Unit 3: Time Value of Money

BY: Pitambar shrestha

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Concept of Time Value of Money

Time value of money is one of the most important concepts in the finance.

The concept of time value of money deals with the facts that a rupee in hand today has more value than a rupee that received one year from now.

It is a concept to understand the value of cash flow occurred at different period of time.

It advises you that "A rupee today is worth more than a rupee to be received after one year". Because, a rupee of today can be invested and due to interest rate becomes more than a rupee after one year.

Significant of Time Value of Money

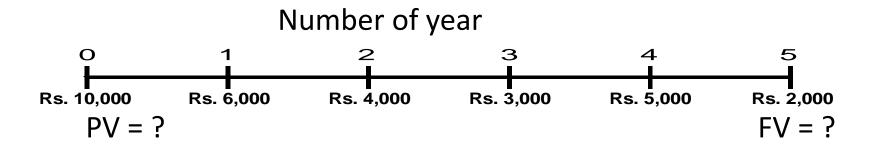
- Valuations of securities and others assets
- Analysis of capital investment project
- The cost of capital analysis
- Short term Assets and Liabilities management
- Lease analysis
- Trade off between risk and return

Cash Flow Time Lines

An important tool used in time value analysis; it is a graphical representation used to show the timing of cash flows.

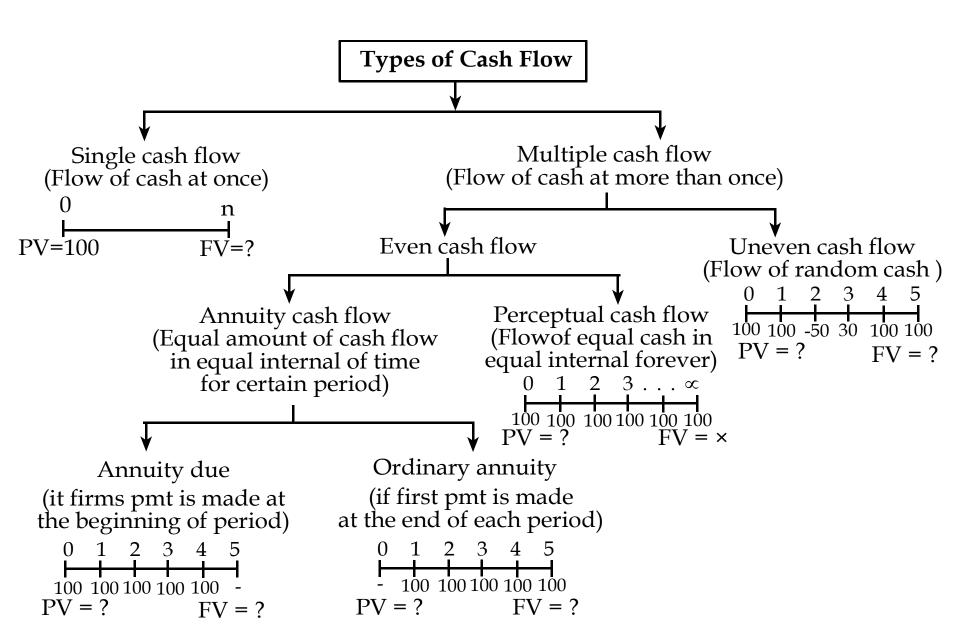
It is a horizontal line on which time zero appears at the left most end and future periods are marked from left to right. A time line covering five period is given in below figure.

Cash flow Time line



The cash flow occurring at time zero and at the end of each year is shown above the line; the negative values represent cash out flows (Rs 10000 at the time zero) and the positive values represent cash inflows (Rs 6000 in flows at the end of year1, Rs 4000 at the end of year 2 and so on).

Cont...



Future value and Present Value

Future Value and Compounding

Future value refers to the value of money at certain period in future or Future Value (FV) is the future value of earlier cash flows. The process of finding the FV of cash flows by considering the effect of time value of money is known as *Compounding*.

The future value of single cash flow (Cash flow receives or pays only once a time during the investment period) can be calculated as follows.

Formula Method		Table Method
$FV_{n} = PV (1+i)^n$	Or	$FV_{n} = PV \times FVIF_{i,n}$

Where,

 FV_n = Future value at the end of n^{th} Year

PV = Present value or beginning amount

i = Interest rate that you have earned. It is expressed as a

decimal

n = number of periods

 $FVIF_{i,n}$ = Future value interest factor for i and n.

Future value and Present Value

Present value and Discounting

Present value refers to the current value of money or value of money at zero years. In other words, present value (PV) is the current value of future cash flows. The process of finding the PV of future cash flows by considering the effect of time value of money is known as discounting

The present value of single cash flow (Cash flow receives or pays only once a time during the investment period) can be calculated as follows.

Formula Method Table Method

PV = FV/ (1+i)ⁿ Or
$$PV_{n} = FV \times PVIF_{i,n}$$

Where,

PVIF_{i.n} = Present value interest factor for i and n.

1.1) Finding present value: If you deposit Rs 10000 in Bank account that pay 10 percent interest annually, how much money will be in your account after 5 year. *Given,*

```
Present Value (PV) = 10000, Future Value (FV) = ?
```

Number of years(n) = 5 years, Interest rate (i) =
$$10 \%$$

Now,

$$FV_{n} = PV (1+i)^n = 10000 (1+0.1)^5 = Rs 16105.10 (Formula Method)$$

Or

$$FV_{n} = PV \times FVIF_{i,n} = 10000 \times FVIF_{10\%,5} = 10000 \times 1.6105 = Rs 16105 \text{ (table method)}$$

1.2) Finding interest rate:

Given,

Present Value (PV) =
$$10000$$
, Future Value (FV) = 16105

Number of years(n) = 5 years, Interest rate (i) = ?

Now,

$$FV_{n} = PV (1+i)^n \text{ or } 16105 = 10000(1+i)^5 \text{ or } = (1+i)^5 = (16105/10000)$$

Required interest Rate (i) =
$$(16105/10000)^{1/5} - 1 = 0.1 = 10\%$$

1.3) Finding number of period (n)

```
Given,
Present Value (PV) = 10000, Future Value (FV) = 16105
Number of years(n) = ?, Interest rate (i) = 10%
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```
Now, FV_{n} = PV (1+i)^n \text{ or } 16105 = 10000(1+0.1)^n \text{ or } = (1.1)^n = (16105/10000)(1.1)^n = 1.6105
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Taking log both side

n log 1.1 = log 1.6105

n = (log 1.6105/log 1.1)

Required number of year (n) = 5 years
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Table Method

$$FV = PV \times FVIF_{i \% n Year}$$
or, $16105 = 10000 \times FVIF_{10\% n year}$
or, $16105/10000 = FVIF_{10\% n year}$

 $FVIF_{10\% \text{ n year}} = 1.6105$

Referring to the FVIF table at 10% column, factor value 1.6105 exactly lies in 5 years.

Multiple Cash Flows

when we pay or receive two or more than two times during the given period of time, such cash flow is know as multiple cash flow. Multiple cash flow can be divided in two even cash flow(Annuity) and uneven cash flow.

- (a)Annuity(Even cash flow): A series of equal payment or receipts occurring in each period of time over a specified number of periods. Annuity can be divided in to Ordinary annuity and Annuity due.
- (i) Ordinary Annuity: A series of equal payment or receipts occurring in at the end each period of time.

The future value of ordinary annuity can be calculated as follows.

Formula Method:

$$FVA_{n} = PMT \left[\frac{(l+i)^n - 1}{i} \right]$$

Table Method:

$$FVA_n = PMT \times FVIFA_{in}$$

Multiple Cash Flows

Where,

PMT = Periodic Payment in each period

FVA = Future value of an annuity

 $FVIFA_{i,n}$ = The future value interest factor for an annuity of n periods compounded at i percent.

The future value of ordinary annuity can be calculated as follows.

Formula Method

$$\mathbf{PVA} = \mathbf{PMT} \begin{bmatrix} 1 - \frac{1}{(l+i)^n} \\ i \end{bmatrix}$$

Table Method

$$PVAn = PMT \times PVIFAi,n$$

PVA = Present value of annuity

PVIFAi,n = present value interest factor for annuity of n periods discounted at i percent.

Multiple Cash Flows

(ii) Annuity Due: A series of equal payment or receipts occurring in at the beginning each period of time.

The future value of ordinary annuity can be calculated as follows.

Formula Method

$$FVA_{n,Due} = PMT \left\lceil \frac{(l+i)^n - 1}{i} \right\rceil [1+i]$$

Table Method:

$$FVA_{n, Due} = PMT \times (FVIFA_{i, n}) [1+i]$$

The present value of ordinary annuity can be calculated as follows.

Formula Method

$$PVA_{n,DUE} = PMT \left[\frac{1 - \frac{1}{(l+i)^n}}{i} \right] [1+i]$$

Table Method

$$PVA_{n.Due} = PMT (PVIFA_{i.n}) [1+i)$$

- (2) Find the present value of the following ordinary annuities.
- (a) Rs. 400 per year for 10 years at 10 percent.
- (b) Rs. 200 per year for 5 years at 5 percent.
- (c) Now rework parts A and B assuming that payments are made at the beginning of each year, that is, they are annuities due.

Solution:

(a) Calculation the present value of ordinary annuity

We have,

$$PV_{A} = PMT \times \left(\frac{1 - \frac{1}{(1+i)^{n}}}{i}\right) = 400 \times \left(\frac{1 - \frac{1}{(1+0.1)^{5}}}{0.1}\right) = 400 \times 6.1446$$
$$= Rs 2457.83$$

Cont...

Or

The present value of annuity can also be calculated by using tabulation method

```
PV_A = PMT [PVIFA<sub>i,n</sub>]

= 400[ PVIFA<sub>10%,10</sub>]

= 400 × 6.1446 = Rs. 2457.83 , The present value is Rs. 2457.83.

(b) PMT = Rs. 200 per year, I = 5%, n = 5 years

we have,

PV_A = PMT[PVIFA<sub>5%,5</sub>]

= 200 × 4.3295 = 865.90
```

∴ Present value = Rs. 865.90

Cont...

- (c) Calculation of present value for annuity due
- (i) PMT = 400 per year, n = 10 years, i = 10% we have,

$$PV_{A(due)} = PMT \times PVIFA_{i,n} \times (1 + i)$$

= $400 \times PVIFA_{10\%,10} \times (1 + 0.10)$
= $400 \times 6.1446 \times 1.10$
= 2703.62

.: Present value of annuity due is Rs. 2703.62

```
(ii) PMT = 200 per year, i = 5%, n = 5 years we have,
```

$$PV_{A(due)} = PMT \times PVIFA_{i,n} \times (1 + i)$$

= 200 [PVIFA_{5%,5}] (1 + 0.05) = 200 × 4.3295 ×1.05 = 909.20

Therefore, Present value of Annuity due is Rs. 909.20

Perpetuity

An annuity that never stops providing its holder with a cash flow at the end of each year. An ordinary annuity whose payments or receipts continue forever. The present value of a perpetuity can be found by.

$$PV_{perpetuity} = \frac{PMT}{i}$$

Example 3:

What is the present value of perpetuity which pays Rs. 1500 per year continuously. The annual interest rate is 8 percent per year.

Given,

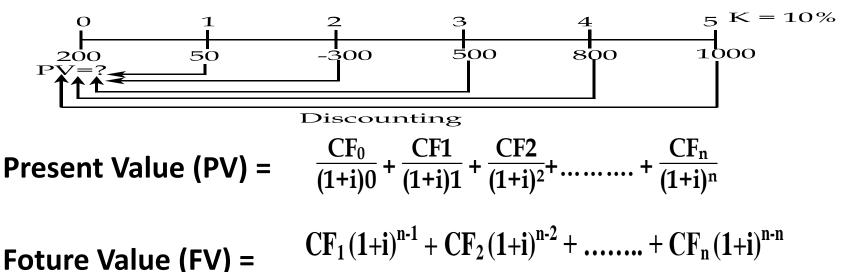
Periodic payment per year (PMT) = 1500

Annual interest rate (i) = 8%

Present value of perpetuity =
$$\frac{PMT}{i} = \frac{1500}{0.08} = Rs. 18750$$

(b)Uneven Cash Flow

If there are multiple cash flows randomly in different period, is called uneven cash flows. Uneven may in terms of time period or in size.



$$CF_{0_r} CF_1 \& CF_2 = Cash flow in year 1 and 2 respectively $CF_n = Cash flow at the end of n Year$$$

Example 4:

Example 4: Mrs. Prerana is considering the following uneven cash flows.

Year	0	1	2	3	4	5
Cash Flows (Rs.)	200	50	-300	500	800	1000

If current market interest rate is 10% per year

- a) Calculate total present value of cash flow
- b)Calculate total Future Value of cash flow

Solution:

(a) Present Value =
$$\frac{CF_0}{(1+i)^0} + \frac{CF_1}{(1+i)^1} + \dots + \frac{CF_n}{(1+i)^n}$$

$$= \frac{Rs.\ 200}{(1+0.1)^1} + \frac{Rs.\ 50}{(1+0.1)^2} + \frac{-Rs.\ 300}{(1+0.1)^3} + \frac{Rs.\ 1000}{(1+0.1)^5} = Rs.$$

$$1605.06$$

Cont...

Under Tabulation Method

Year	Cash flows (CF)	PVIFi _{i,n} = PVIFi _{@ 10%}	FV= CF _n ×Pv Factors
0	200	1.000	Rs 200
1	Rs. 50	0.9091	Rs.45.455
2	-300	0.8264	-247.92
3	500	0.7513	375.65
4	800	0.6830	546.41
5	1000	0.6209	620.9
		Total PV = $\sum CF_n \times Dis_n$	Rs. 1605.06

(b) Future Value =
$$CF_0(1+i)^{n-0} + CF_1(1+i)^{n-1} + \dots + CF_n(1+i)^{n-n}$$

= $200(1.1)^5 + 50(1.1)^4 - 300(1.1)^3 + 500(1.1)^2 + 800(1.1)^1 + 1000(1.1)^0 = Rs. 1875.37$

Types of Interest

Interest is divided into two types;

- a) Nominal interest rate/Quoted/Annual percentage rate (APR)
- b) Effective Interest rate (EAR)

Nominal interest rate/Quoted interest arte: When the rate is annualized without considering compounding effect, is known as nominal interest rate or annual percentage rate (APR).

Nominal Interest rate/ annual percentage rate (APR) = Periodic interest rate × Number of period during the year (m)

Effective Annual rate (EAR): When the interest rate is annualized with considering compounding period or reinvestment opportunity, is known as effective annual rate (EAR).

(EAR).

Effective Annual Rate (EAR) = $\left(1 + \frac{\mathbf{i}}{\mathbf{m}}\right)^{m} - 1$ Or, EAR = $(1 + \text{periodic rate})^{m} - 1$ m = No. of compounding period in a year

EAR of Continuous compounding period = e^{i} -1

E = exponential factor factors which value = 2.7183

Semi-annual and other compounding Period

Future value can be computed when compounding occurs more than once a year such as semiannually, quarterly , monthly or daily by using the following formula:-

the following formula:- Future Value (FV) = PV
$$\times \left(1 + \frac{i}{m}\right)^{n \times m}$$
 for single cash flow

Note: If cash flow is even and given in annual basis but interest is compounded less than one year period such as monthly quarterly or semiannually, in this condition 1st we need to calculate effective annual rate (EAR) and use this EAR to calculate PV and FV. Such as;

$$FVA = PMT \left\lceil \frac{(l + EAR)^n - 1}{EAR} \right\rceil$$

If interest is compounding continuously,

Future value (FV) =
$$PV \times e^{i \times n}$$

Example 5:

Example 5: You are planning to deposit Rs. 100 in bank a/c which has promised to pay 10% interest on your money. If so, find the future value under each of the following conditions.

a)Interest compounded annually (b)Semi-annual compounding (f)Daily compounding.

Solution:

Present value (PV) = 100, Interest (i) = 10%, Future value (FV) = ?, No. of year (n) = 1 year

(a) If interest is compounded annually, the number of compounding period in a year (m) = 1

$$FV = PV (1 + i/m)^{n \times m} = 100 (1 + 0.1/1)^{1 \times 1} = 110$$

(b) If interest is compounded semi-annually, the number of compounding period in a year (m) = 2

$$FV = PV (1 + i/m)^{n \times m} = 100 (1 + .1/2)^{1 \times 2} = 110.25$$

(c) If interest is compounded daily, the number of compounding period in a year (m) = 365

$$FV = PV (1 + i/m)^{n \times m} = 100 (1 + .1/365)^{1 \times 365} = 110.52$$

Amortized Ioan and Ioan amortization Schedule

The term loan amortization refers to the determination of the equal periodic loan payments necessary to provide a lender with a specified interest return and repay the loan principal over a specified period of time.

Loan amortization schedule showing the repayment schedule of interest and principal necessary to pay off a loan by maturity period is called amortization schedule.

Periodic payment or periodic Installments =
$$\frac{\frac{Loan\ amount}{PVIFA_{i\%,n}}}{\frac{1-\frac{1}{(1+i)^n}}{i}} or, \frac{\frac{Loan\ Amount}{n}}{\frac{1-\frac{1}{(1+i)^n}}{n}}$$

The loan amortization Schedule can be presented as follows.

Cont...

Loan Amortization schedule

Year (a)	Beginning Amount (b)	Installment Payment (c)	Interest d= (b × i)	Principal $e = (c - d)$	Ending Balance $f = (b - e)$
0					XXXX
1	XXX	XXX	XXX	XXX	XXX
2	XXX	XXX	XXX	XXX	XXX
n	XXX	XXX	XXX	XXX	0

Example 6: Suppose Mr. Dipak borrowed loan Rs. 1,00,000 for five years at 10% annual interest rate. The loan is repaid on annual instalment. Calculate annual instalment and prepare loan amortization schedule.

Cont...

Solution:

Loan installment =
$$\frac{\text{Loan amount}}{\text{PVIFA}_{i\%,n}} = \frac{\text{Rs.1,00,000}}{\text{PVIFA10\%,5 years}} = \frac{\text{Rs.1,00,000}}{3.7908} = \text{Rs. 26379.75}$$

Loan Amortization Schedule

(a) Year	(b) Loan amount	(c) Instalment	(d) = (b) × 15% Interest	(e) = (c) – (d) Principal	(f) = (b) – (f) Remaining
0	Rs. 1,00,000				Rs. 1,00,000
1	1,00,000	26,379.75	10,000	16379.75	83,620.25
2	83,620.25	26,379.75	8,362.03	18,017.73	65,602.53
3	65602.53	26,379.75	6,560.25	19,819.50	45,783.03
4	45,783.03	26,379.75	4,578.30	21,801.45	23,981.58
5	23,981.58	26,379.75	2,398.16	23,981.59	0.01

Example 7: Assume that it is now January 1, 1998, and you will need Rs. 1000 on January 1, 2002. Your bank compounds interest at an 8% rate annually.

- (a) How much must you deposit on January 1, 1999 to have a balance of 1,000 on January 1, 2002?
- (b)If you want to make equal payments on each January 1 from 1999 through 2002 to accumulate the Rs. 1000, how large must each of the four payments be?
- (c)If your father were to offer either to make the payments calculated in Part B or to give you a lump sum of Rs. 750 on January 1, 1999, which would you choose?
- (d)If you have only Rs. 750 on January 1, 1999, what interest rate, compounded annually, would you have to earn to have the necessary Rs.1000 on January 1, 2002?

Solution:

(a) Interest rate (i) = 8%, Needed fund on Jan. 1, 2002 (FV) = Rs. 1000, Initial deposit(PV) = ?, Time period (n) = 3 years

Present value of deposit (PV) =
$$\frac{FV}{(1+i)^n} = \frac{1000}{(1+0.8)^3} = Rs. 793.83$$

Therefore, the initial deposit is Rs. 793.83 on Jan 1, 1999 to have a balance of Rs. 1000 on Jan. 1, 2002.

(b) Here, we are dealing with a 4 year annuity whose first payment occurs one year from today. On Jan. 1, 1999 and whose future value must equal Rs. 1000.

Given,

Time period (n) = 4 years, Interest rate (i) = 8%, Future value (FV_A) = Rs. 1000, Periodic payment (PMT) =?

We have,

$$FV_A = PMT [FVIFA_{i,n}]$$

 $1000 = PMT[FVIFA_{8\%,4}]$
 $PMT = \frac{1000}{4.5061} = Rs. 221.92.$

Therefore, the yearly payment would be Rs. 221.92 from Jan. 1, 1999 through 2002

(c) Interest rate (i) = 8%, Present value (PV) = Rs. 750, Future value (FV₃) = ? We have,

FV_n = PV
$$(1 + i)^n$$

= 750 $(1 + 0.08)^4$
= 1020.36

Here, future value of lump sum amount (i.e. Rs. 750) is Rs.1020.36 which is greater than Rs. 1000. Therefore I must choose lump sum amount rather than periodic payment of Rs. 221.92 of Rs. 750.

(d) Present value (PV) = Rs. 750, Future value (FV) = Rs. 1000, Time period (n) = 3 years, Int We have,

FV_n = PV
$$(1 + i)^n$$
 or $(1 + i)^n = \frac{FV}{PV}$
 $i = (\frac{FV}{PV})^{1/n} - 1 = (\frac{1000}{750})^{1/3} - 1 = 0.10064 = 10.06\%$

Therefore, interest rate (i) is 10.06% to have the Rs. 1000 on Jan 1, 2002

Example 8: To complete your last year in Bachelor on Business Studies and then go through Chartered Accountancy you will need Rs. 60,000 per year for four years, starting next year. Your father offers to put you through in the Chartered Accountancy Institute, and he will deposit in a bank time deposit paying 8% interest a sum of money that is sufficient to provide the four payments of Rs. 60,000 each. His deposit will be made today. How large must the deposit be?

Solution:

Amount needed per year (PMT) = Rs. 60,000 per year, Interest rate (i) = 8%, Time period (n) = 4 years, Amount that must be deposit today (PV_A) = ? We have.

PVA = PMT [PVIFA_{i,n}] = 60,000 [PVIFA_{8%,4}] = $60,000 \times 3.3121$ = Rs. 1,98,726 Therefore, he must deposit Rs. 1,98,7260 today to complete his study.

Example 9: Assume that it is now January 1, 1998, and you will need Rs. 1000 on January 1, 2002. Your bank compounds interest at an 8% rate annually.

- (a) How much must you deposit on January 1, 1999 to have a balance of 1,000 on January 1, 2002?
- (b) If you want to make equal payments on each January 1 from 1999 through 2002 to accumulate the Rs. 1000, how large must each of the four payments be?
- (c) If your father were to offer either to make the payments calculated in Part "b" or to give you a lump sum of Rs. 750 on January 1, 1999, which would you choose?
- (d) If you have only Rs. 750 on January 1, 1999, what interest rate, compounded annually, would you have to earn to have the necessary Rs.1000 on January 1, 2002?

Solution:

Interest rate (i) = 8%, Needed fund on Jan. 1, 2002 (FV) = Rs. 1000

(a) Initial deposit in Jan. 1, 1999(PV) = ? to get Rs.1000 in Jan. 1,2002 Given, deposit will grow for 3 years at 8 percent

 \therefore Time period (n) = 3 years

We have,

PV =
$$\frac{\text{FV}}{(1+i)^n} = \frac{1000}{(1+0.8)^3} = \text{Rs. } 793.83$$

Therefore, the initial deposit is Rs. 793.83 on Jan 1, 1999 to have a balance of Rs. 1000 on Jan. 1, 2002.

(b) Here, we are dealing with a 4 year annuity whose first payment occurs one year from today. On Jan. 1, 1999 and whose future value must equal Rs. 1000

Given,

Time period (n) = 4 years , Interest rate (i) = 8% , Future value (FV_A) = Rs. 1000, Periodic payment (PMT) =?

We have,

 $FV_A = PMT [FVIFA_{i,n}] \text{ or } 1000 = PMT[FVIFA_{8\%,4}] = 1000 = PMT \times 4.5061$ PMT = Rs. 221.92

Therefore, the each payment would be Rs. 221.92 from Jan. 1, 1999 through 2002.

(c) Calculation of future value

Time period (n) = 4 years , Interest rate (i) = 8%, Present value (PV) = Rs. 750, Future value (FV) = ?

We have,

$$FV_n = PV (1 + i)^n = 750 (1 + 0.08)^4 = 1020.36$$

Here, future value of lump sum amount (i.e. Rs. 750) is Rs. 1020.36 which is greater than Rs. 1000. Therefore I must choose lump sum amount rather than periodic payment of Rs. 221.92 of Rs. 750.

(d) Calculation of interest rate

Present value (PV) = Rs. 750

Future value (FV) = Rs. 1000

Time period (n) = 3 years

Interest rate (i) = ?

We have,

$$FV_n = PV (1 + i)^n$$

 $(1 + i)^n = \frac{FV}{PV} i = (\frac{FV}{PV})^{1/n} - 1 = (\frac{1000}{750})^{1/3} - 1 = 0.10064 = 10.06\%$

Therefore, interest rate (i) is 10.06% to have the Rs. 1000 on Jan 1, 2002.

Example 10: While Shyam Thapa was a student at the University, he borrowed Rs. 12,000 at an annual interest rate of 9%. If he repays Rs. 1500 per year, how long to the nearest year will it take him to repay the loan?

Solution:

Given,

```
Present value (PV) = Rs. 12,000, Regular payment (PMT) = Rs. 1500 per year, Interest rate (i) = 9%, Time period (n) = ?

We have, PV_A = PMT [PVIFA_{i,n}] = 1500 [PVIFA_{9%,n}]
```

12000 = 1500 [PVIFA_{9%,n}]
PVIFA_{9%,n} =
$$\frac{12000}{1500}$$
 = 8.00

Looking at PVIFA table the factor 8 in 9 percent rate lies nearly at 15 years. Therefore it is nearly 15 years to repay the loan.

Example 11: The Lalitupur Textile Company buys a machine for Rs. 50,000 and expects return of Rs. 11,511.19 per year for the next 8 years. What is the expected rate of return?

Solution:

Given,

Present value (PV) = Rs. 50,000, Annual return (PMT) = Rs. 11,511.19 per year, Time period (n) = 8 years, Interest rate (i) = ?

We have,

 $PV_A = PMT [PVIFA_{i,n}]$

50,000= 11,511.19 [PVIFA_{i.8}]

 $PVIFA_{i,8} = 50000/11511.19 = 4.3436$

Now,

In PVIFA table, factor 4.3436 for 8 years exactly lies at 16% interest rate. Therefore, interest rate is 16%

Thank You