CS-E4850 Computer Vision
Exercise Round 10

The problems should be solved before the exercise session and solutions returned via the MyCourses page. In this round, there are also programming tasks for which the instructions are given separately. Return the solutions to pen and paper problems in a single PDF file, and another file for programming tasks.

Exercise 1. Epipolar geometry. (Pen & paper problem)
Let's assume that the camera projection matrices of two cameras are $\mathbf{P} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$ and $\mathbf{P}' = \begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$,where $\mathbf{R}$ is a rotation matrix and $\mathbf{t} = (t_1, t_2, t_3)^\top$ describes the translation between the cameras. Hence, the cameras have identical internal parameters and the image points are given in the normalized image coordinates (the origin of the image coordinate frame is at the principal point and the focal length is 1).

The epipolar constraint is illustrated in the figure below and it implies that if p and p' are corresponding image points then the vectors $\overrightarrow{Op}$, $\overrightarrow{O'p'}$ and $\overrightarrow{O'O}$ are coplanar, i.e.

$$\overrightarrow{O'p'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op}\right) = 0 \tag{1}$$

Let $\mathbf{x} = (x, y, 1)^\top$ and $\mathbf{x}' = (x', y', 1)^\top$ denote the homogeneous image coordinate vectors of p and p'.

Show that the equation (1) can be written in the form

$$\mathbf{x}'^\top \mathbf{E} \mathbf{x} = 0, \tag{2}$$

where matrix $\mathbf{E}$ is the essential matrix $\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$ (as defined on slide 21 of Lecture 9).

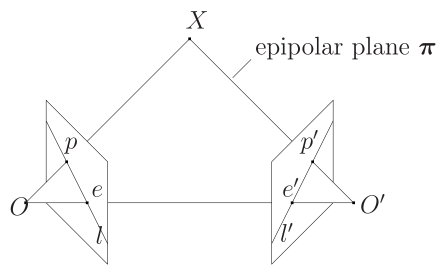

Figure 1: Epipolar geometry. Given a point p in the first image its corresponding point in the second image is constrained to lie on the line l' which is the epipolar line of p. Correspondingly, the line l is the epipolar line of p'. Points e and e' are the epipoles.

Exercise 2. Stereo vision. (Pen & paper problem)

In Figure 2 there is a picture of a typical stereo configuration, where two similar pinhole cameras are placed side by side. The focal length of the cameras is f and the distance between the camera centers is b. The point P is located in front of the cameras and its disparity d is the distance between the corresponding image points (i.e. $d = |x_\mathrm{l} - x_\mathrm{r}|$). The disparity depends only on the parameters b and f and the Z-coordinate of P.

a) Assume that $d = 1\ cm$, $b = 6\ cm$ and $f = 1\ cm$. Compute Z_P.

b) Assume that the smallest measurable disparity is 1 pixel and the pixel width is 0.01 mm. What is the range of Z-coordinates for those points for which the disparity is below 1 pixel?

c) In the configuration illustrated in Figure 2 the camera matrices are $\mathbf{P}_\mathrm{l} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$ and $\mathbf{P}_\mathrm{r} = \begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}$, where $\mathbf{I}$ is the identity matrix and $\mathbf{t} = (-6, 0, 0)^\top$. The point Q has coordinates $(3, 0, 3)$. Compute the image of Q on the image plane of the camera on the left and the corresponding epipolar line on the image plane of the camera on the right. (Hint: The epipolar line is computed using the essential matrix.)

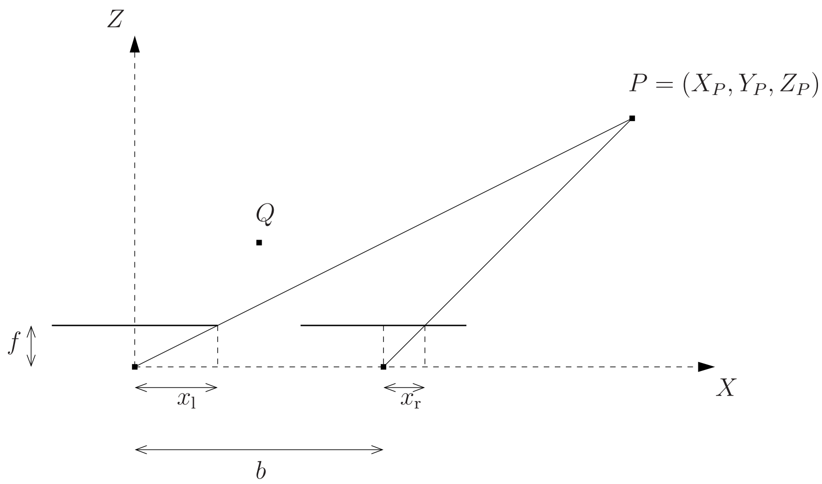

Figure 2: Top view of a stereo configuration where two pinhole cameras are placed side by side.

Exercise 3. Fundamental matrix estimation. (Python exercise)

See separate instructions in github.

Demo 1. Stereo disparity computation. (Just a demo, no points given)

See the example in github.