The Odd Questions of Chapter 9

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17 November, 2023

9.1: Using just $\neg$ and $\vee$, rewrite the following equations

a: $p \wedge q$

$\neg(\neg p \vee \neg q)$

b: $p \oplus q$

$\neg(\neg(p \vee q) \vee \neg(\neg p \vee \neg q))$

c: $p \Leftrightarrow q$

$\neg p \vee q$

9.3: Prove the associative laws by comparing truth tables for the following expressions

$(\alpha \vee \beta) \vee \gamma \equiv \alpha \vee (\beta \vee \gamma)$

α	β	γ	$(\alpha \vee \beta)$	$(\alpha \vee \beta) \vee \gamma$	$(\beta \vee \gamma)$	$\alpha \vee (\beta \vee \gamma)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

The 2nd and 4th columns are equivalent

$(\alpha \wedge \beta) \wedge \gamma \equiv \alpha \wedge (\beta \wedge \gamma)$

α	β	γ	$(\alpha \wedge \beta)$	$(\alpha \wedge \beta) \wedge \gamma$	$(\beta \wedge \gamma)$	$\alpha \wedge (\beta \wedge \gamma)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

The 2nd and 4th columns are equivalent

9.5: With truth tables determine if the following propositions are satisfiable, a tautlogy, or unsatisfiable

a: $p \Rightarrow (p \vee q)$

p	q	$(p \vee q)$	$p \implies (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

$p \implies (p \vee q)$ is a tautalogy

b: $\neg(p \Rightarrow (p \vee q))$

p	q	$(p \vee q)$	$p \implies (p \vee q)$	$\neg(p \implies (p \vee q))$
0	0	0	1	0
0	1	1	1	0
1	0	1	1	0
1	1	1	1	0

$\neg(p \implies (p \vee q))$ is unsatisfiable

c: $p \Rightarrow (p \Rightarrow q)$

p	q	$(p \implies q)$	$p \implies (p \implies q)$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

$p \implies (p \implies q)$ is satisfiable

9.7: Prove that $\alpha \equiv \beta$ iff $\alpha \Leftrightarrow \beta$ is a tautology

$\alpha \Leftrightarrow \beta$ checks that the values in α and β are either both true or both false. If it results a tautology then it means the values are equivalent in all possible combinations. If it does not return a tautology, then atleast one combination of α and β are not the same, and so then α and β are not equivalent

9.9: Give a real world example of p, q, r where $(p \wedge q) \Rightarrow r$ and $p \wedge (q \Rightarrow r)$ are not equivalent

Let's make the scenerio that where p is happiness, q is hungryness, and r is if you went a restaurant. The first proposition would read "If I am happy and hungry, then I will go to a restaurant" while the second proposition says "If I am hungry, I will go to a restaurant and I will be happy". The propositions will not be equivalent any scenerio I am not happy

9.11:

a: Write $p \Leftrightarrow q$ using $\oplus$ and the constant T

$p \oplus q \oplus T$

b: Show that $\oplus$ and $\Leftrightarrow$ are associative

q	p	r	$(q \oplus p)$	$(q \oplus p) \oplus r$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

q	p	r	$(p \oplus r)$	$q \oplus (p \oplus r)$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

They have the same final result

q	p	r	$(q \Leftrightarrow p)$	$(q \Leftrightarrow p) \Leftrightarrow r$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

q	p	r	$(p \Leftrightarrow r)$	$q \Leftrightarrow (p \Leftrightarrow r)$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

They have the same final result

c: Show that $\{\oplus, \Leftrightarrow, \neg, T, F\}$ is not a complete set of logical operators

The set of operators $\{\oplus, \Leftrightarrow, \neg, T, F\}$ cannot be used to create $\wedge$, $\vee$, or $\implies$ so it cannot be functionally complete. I have no clue how to prove this. In my head it makes sense to me that $\Leftrightarrow$ cannot diffentiate between two same 0s and 1s, but I am unsure if this has any real weight.
It would be good to note that $\oplus$ and $\Leftrightarrow$ are just negations of each other, and same with just T and F, and checking $\oplus$ or $\Leftrightarrow$ with the constants T and F is only going to result itself or its negation. So the set can be reduced to only 2 items, one of the operators and negation. Hopefully this makes my previous conclusion easier to reach.