The Odd Questions of Chapter 6

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6.1: Let f be any function. Suppose that the inverse relation $f^{-1} = \{\langle y, x\rangle : y = f(x)\}$ is a function. Is f^{-1} a bijection?

Yes, it is a bijection because every ordered pair will have an inverse pair and no 2 ordered pairs will share an inverse unless they themselves where the same pair.
If f^{-1} was not injective then more than one y maps to a singular x, which is not possible because f is a function. If f^-1 was not surjective then no y would map to an x, which is not possible beceause f is a function.

6.3:

a: If 2 finite sets A and B are the same size and r is an injective function from A to B, show that r is also surjective:

Injective means that the codomain has a element for atleast every element of the domain;—B— ¿= —A—. If the sets must be the same size then we know there are not any extra elements in the codomain so the function must also be surjective.

b: Give a counterexample showing part (a) does not necessarily hold if the sets are bijectively related infinite sets:

Suppose we have a square root function that relates a set of all perfect squares to a set of every real number. The function may seem bijective as each perfect square has a distinct integer (injective) and both sets have a cardinality of infinite, but there are numbers in the second set that cannot be mapped to, so the function is not surjective.

6.5: Suppose $f : A \mapsto B, g : C \mapsto D$, and $A \subseteq D$. Explain when $(f \circ g)^{-1}$ exists as a function from a subset of B to C. and express it in terms of f^{-1} and g^{-1}

$(f \circ g)$ goes from $C \mapsto A \mapsto B$ so $(f \circ g)^{-1}$ must go from $B \mapsto A \mapsto C$. Expressed in proper notation starting from a subset of B it would be $g^{-1}(f^{-1}(B'))$. This only represents a function though when functions f and g are injetive so they're inverses remain functions. g may be surjective but f may not as extra elements must in B so that the inverse of its subset may map to the entire set A.

6.7: The function $f(n) = 2n$ is a bijection from $\mathbb{Z}$ to the even integers and the function $g(n) = 2n + 1$ is a bijection from $\mathbb{Z}$ to the odd integers. What are f^{-1}, g^{-1} and the function h of Theorem 6.4?

$$f^{-1} = \frac{1}{2}n$$
$$g^{-1} = \frac{n-1}{2}$$

$$\begin{aligned} h(n) &= g^{-1}(f^{-1}(n)) \\ &= g^{-1}(\frac{1}{2}n) \\ &= \frac{\frac{1}{2}n - 1}{2} \\ &= \frac{n-2}{4} \end{aligned}$$