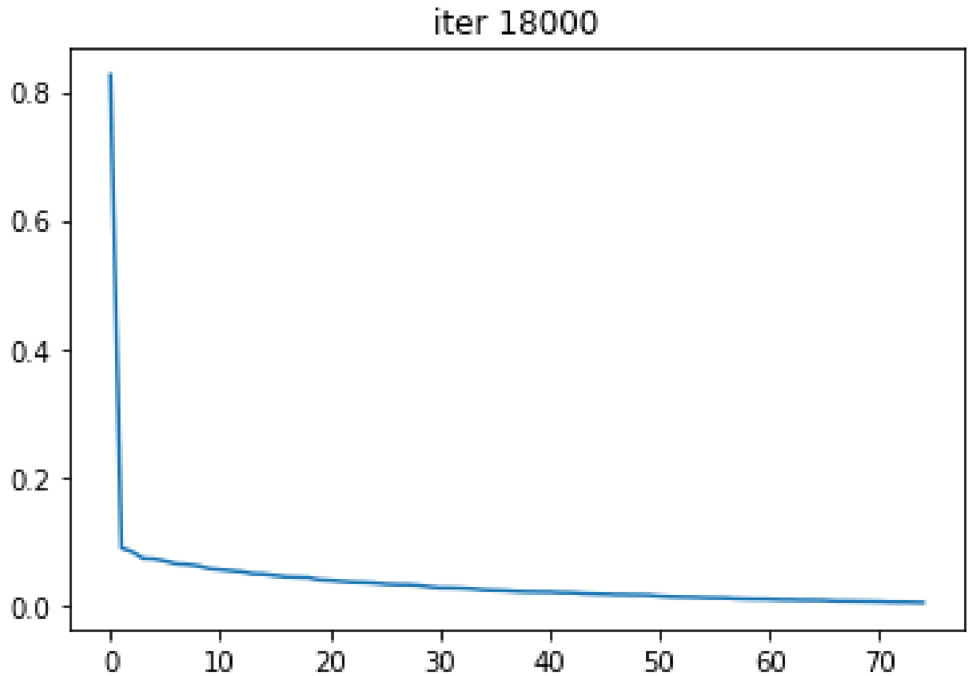
```
i = 14000, discriminant loss = 1.3803, generator loss =0.7034
```



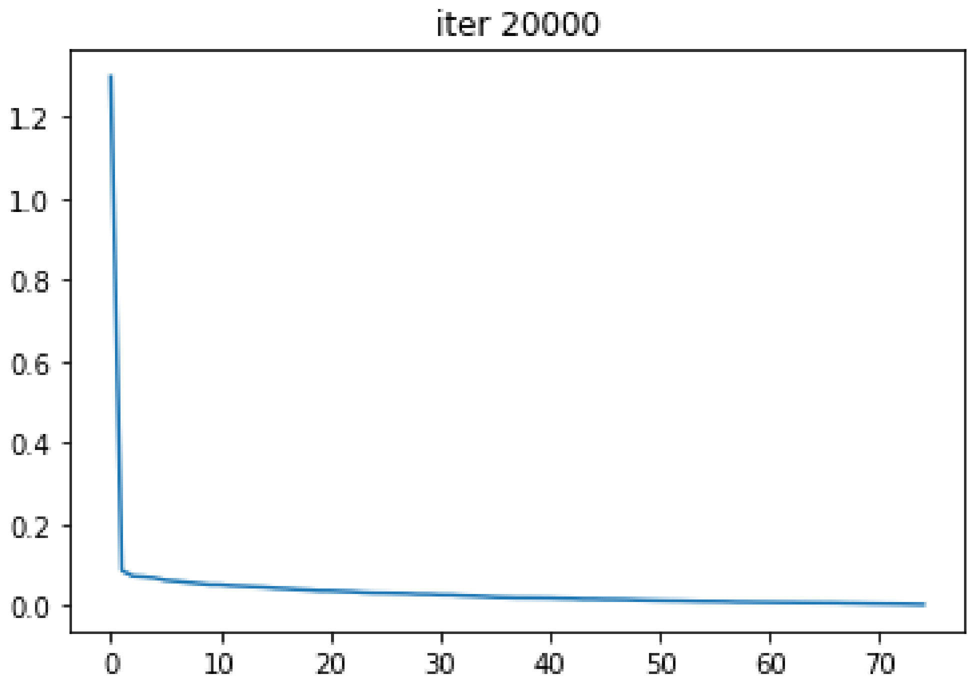

```
i = 16000, discriminant loss = 1.4275, generator loss =0.7320
```

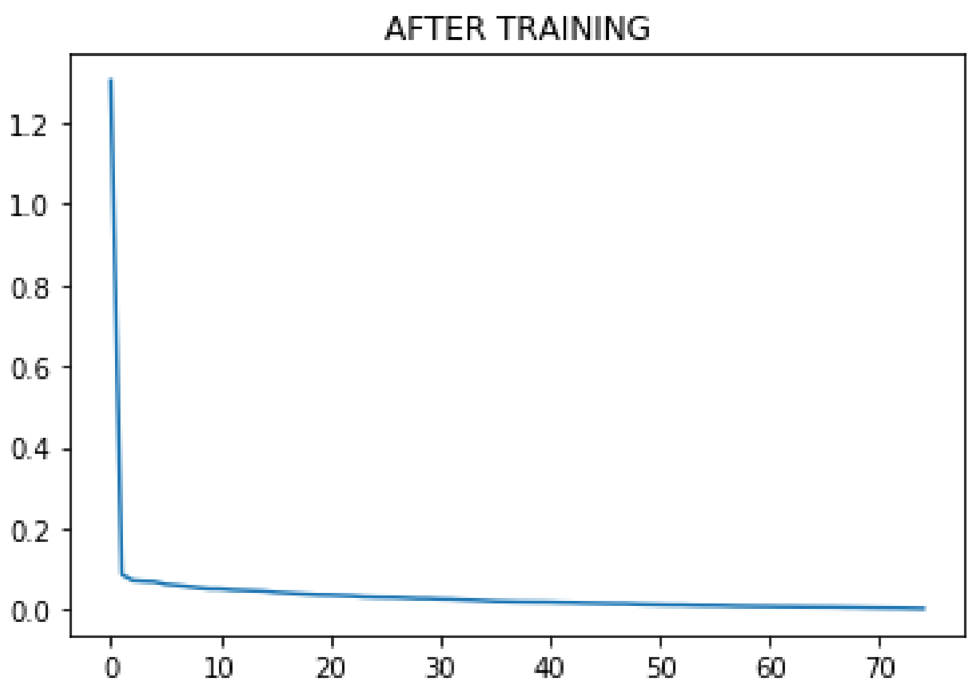


```
i = 18000, discriminant loss = 1.3801, generator loss =0.6883
```




```
i = 20000, discriminant loss = 1.4164, generator loss =0.7083
```

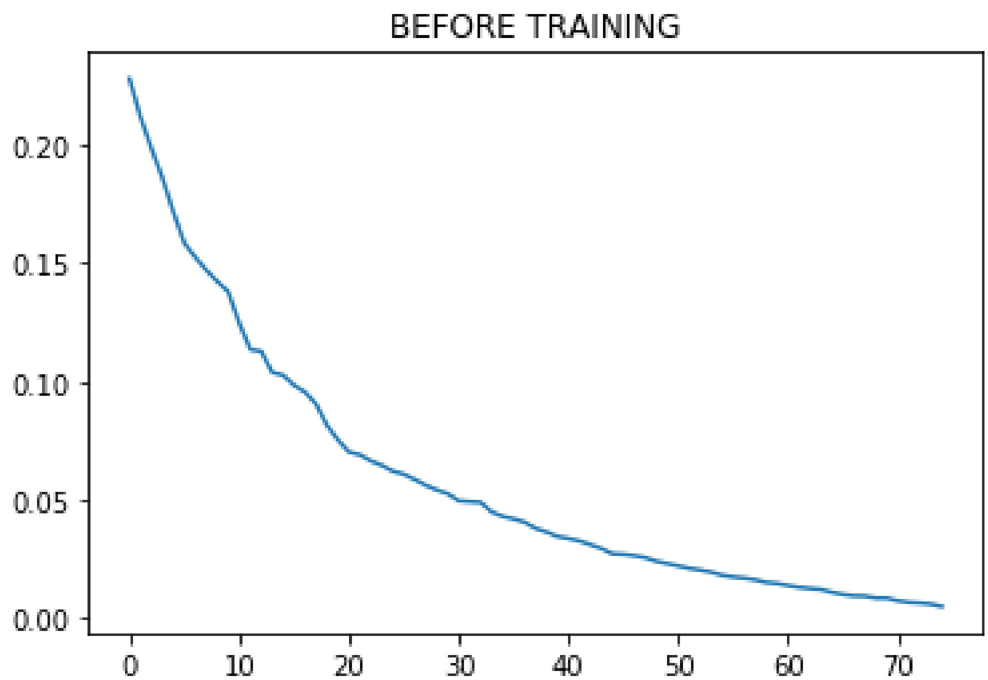

iter 20000
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
60
70


AFTER TRAINING
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
60
70

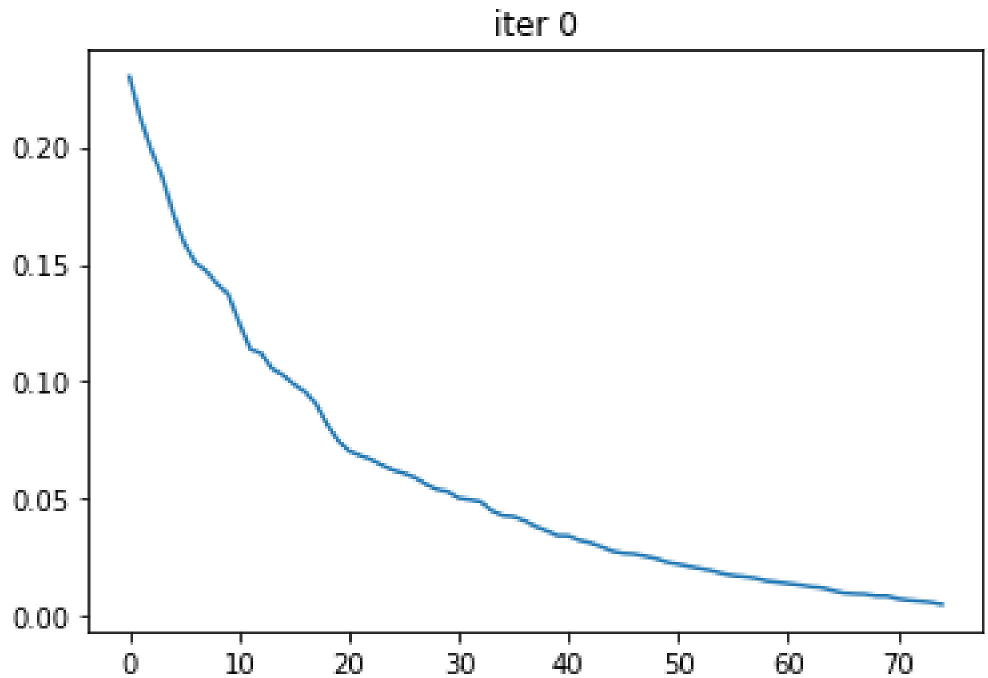
1.9.2 Using SGA

```
In [0]: # Use Symplectic Gradient Adjustment to optimise the GAN parameters
        high_dim_gaussian_experiment('SGA')
```

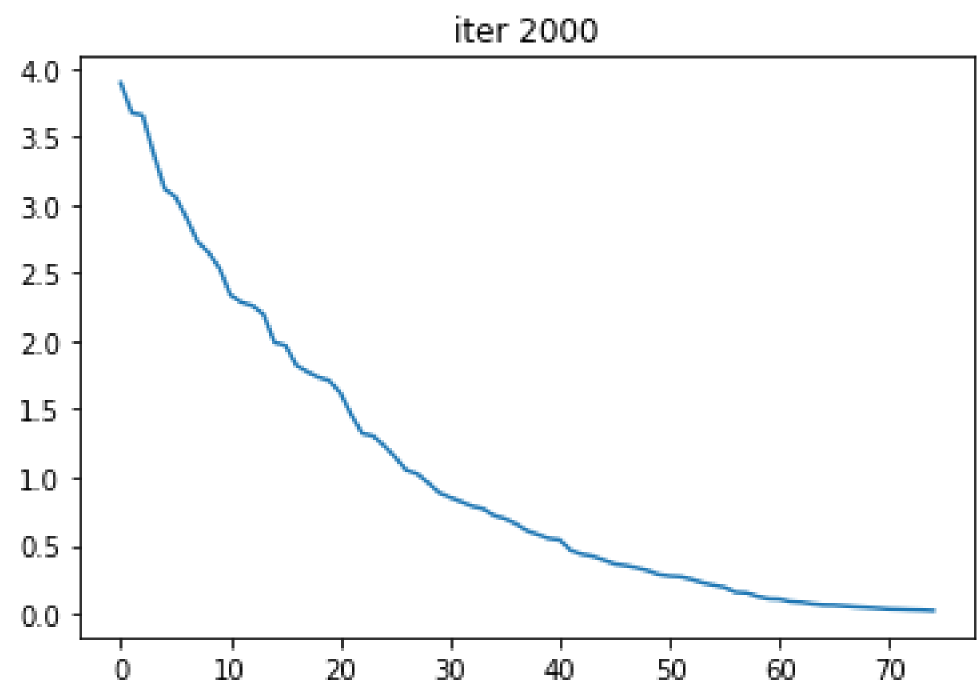
```
SGA
```



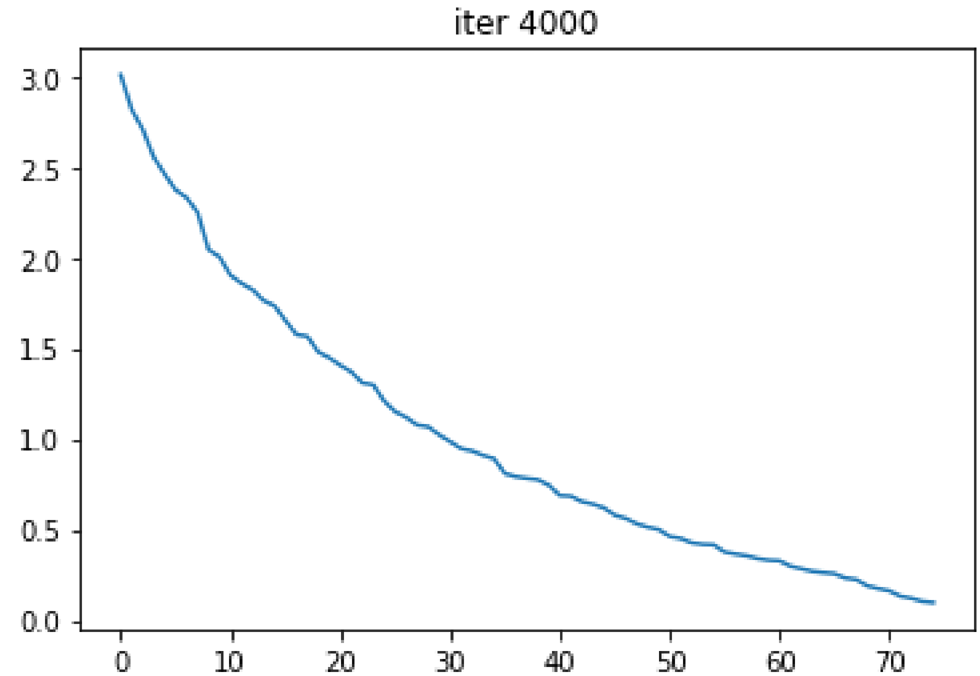

```
i = 0, discriminant loss = 1.5273, generator loss =0.7279
```



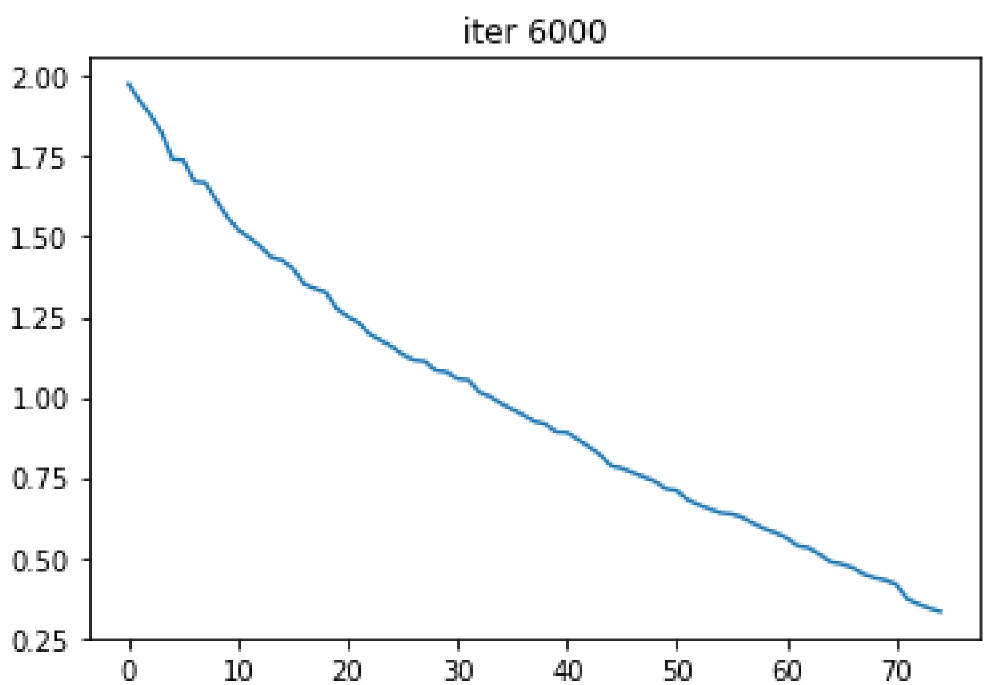

```
i = 2000, discriminant loss = 1.2478, generator loss =0.7696
```



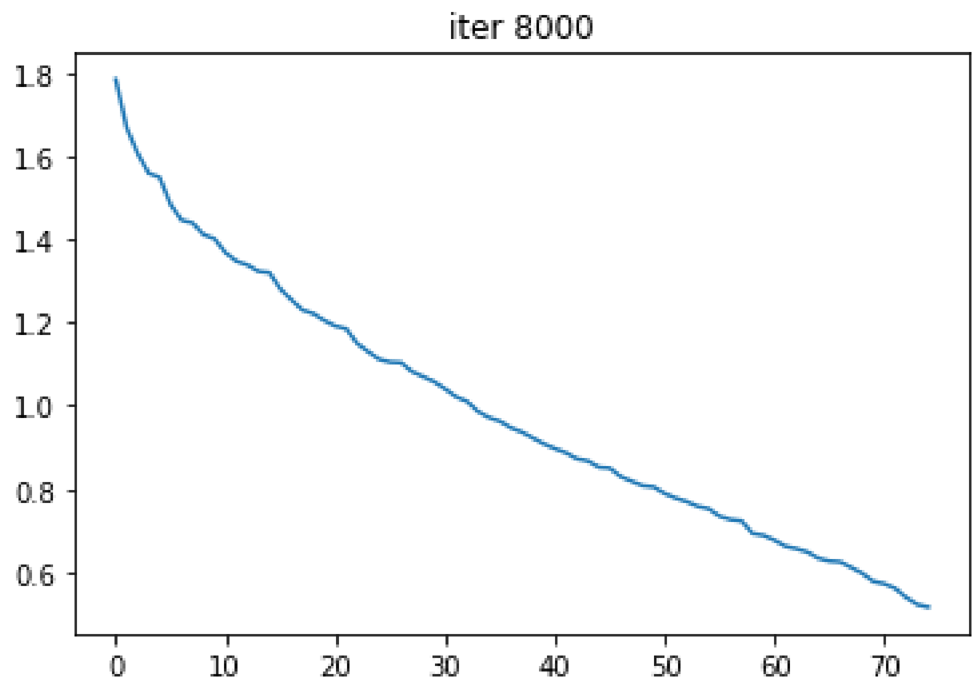
```
i = 4000, discriminant loss = 1.3265, generator loss =0.7274
```



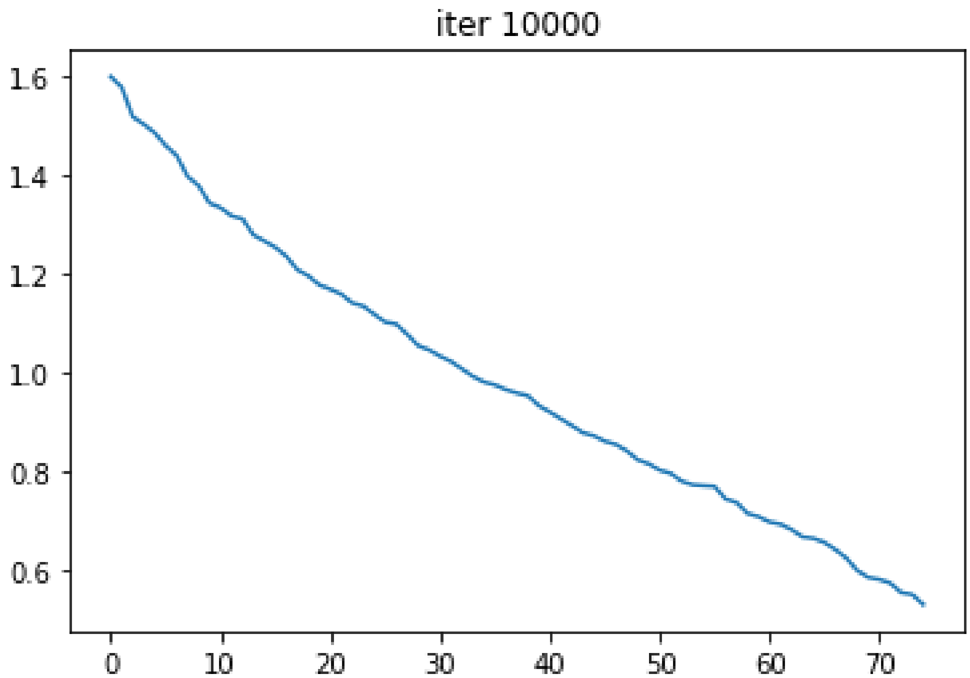

```
i = 6000, discriminant loss = 1.3730, generator loss =0.6980
```



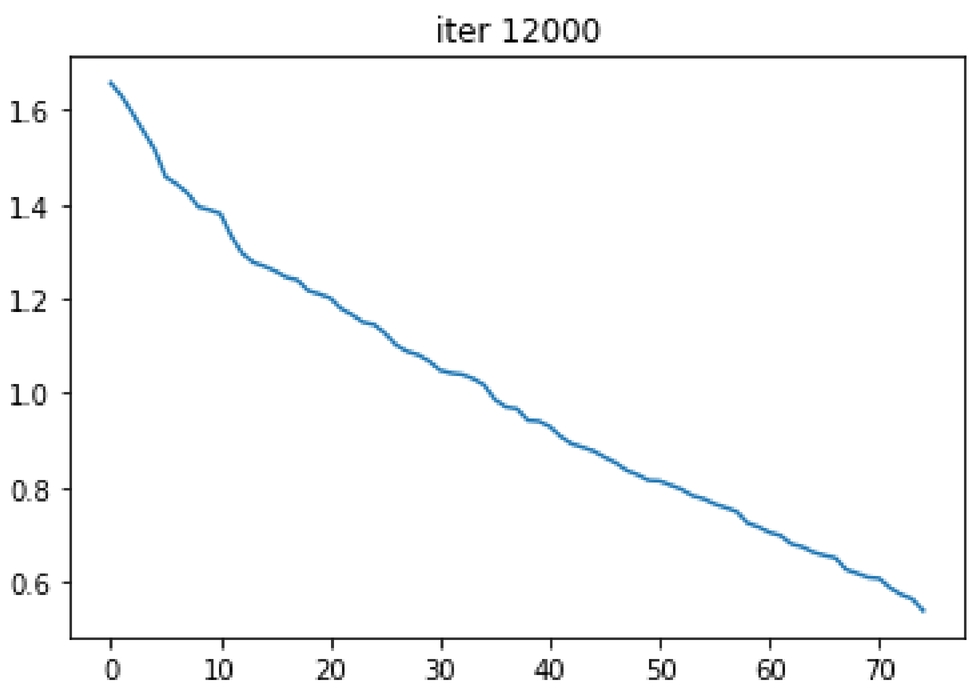
```
i = 8000, discriminant loss = 1.3824, generator loss =0.6947
```




```
i = 10000, discriminant loss = 1.3806, generator loss =0.6989
```



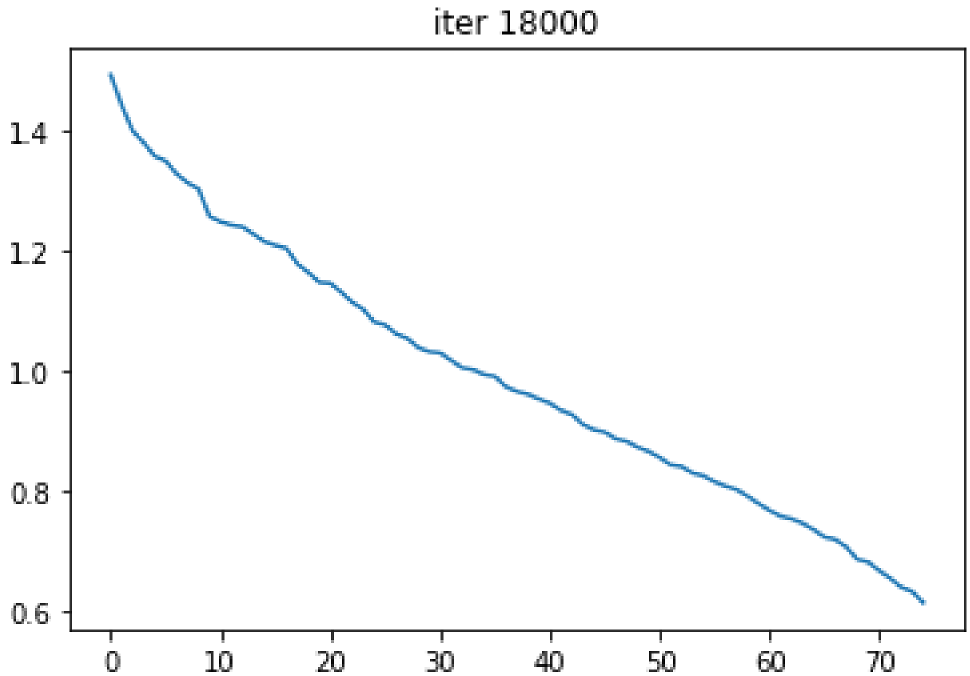

```
i = 12000, discriminant loss = 1.3703, generator loss =0.7037
```



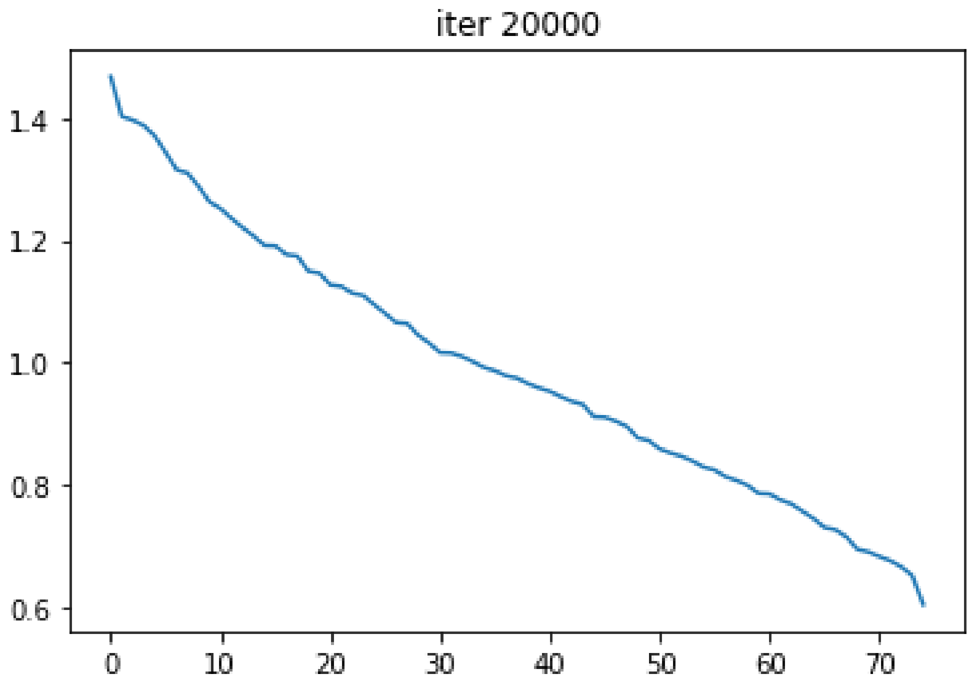
```
i = 14000, discriminant loss = 1.3645, generator loss =0.7058
```

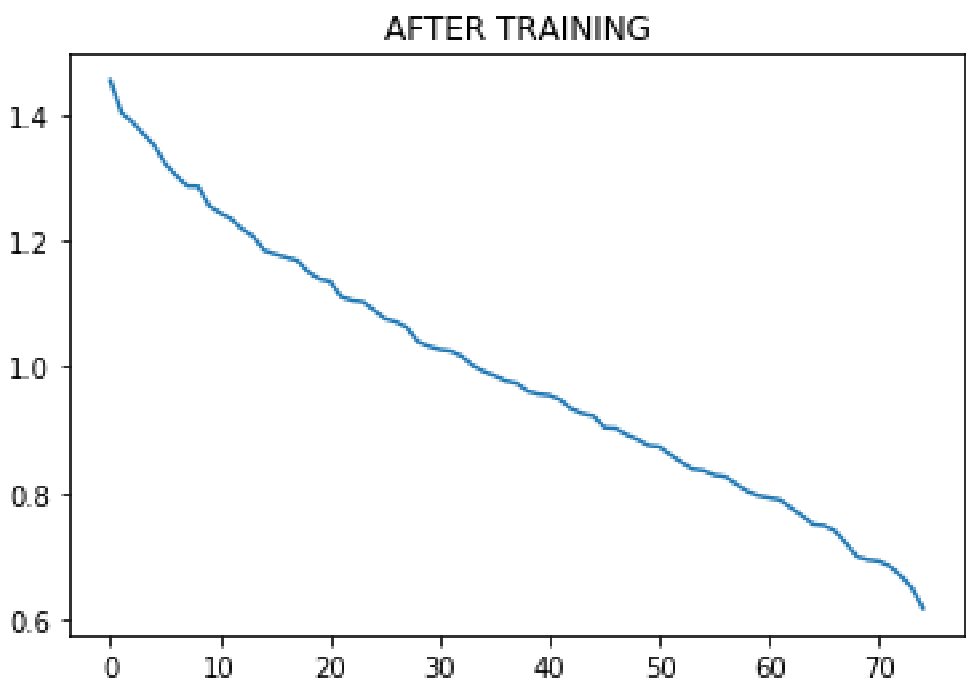

```
i = 16000, discriminant loss = 1.3513, generator loss =0.7180
```

```
i = 18000, discriminant loss = 1.3613, generator loss =0.7066
```




```
i = 20000, discriminant loss = 1.3681, generator loss =0.6979
```


iter 20000
1.4
1.2
1.0
0.8
0.6
0
10
20
30
40
50
60
70


AFTER TRAINING
1.4
1.2
1.0
0.8
0.6
0
10
20
30
40
50
60
70

1.10 Experiment 8: An SGA implementation in PyTorch

```
In [0]: def list_divide_scalar(xs, y):
            return [x / y for x in xs]

        def list_subtract(xs, ys):
            return [x - y for (x, y) in zip(xs, ys)]

        def jacobian_vec(ys, xs, vs):
            us = [torch.zeros_like(y) + float("nan") for y in ys]
            #print(us)
            dydxs = torch.autograd.grad(ys, xs, grad_outputs=us)
            print(dydxs)
            dydxs = [torch.zeros_like(x) if dydx is None else dydx for x, dydx in zip(xs, dydxs
            dysdx = torch.autograd.grad(dydxs, us, grad_outputs=vs)
            return dysdx

        def jacobian_transpose_vec(ys, xs, vs):
            dydxs = torch.autograd.grad(ys, xs, grad_outputs==vs)
            dydxs = [torch.zeros_like(x) if dydx is None else dydx for x, dydx in zip(xs, dydxs
            return dydxs

        def _dot(x, y):
            return torch.mm(x, y)
```

```
In [0]: class my_SGA(Optimizer):
            def __init__(self, params, lr=required, reg_params=1., use_signs=True, use_locking=
                if lr is not required and lr < 0.0:
                    raise ValueError("Invalid learning rate: {}".format(lr))

                defaults = dict(lr=lr, reg_params=reg_params, use_signs=use_signs,
                                use_locking=use_locking)
                super(my_SGA, self).__init__(params, defaults)

            def __setstate__(self, state):
                super(my_SGA, self).__setstate__(state)

            def step(self, closure=None):

                loss = None
                if closure is not None:
                    loss = closure()

                grads = []
                vars_ = []
                for group in self.param_groups:
                    for p in group['params']:
                        print(p)
                        grads.append(p.grad.data)
```

```
        print(p.grad.data)
        vars_.append(p.data)
#print(grads)
#print(vars_)
n = len(vars_)
h_v = jacobian_vec(grads, vars_, grads)
ht_v = jacobian_transpose_vec(grads, vars_, grads)
at_v = list_divide_scalar(list_subtract(ht_v, h_v), 2.)
if group['use_signs']:
    grad_dot_h = _dot(grads, ht_v)
    at_v_dot_h = _dot(at_v, ht_v)
    mult = grad_dot_h * at_v_dot_h
    lambda_ = torch.sign(mult / n + 0.1) * group['reg_params']
else:
    lambda_ = group['reg_params']

apply_vec = [(g + lambda_ * ag, x) for (g, ag, x) in zip(grads, at_v, vars_) i


for group in self.param_groups:
    for p in group['params']:
        if p.grad is None:
            continue
        d_p = p.grad.data
        p.data.add_(-group['lr'], d_p)


return loss
```