$$\frac{1}{\binom{N}{n}}\sum_{x=2}^{n}\frac{x(x-1)\,a(a-1)}{x(x-1)(x-2)!\,(a-x)!}\binom{N-a}{n-x}+\frac{na}{N}$$

$$\frac{a(a-1)}{\binom{N}{n}}\sum_{x=2}^{n}\frac{(a-2)!}{(x-2)!\,(a-x)!}\binom{N-a}{n-x}+\frac{na}{N}$$

$y=x-2$, $x=y+2$ şeklinde değişken değiştirelim

$$\frac{(a(a-1))}{\binom{N}{n}}\sum_{y=0}^{n-2}\binom{a-2}{x-2}\binom{N-a}{n-x}+\frac{na}{N}=\frac{a(a-1)}{\binom{N}{n}}\sum_{y=0}^{n-2}\binom{a-2}{y}\binom{N-a}{n-(y+2)}$$

$$+\frac{na}{N}=\frac{a(a-1)}{\binom{N}{n}}\binom{N-2}{n-2}+\frac{na}{N}$$

$$=\frac{a(a-1)n(n-1)}{N(N-1)}+\frac{na}{N}=\frac{na[(a-1)(n-1)(N-1)]}{N(N-1)}$$

$$Var(x)=\frac{na(na-a-n+N)}{N(N-1)}-\frac{n^2a^2}{N^2}=\frac{N-n}{N-1}\,\frac{na}{N}\left(1-\frac{a}{N}\right)$$

POISSON DAĞILIMI

O.F

$$f(x)=\begin{cases}\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} & \text{için } x\in R\\ 0 & \text{d.d}\end{cases}$$

Beklenen Değer

$$E(x)=\sum_{n=0}^{\infty}x\,\frac{e^{-\lambda}\lambda^x}{x!}=e^{-\lambda}\sum_{n=0}^{\infty}x\,\frac{\lambda^x}{x!}=e^{-\lambda}\lambda\sum_{x=1}^{\infty}x$$

$$\frac{\lambda^{x-1}}{x(x-1)!}=e^{-\lambda}\lambda\sum_{x=1}^{\infty}\frac{\lambda^{x-1}}{(x-1)!}=e^{-\lambda}\lambda\cdot e^{\lambda}=\lambda$$

Varyans

$$E(x^2) = \sum_{n=0}^{\infty} x^2 \frac{e^{-\lambda} \cdot \lambda^x}{x!} = e^{-\lambda} \sum_{n=0}^{\infty} x \frac{\lambda^x}{x!} = e^{-\lambda} \lambda \sum_{n=1}^{\infty} x \frac{\lambda^{x-1}}{x(x-1)!}$$

$$= e^{-\lambda} \lambda \sum_{n=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} = e^{-\lambda} \lambda e^{\lambda} = \lambda$$

MOMENT ÜRETEN FONK

$$M_x(t) = E[e^{xt}] = \sum_{x=0}^{\infty} e^{xt} \frac{e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e\lambda^t)^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}$$

Kümilant ÜRETEN FONK

$$K(t) = \frac{\ln(M_x t)}{n} = \frac{d^n \ln(M_x t)}{dt^n} = \lambda(e^t - 1)$$

ÇARPIMSAL ~~KARAKTERİSTİK~~ FONK $t=1$

$$N_x(t) = E(t^x) = \sum_{x \in R} t^x \cdot \frac{e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x \in R} \frac{(\lambda t)^x}{x!} = e^{\lambda t}$$

$$= e^{-\lambda} e^{\lambda t} = e^{\lambda(t-1)}$$

$$\frac{d^n N_x(t)}{dt^n} = \frac{d^n}{dt^n} \left(e^{\lambda(t-1)} \right) \Big|_{t=1} = \lambda^n$$

ÇARPIKLIK

$$\frac{k_3(t)}{k_2(t)} = \frac{\lambda}{\lambda^{3/2}} = \frac{1}{\sqrt{\lambda}}$$

BASIKLIK KURTOSIS

$$K_4(t) \neq E((x - E(x))^4)$$

Hepsini aç.

$E[X^4] - 4E[X^3]E[X] + 6E(X^2)(E(X))^2 - 3(E(X))^4$

$= \frac{\lambda + 3\lambda^2}{\lambda^2} = \frac{1}{\lambda} + 3$

Bir birim zaman veya alanda X rd λ parametresi ile poisson dağılıyor ise a birim zamanında yada alanda X rd $a.\lambda$ parametresi ile poisson Dağılır.

UNiFORM(DÜZGÜN DAĞILIM)

$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{dd} \end{cases}$ şeklinde verilmiş ise X rd (a,b) kapalı aralığında düzgün veya uniform dağ. sahiptir denir.

i) $x \in (a,b)$ ise $f(x) = \frac{1}{b-a} > 0$ $b > a$ old göre

ii) $\int_{-\infty}^{\infty} f(x)dx = \int_a^b \frac{1}{b-a} = \frac{x}{b-a}\Big|_{x=a}^{b} = \frac{b}{b-a} - \frac{a}{b-a} = 1$

O.y.f ispatı

$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_a^b x \frac{1}{b-a} = \frac{x^2}{2(b-a)}\Big|_{x=a}^{b} = \frac{b^2-a^2}{2(a-b)} = \frac{\cancel{(a-b)}(a+b)}{2\cancel{(a-b)}}$

$= \frac{a+b}{2}$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_a^b x^2 \frac{1}{b-a} = \frac{x^3}{3} \frac{1}{b-a}\Big|_{x=a}^{b} = \frac{b^3-a^3}{3(b-a)}$

$= \frac{\cancel{(b-a)}(a^2+ab+b^2)}{3\cancel{(b-a)}} = \frac{a^2+ab+b^2}{3}$

$\text{var}(x) = \frac{a^2+ab+b^2}{\underset{(4)}{3}} - \frac{a^2+b^2}{\underset{(3)}{4}} = \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12}$

MOMENT ÜRETEN FONK

$$M_X(t) = E(e^{xt}) = \int_a^b e^{xt} \frac{1}{b-a} = \int_a^b \frac{e^{xt}}{b-a} = \frac{e^{xt}}{t} \cdot \frac{1}{b-a} \Big|_{x=a}^{b}$$

$$= \frac{e^{bt} - e^{at}}{t(b-a)}$$

(α, β)
şekil ölçek

GAMA DAĞILIMI

Olasılık Yoğunluk Fonk

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} & \text{için } \alpha>0, x>0, \beta>0 \\ 0 & \text{dd} \end{cases}$$

BEKLENEN DEĞER

$$E(x) = \int_0^\infty x \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta} dx = \frac{1}{\Gamma(\alpha)\beta^\alpha} \int_0^\infty x^\alpha \cdot e^{-x/\beta} dx$$

* $\frac{x}{\beta} = v$, $x = v\beta$, $dx = \beta dv$ değişkenleri değiştirelim

$$\frac{1}{\Gamma(\alpha)\beta^\alpha} \int_0^\infty (v\beta)^{\alpha+1} e^{-v} \beta dv = \frac{1}{\Gamma(\alpha)\beta^\alpha} \beta^{\alpha+1} \int_0^\infty v^\alpha \cdot e^{-v} dv$$

$$\frac{\beta^{\alpha+1}\Gamma(\alpha+1)}{\Gamma(\alpha)\beta^\alpha} = \frac{\beta^\alpha \cdot \beta \cdot \alpha\Gamma(\alpha)}{\Gamma(\alpha)\beta^\alpha} = \alpha\beta$$

VARYANS

$$E(x^2) = \int_0^\infty x^2 \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} \cdot e^{-x/\beta} dx = \frac{1}{\Gamma(\alpha)\beta^\alpha} \int_0^\infty x^{\alpha+1} e^{-x/\beta} dx$$

* $v = \frac{x}{\beta}$ $\quad \beta v = x \quad dx = \beta dv$ *

$$\frac{1}{\Gamma(\alpha)\beta^\alpha} \int_0^\infty (v\beta)^{\alpha+1} e^{-v} \beta dv = \frac{1}{\Gamma(\alpha)\beta^\alpha} \beta^{\alpha+2} \int_0^\infty v^{\alpha+1} e^{-v} dv$$

$$\Gamma(\alpha+2)$$

$$\frac{\beta^\alpha \cdot \beta^2}{\Gamma(\alpha)\beta^\alpha} \cdot (\alpha+1)\alpha\Gamma(\alpha) = (\alpha^2+\alpha)\beta^2 = \alpha^2\beta^2 + \alpha\beta^2$$

$$Var(x) = \alpha^2\beta^2 + \alpha\beta^2 - \alpha^2\beta^2 = \alpha\beta^2$$

MOMENT ÜRETEN FONKSİYON

$$M_x(t) = E(e^{xt}) = \int_0^\infty e^{xt} \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} \cdot e^{-x/\beta}\, dx$$

$$\frac{1}{\Gamma(\alpha)\beta^\alpha}\int_0^\infty x^{\alpha-1} \cdot e^{xt - \frac{x}{\beta}}\, dx = \frac{1}{\Gamma(\alpha)\beta^\alpha}\int_0^\infty x^{\alpha-1} \cdot e^{-x(1-t\beta)/\beta}\, dx$$

$$v = \frac{x(1-t\beta)}{\beta}, \quad \frac{\beta v}{(1-t\beta)} = x, \quad dx = \frac{\beta\, dv}{(1-t\beta)}$$

$$\frac{1}{\Gamma(\alpha)\beta^\alpha}\int_0^\infty \left(\frac{\beta v}{1-t\beta}\right)^{\alpha-1} \cdot e^{-v} \frac{\beta\, dv}{(1-t\beta)} = \frac{1}{\Gamma(\alpha)\beta^\alpha} \cdot \frac{(1-t\beta)}{\beta} \cdot \frac{\beta}{(1-t\beta)} \cdot \frac{\beta^\alpha}{(1-t\beta)^\alpha} \int_0^\infty v^{\alpha-1} e^{-v}\, dv$$

$$\frac{1}{\Gamma(\alpha)\beta^\alpha} \cdot \frac{\beta^\alpha}{(1-t\beta)^\alpha} \cdot \Gamma(\alpha) = \left(\frac{1}{1-t\beta}\right)^\alpha$$

$M_X(t)$ ile 1 ve 2. momentleri ile varyans Bulmak

1. MOMENT

$$\frac{d^1 M_x(t)}{dt} = \frac{d^1 (1-t\beta)^{-\alpha}}{dt} = -\alpha(1-t\beta)^{-\alpha-1} \cdot (0-\beta) = \alpha\beta(1-t\beta)^{-\alpha-1}\Big|_{t=0} = \alpha\beta$$

2. MOMENT

$$\frac{d^2 M_x t}{dt} = \frac{d^1 \alpha\beta(1-t\beta)^{-\alpha-1}}{dt} = \alpha\beta(-\alpha-1)(1-t\beta)^{-\alpha-2} \cdot -\beta = -\alpha\beta^2(-\alpha-1)(1-t\beta)^{-\alpha-2}$$

$$\alpha^2\beta^2 - \alpha\beta^2(1-t\beta)^{-\alpha-2}\Big|_{t=0} = \alpha^2\beta^2 - \alpha\beta^2$$

VARYANS

$$Var(x) = \alpha^3\beta^2 - \alpha\beta^2 - \alpha^2\beta^2 = \alpha\beta^2$$

Gama fonk ...

$$f(x) = \int x^{\alpha-1} \cdot e^{-x/\beta}$$

Gama fonk

ÇARPIKLIK (SKEWNESS)

KARAKTERİSTİK FONK

$$\varphi(t) = E(e^{ixt}) = \int_0^{\infty} e^{ixt} \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} dx$$

$$\frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} x^{\alpha-1} e^{ixt - \frac{x}{\beta}} dx = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} x^{\alpha-1} e^{-x(1-it\beta)/\beta} dx$$

$$u = \frac{x(1-it\beta)}{\beta} \qquad \frac{u\beta}{(1-it\beta)} = x \qquad dx = \frac{\beta du}{(1-it\beta)}$$

$$\frac{1}{\Gamma(\alpha)\beta^{\alpha}} \int_0^{\infty} \left(\frac{\beta u}{(1-it\beta)}\right)^{\alpha-1} e^{-u} \frac{\beta du}{(1-it\beta)} = \frac{\beta}{(1-it\beta)} \frac{(1-it\beta)}{\beta} \cdot \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \cdot \frac{\beta^{\alpha}}{(1-it\beta)^{\alpha}}$$

$$\underbrace{\int_0^{\infty} u^{\alpha-1} e^{-u} du}_{\Gamma(\alpha)} = \left(\frac{1}{(1-it\beta)}\right)^{\alpha}$$

KÜMİLANT ÜRETEN FONK

$$\ln\left(\ln\left(\frac{1}{1-t\beta}\right)^{\alpha}\right) = \ln(\ln(1-t\beta)^{-\alpha})$$

(E(X)) Kümilant üreten fonk 1. türevi

$$k'(t) = \frac{M_x'(t)}{M_x(t)} \qquad M_x(t) = (1-t\beta)^{-\alpha}$$

$$M_x'(t) = -\alpha(1-t\beta)^{-\alpha-1} \cdot -\beta = \alpha\beta(1-t\beta)^{-\alpha-1}$$

$$k'(t) = \frac{M_x'(t)}{M_x(t)} = \frac{\alpha\beta(1-t\beta)^{-\alpha-1}}{(1-t\beta)^{-\alpha}} = \frac{\alpha\beta}{(1-t\beta)}\Bigg|_{t=0} = \alpha\beta$$

Var(X) Kümilant üreten fonk 2. türevi

$$k^2(t) = \frac{d}{dt}\frac{\alpha\beta}{1-t\beta} = \frac{0(1-t\beta) - -\beta\alpha\beta}{(1-t\beta)^2}\Bigg|_{t=0} = \alpha\beta^2$$

Çarpıklık

$$Var(x) = \alpha\beta^2 \quad , k^2(t) = \frac{\alpha\beta^2}{(1-t\beta)^2}$$

$$k^3(t) = \frac{d}{dt}\frac{\alpha\beta^2}{(1-t\beta)^2} = \frac{0\cdot(1-t\beta)^2 - 2(1-t\beta)\alpha\beta^2 \cdot -\beta}{(1-t\beta)^4}$$

$$= \frac{2\alpha\beta^3}{(1-t\beta)^3}\Bigg|_{t=0} = 2\alpha\beta^3$$

BASIKLIK

$$k^4(t) = \frac{d}{dt}\frac{2\alpha\beta^3}{(1-t\beta)^3} = \frac{0(1-t\beta)^3 - 3(1-t\beta)^2 \cdot -\beta \cdot 2\alpha\beta^3}{(1-t\beta)^6}$$

$$= \frac{2\alpha\beta^4 \cdot 3(1-t\beta)^2}{(1-t\beta)^6} = \frac{6\alpha\beta^4}{(1-t\beta)^4}\Bigg|_{t=0} = 6\alpha\beta^4$$

$$\text{Basıklık} = \frac{k^4(t)}{(Var(x))^2} = \frac{6\alpha\beta^4}{\alpha^2\beta^4} = \frac{6\alpha}{\alpha^2} = \frac{6}{\alpha}$$

ÜSTEL DAĞILIM

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & x > 0 \quad \theta > 0 \\ 0 & d.d. \end{cases}$$

GAMA ~ ÜSTEL İLİŞKİSİ

$$Gama(\alpha, \beta) \Rightarrow \alpha = 1, \beta = \theta = \text{üstel}(\theta)$$

$$x \quad \frac{1}{\Gamma(1)\theta^1} \cdot e^{-x/\theta} = \text{üstel Dağılım} \qquad Var(\theta^2)$$

Gama		Üstel
$E(X) = \alpha\beta$	$\alpha = 1$	$E(X) = \theta$
$E(X^3) = \alpha(\alpha+1)(\alpha+2)\beta^3$	$\beta = \theta$	$E(X^3) = 6\theta^3$
$Skew = \frac{2}{\sqrt{\alpha}}$		$Skew = 2$
$K(t) = \ln(1-\beta t)^{-\alpha}$		$K(t) = -\ln(1-\theta t)$

İstatistik vs Mühendislik

$X \sim \text{üstel}(\theta)$

$$f(x) = \begin{cases} \frac{1}{\theta} \cdot e^{-\frac{x}{\theta}} & x>0, \theta>0 \\ 0 & dd \end{cases}$$

$X \sim \text{üstel}\left(\frac{1}{\theta}\right)$

$$f(x) = \begin{cases} \theta e^{-x\theta} & x>0, \theta>0 \\ 0 & dd \end{cases}$$

$X \sim \text{Gama}(\alpha, \beta)$

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} & \alpha>0, \beta>0, x>0 \\ 0 & dd \end{cases}$$

$X \sim \text{Gama}\left(\alpha, \frac{1}{\beta}\right)$

$$\frac{1}{\Gamma(\alpha)\frac{1}{\beta^{\alpha}}} x^{\alpha-1} e^{-xB}$$

$$f(x) = \begin{cases} \frac{B^{\alpha}}{\Gamma(\alpha)} \cdot x^{\alpha-1} \cdot e^{-x/B} & x>0, \beta>0, x>0 \\ 0 & dd \end{cases}$$

ÜSTEL DAĞILIM · DAĞILIM FONK

$$f(x) = \frac{dF(x)}{dx} \qquad F(x) = P(X \leq x) = \int_0^x \frac{1}{\theta} e^{-\frac{t}{\theta}} dt$$

$\frac{t}{\theta} = v \quad dt = \theta dv \quad t \to 0 \to v = 0 \quad t = x \to v = \frac{x}{\theta}$

$$\frac{1}{\theta} \int_0^{x/\theta} e^{-v} \theta dv = -e^{-v} \Big|_{v=0}^{x/\theta} = 1 - e^{-\frac{x}{\theta}}$$

$$F(x) = \begin{cases} 0 & x<0 \\ 1 - e^{-\frac{x}{\theta}} & 0<x<\infty \\ 1 & x \to \infty \end{cases}$$

$X \sim \text{ÜSTEL}(\theta) \cong \text{Gama}(1, \theta)$

Ki-Kare Dağ (χ^2)

X r.d. ayt
$$f(x) = \begin{cases} \frac{1}{\Gamma\left(\frac{r}{2}\right) 2^{\frac{r}{2}}} \cdot x^{\frac{r}{2}-1} e^{-\frac{x}{2}} & x>0, r>0 \\ 0 & dd \end{cases}$$

şeklinde ise ki-kare Dağılımına sahiptir denir ve $X \sim \chi^2(r)$ ifade edilir

$\text{Gama}(\alpha, \beta) \cong \alpha = \frac{r}{2} \quad \beta = 2 \quad \text{Gama}\left(\frac{r}{2}, 2\right) \cong \chi^2(r)$

$\text{Gama}(1, 2) \cong \chi^2(2) \quad \text{Gama}\left(\frac{4}{2}, 2\right) = \chi^2(2)$

- NOT! Tam sayı çıkmasının sebebi örnekleme dağılımı olmasıdır, chi², normal, t, F dağ

$\text{Gama}(2, 4) = \quad \beta$ hep 2 olmalı

NOT: analitik olarak f(x)'i yoktur

Gama – Kikare

Gama(α β)

$E(X) = \alpha \cdot \beta$

$E(X^2) = \alpha(\alpha+1)\beta^2$

$Var(x) = \alpha\beta^2$

$M_X(t) = (1-\beta t)^{-\alpha}$

$k_X(t) = -\alpha(1-\beta t) =$

$\text{Skew}(x) = \frac{2}{\sqrt{\alpha}}$

$\text{kurt}(x) = \frac{6}{\alpha}$

ki-kare $\chi^2(r) \cong \text{Gam}\left(\frac{r}{2}, 2\right)$

$E(x) = \frac{r}{2} \cdot 2 = r$

$E(x^2) = \frac{r}{2}\left(\frac{r}{2}+1\right)2^2 = r(r+2)$

$E(x^3) = \alpha(\alpha+1)(\alpha+2)\beta^3$

$Var(x) = 2r$

$K_X(t) = \frac{r}{2}(1-2t)$

$M_X(t) = (1-2t)^{-\frac{r}{2}}$

$\text{Skew}(x) = \frac{1}{\sqrt{\frac{r}{2}}}$

NOT! Ki-karenin F(x)'i yoktur

Beta Dağılımı

Beta Fonk

$$Beta(\alpha,\beta) = \begin{cases} \int_0^1 x^{\alpha-1}.(1-x)^{\beta-1}dx & x>0\ \beta>0 \\ 0 & d.d. \end{cases}$$

$$= \frac{\Gamma(\alpha).\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$Beta(\alpha,\beta) = \frac{\Gamma(\alpha).\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$$\int_0^1 x^{\alpha-1}.(1-x)^{\beta-1}dx \quad \downarrow 0$$

$$Beta(\alpha+1,\beta) = \frac{\Gamma(\alpha+1).\Gamma(\beta)}{\Gamma(\alpha+\beta+1)}$$

$$(a+b)^n = \sum_{x=0}^{n}\binom{n}{x}a^x.b^{n-x}$$

$$\frac{\alpha\Gamma(\alpha)\Gamma(\beta)}{(\alpha+\beta)\Gamma(\alpha+\beta)} \quad Beta(\alpha,\beta)$$

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx \qquad \frac{\Gamma(\alpha)}{\Gamma(\alpha)} = \frac{\int_0^\infty x^{\alpha-1}e^{-x}dx}{\Gamma(\alpha)}$$

$$1 = \frac{\int x^{\alpha-1}.e^{-x}dx}{\Gamma(\alpha)}$$

Bu şekilde Beta fonk. oy. f elde edebilir.

Beta Dağılımı

$$f(x) = \begin{cases} \frac{1}{Beta(\alpha,\beta)}x^{\alpha-1}.(1-x)^{\beta-1} & 0<x<1,\ \alpha>0,\ \beta>0 \\ 0 & d.d. \end{cases}$$

X r.d. Beta dağ. sahiptir denir ve kısaca $X \sim Beta(\alpha,\beta)$ şeklinde gösterilir.

Buradaki α parametresi şekil parametresi.

Beta dağ. en büyük özelliği 0,1 gibi sonlu aralıklarda tanımlı olmasıdır.

BETA DAĞ. E(x), Var(x),

$$E(x^r) = \int_0^1 x^r.\frac{1}{Beta(\alpha,\beta)}.x^{\alpha-1}.(1-x)^{\beta-1}dx$$

$$\frac{1}{Beta(\alpha,\beta)}\underbrace{\int_0^1 x^{\alpha+r-1}.(1-x)^{\beta-1}dx}_{Beta(\alpha+r,\beta)} = \frac{Beta(\alpha+r,\beta)}{Beta(\alpha,\beta)}$$

$$\frac{Beta(\alpha+r,\beta)}{Beta(\alpha,\beta)} = \frac{\Gamma(\alpha+r).\Gamma(\beta)}{\Gamma(\alpha+r+\beta)}.\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\Gamma(\beta)}$$

$$= \frac{\Gamma(\alpha+r).\Gamma(\alpha+\beta)}{\Gamma(\alpha+r+\beta)\Gamma(\alpha)}$$

$E(x^1)$ için

$$\frac{\Gamma(\alpha+1)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+1)\Gamma(\alpha)} = \frac{\alpha\Gamma(\alpha)\Gamma(\alpha+\beta)}{(\alpha+\beta)\Gamma(\alpha+\beta)\Gamma(\alpha)} = \frac{\alpha}{(\alpha+\beta)}$$

$E(x^2)$ için

$$\frac{\Gamma(\alpha+2).\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+2).\Gamma(\alpha)} = \frac{\alpha(\alpha+1)\Gamma(\alpha).\Gamma(\alpha+\beta)}{(\alpha+\beta)(\alpha+\beta+1)\Gamma(\alpha+\beta).\Gamma(\alpha)} = \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}$$

$$Var(x) = \underset{(\alpha+\beta)}{\frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}} - \underset{(\alpha+\beta+1)}{\frac{\alpha^2}{(\alpha+\beta)^2}} = \frac{\alpha}{}$$

$$\frac{\alpha(\alpha+1)(\alpha+\beta)}{(\alpha+\beta)^2(\alpha+\beta+1)} - \frac{\alpha^2(\alpha+\beta+1)}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$(\alpha^2+\alpha)(\alpha+\beta)$$

$$\cancel{\alpha^3}+\beta\alpha^2+\alpha^2+\beta\alpha - \cancel{\alpha^3}+\alpha^2\beta\ \alpha^2$$

$$X \sim Beta(\alpha,\beta) \approx U(0,1)$$

$$\alpha=1,\ \beta=1$$

CAUCHY DAĞ

ÖYF

$$f(x) = \begin{cases} \frac{1}{\pi} \cdot \frac{1}{1-(x-\theta)^2} & -\infty < x < \infty,\ x \in R \\ 0 & \text{dd} \end{cases}$$

şeklinde ise X r.d cauchy dağ sahiptir.

$\int \frac{1}{1+x^2} = \arctan(x)$ $\quad \int \frac{1}{1+(x-\theta)^2} dx = \arctan(x-\theta)$

i) $f(x) \geq 0$

$$f(x) = \frac{1}{\pi}\overset{>0}{\cdot} \frac{1}{1+(x-\theta)^2} > 0 \quad x > 0 \quad ; \quad 0 \quad \text{dd} \quad \Big\} \ f(x) \geq 0,\ x \in R$$

ii) $\int_{-\infty}^{\infty} f(x) = 1$

$$\int_{-\infty}^{\infty} \frac{1}{\pi} \frac{1}{1+(x-\theta)^2} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{1+(x-\theta)^2} = \frac{1}{\pi} \arctan(x-\theta) \Big|_{-\infty}^{\infty}$$

$$\lim_{x \to \infty} \frac{1}{\pi} \left[\arctan(x-\theta)\right] \quad \lim_{x \to -\infty} \frac{1}{\pi} \left[\arctan(x-\theta)\right]$$

$$= \frac{1}{\pi} \left[\frac{\pi}{2} - -\frac{\pi}{2}\right] = \frac{\pi}{\pi} = 1$$

$\tan x = y$
$\arctan x = y$
$x = \arctan(y)$
$\cos x (90°) = \frac{\pi}{2} = 0$
$\arctan(90°) = \frac{\pi}{2}$

Cauchy F(x)

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt = \int_{-\infty}^{x} \frac{1}{\pi} \cdot \frac{1}{1+(x-\theta)^2} dt$$

$$= \frac{1}{\pi} (\arctan(x-\beta)) \Big|_{-\infty}^{x} = \frac{1}{\pi} \left[\arctan(x-\theta) - \left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{\pi} \arctan(x-\theta) + \frac{1}{2}$$

$$F(x) = \begin{cases} 0 & x \to -\infty \\ \frac{1}{\pi} \arctan(x-\theta) + \frac{1}{2} & x \in R \\ 1 & x \to \infty \end{cases}$$

θ parametresine göre simetrik

. E(x), Var(x) çarpıklık kümlet yok

$X \sim \chi^2_{(r)}$ Gamma$(\frac{r}{2}, 2)$	$X \sim$ Gamma(α, β)	Üstel$(\theta) \sim$ Gamma$(1, \theta)$
$E(x) = \frac{r}{2} \cdot 2 = r$	$E(x) = \alpha\beta$	$E(x) = 1 \cdot \theta = \theta$
$Var(x) = \frac{r}{2} \cdot 4 = 2r$	$Var(x) = \alpha\beta^2$	$Var(x) = 1 \cdot \theta^2 = \theta^2$
$M_x(t) = (1-2t)^{-\frac{r}{2}}$	$M_x(t) = (1-t\beta)^{-\alpha}$	$M_x(t) = (1-\theta t)^{-1}$
$K(t) = -\frac{r}{2} \ln(1-2t)$	$K(t) = -\alpha \ln(1-\beta t)$	$K_x(t) = -1 \ln(1-\theta t)$
$Skew = \frac{2\sqrt{2}}{\sqrt{r}}$	$Skew = \frac{2}{\sqrt{\alpha}}$	$skew(x) = \frac{2}{\sqrt{1}} = 2$
$kurt_0 = \frac{12}{r}$	$kurt = \frac{6}{\alpha}$	$kurtosis = \frac{6}{1} = 6$

NORMAL DAĞILIM

O.Y.F

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

STANDART NORMAL DAĞILIM

$Z \sim N(0,1)$

$\mu = 0$, varyansı 1 den, $z = \frac{x-0}{1}$ şeklinde standartlaştırınız. $\sigma^2 = 1$, $\mu = 0$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad z \in \mathbb{R}$$

BEKLENEN DEĞER

$$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx \qquad \int_{-\infty}^{\infty} x\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx$$

$$\frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} x\cdot e^{-\frac{x^2}{2}}dx + \int_{0}^{\infty} x\cdot e^{-\frac{x^2}{2}}dx\right] \qquad e^{-\infty} \quad x=\infty = 0 \qquad e^{0} \quad x=0 = e^{0} = 1$$

$$\frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} x\cdot e^{u}\frac{du}{x} + \int_{0}^{\infty} x\, e^{u}\frac{du}{x}\right] = \frac{1}{\sqrt{2\pi}} = 0$$

$$[0-1] + [1+0] = 0$$

$$E[X] = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} x e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \quad t = \frac{x-\mu}{\sqrt{2}\sigma} \quad \sqrt{2}\sigma t + \mu = x$$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sqrt{2}\sigma t + \mu) e^{-t^2} dt = \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}}\underbrace{\int_{-\infty}^{\infty} t e^{-t^2} dt}_{1} + \mu \int_{-\infty}^{\infty} e^{-t^2} dt$$

(2) $\int e^{-t^2} dt = \sqrt{\pi}$

$$= \frac{\mu\sqrt{\pi}}{\sqrt{\pi}} = \mu$$

Moment Üreten Fonksiyon

$$M_X(t) = E(e^{xt}) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} e^{xt}\,dx \qquad \sqrt{2}\sigma u + \mu = x \quad t = \frac{x-\mu}{\sqrt{2}\sigma}$$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{xt - \frac{(x-\mu)^2}{\sigma^2}} dx$$

$$\frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{xt - \frac{(x-\mu)^2}{\sigma^2}} du = \qquad \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(u^2 - 2\sigma u t)} du$$

$$\frac{e^{\mu t}}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-(u-\frac{\sqrt{2}}{2}\sigma t)^2 + \frac{\sigma^2 t^2}{2}} du$$

$$\frac{\sqrt{\pi}\, e^{\mu t + \frac{\sigma^2 t^2}{2}}}{\sqrt{\pi}} = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

Kümilant Üreten Fonksiyon

X in kümilant üreten fonk. $K_X(t)$

$K_X(t) = \ln(M_X(t))$ olarak tanımlanmaktadır.

$X \sim N(\mu, \sigma^2)$ için $M_X(t) = e^{t\mu + \frac{t^2\sigma^2}{2}}$

$$K_X(t) = \ln(M_X(t)) = \ln\left(e^{t\mu + \frac{t^2\sigma^2}{2}}\right) = t\mu + \frac{t^2\sigma^2}{2}$$

Tanımlayıcı Momentler

$$\left.\frac{d^k(M_X(t))}{dt^k}\right|_{t=0} = E(X^k) \text{ olarak elde edilir}$$

$N \sim N(\mu, \sigma^2)$ için $M_X(t) = e^{t\mu + \frac{t^2\sigma^2}{2}}$ old

$$\frac{d(M_X(t))}{dt} = (\mu + t\sigma^2)\, e^{t\mu + \frac{t^2\sigma^2}{2}}$$

$$E(X) = \left.\frac{d(M_X(t))}{dt}\right|_{t=0} = \left.(\mu + t\sigma^2)\, e^{t\mu + \frac{t^2\sigma^2}{2}}\right|_{t=0} = \mu$$

X rassal değişkenin eğer $k_x(t)$'si var ise

$$\left.\frac{d^k(k_x(t))}{dt^k}\right|_{t=0} = k_k \qquad E(X)=k_1 \quad Var(X)=k_2$$

$$E(X-E(X))^3 = k_3 \qquad E(X-E(X))^4 = k_4 + 3(k_2)^2$$

$X \sim N(\mu_x, \sigma^2)$ için

(Çarpıklık) (Skewness)(x) $= \frac{E(X-E(X))^3}{Var(X)^{3/2}} = \frac{0}{(\sigma^2)^{3/2}}$

Basıklık $= \frac{E(X-E(X))^4}{(Var(X))^2} = \frac{3\sigma^2}{(\sigma^2)^2} - 3$

Birikimli Dağılım Fonk.

$$F(x) = p(X \le x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

$$F(x) = p(X \le x) = \int_{-\infty}^{x} f(t)dt$$

ORTAK TANIMLAR

ORTAK OLASILIK DAĞILIM FONKSİYONUDUR.

$$F(x,y) = \begin{cases} \frac{\partial^2 f(x,y)}{\partial x \partial y} & \text{f'in türevlenebilir olduğu noktalar} \\ 0 & \text{dd} \end{cases}$$

$$\int_{-\infty}^{x} f(s)ds \quad F(x) = 1$$

ORTAK OLASILIK FONKSİYONU / YOĞUNLUK FONKSİYONU

ORTAK OLASILIK FONKSİYONU (kesikli)

i) $\forall x \in R$, $y \in R$ için $f(x,y) \ge 0$

ii) $\sum_{x \in R} \sum_{y \in R} f(x,y) = 1$

ORTAK OLASILIK YOĞUNLUK FONKSİYONU (sürekli)

i) $\forall x \in R$, $y \in R$ için $f(x,y) \ge 0$

ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1 \qquad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dydx$

MARJİNAL OLASILIK FONKSİYONLARI

Kesikli marjinal olasılık fonksiyonlar

$$f(x) = \begin{cases} \sum_{y \in D_y} f(x,y) & \forall x \in D_x \text{ için} \\ 0 & \text{dd} \end{cases} \qquad f(y) = \begin{cases} \sum_{x \in D_x} f(x,y) & \forall y \in D_y \text{ için} \\ 0 & \text{dd} \end{cases}$$

Sürekli marjinal olasılık yoğunluk fonksiyonları

$$f(x) = \begin{cases} \int_{-\infty}^{\infty} f(x,y)dy & y \in D_y \\ 0 & \text{dd} \end{cases} \qquad f(y) = \begin{cases} \int_{-\infty}^{\infty} f(x,y)dx & y \in D_y \\ 0 & \text{dd} \end{cases}$$

KOŞULLU OLASILIK / OYF

$$f_{X|Y=y} \quad f(x|y) = \begin{cases} \dfrac{f(x,y)}{f(y)} & f(y) > 0 \\ 0 & dd \end{cases}$$

$$f_{Y|X} = f(y|x) = \begin{cases} \dfrac{f(x,y)}{f(x)} & f(x) > 0 \text{ ve } y \in D_{y|x} \\ 0 & dd \end{cases}$$

KOVARYANS

(iki değişkenin birbirine göre değişimi)

$$COV(X,Y) = E[(X - E(X))(Y - E(Y))]$$

$E[XY] - E(X)E(Y)$ bağımsız ise 0

KORELASYON KATSAYISI

doğrusal ilişki

$$P_{x,y} = \frac{COV(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}}$$

$|P_{x,y}| \leq 1$ ise ilişki vardır.

$-1 < P_{x,y} < 1$ 0 ise bağımsız eğer 0 ise veya yaklaşıyorsa ilişki yok.

Bağımsızlık: $f(x,y) = f(x) \cdot f(y)$

Varyans: bir değişkenin varyansı

verinin ortalamadan sapmalarının karesidir