The Odd Questions of Chapter 2

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2.1: How we have it defined, is -1 a negative number?

$$\begin{aligned} 2k + 1 &= n \\ 2k + 1 &= -1 \\ 2k &= -2 \\ k &= -1 \end{aligned}$$

We can get -1 as a negative number when k = -1

2.3: If m & n are odd, prove mn is odd

m = (2a+1); n = (2b+1); c, d $\in \mathbb{Z}$

$$\begin{aligned} 2k + 1 &= (2a + 1)(2b + 1) \\ 2k + 1 &= 4ab + 2a + 2b + 1 \\ 2k + 1 &= 2(2ab + a + b) + 1 \\ k &= (2ab + a + b) \end{aligned}$$

mn is an odd number when its k is equal to the integer (2ab + a + b)

2.5: Prove that $\sqrt[3]{2}$ is irrational

Every rational number can be written as a simplified fraction $\frac{a}{b}$

$$\begin{aligned} \sqrt[3]{2} &= \frac{a}{b} \\ \sqrt[3]{2}b &= a \\ 2b^3 &= a^3 \\ 2b^3 &= (2k)^3 \\ 2b^3 &= 8k^3 \\ b^3 &= 4k^3 \\ (2j)^3 &= 4k^3 \\ &\downarrow \end{aligned}$$

Both a & b were proven to be even (line 3 and line 6), meaning the fraction was not simplified and $\sqrt[3]{2}$ must be irrational. Also, the reduction of the problem can continue forever

2.7: Prove that a fair 7-sided dice is possible

A dices' chances rely on the area of each side.
For a pentagonal prism, there must be some lateral length where the 5 rectangular sides each have an area equal to the area of the pentagon bases

2.9:

a: Prove or provide counterexample: if c & d are perfect squares, then so is cd

c = x * x;   d = y * y;   c, d $\in \mathbb{Z}$

$$\begin{aligned} c * d &= cd \\ (x * x) * (y * y) &= cd \\ xy * xy &= cd \end{aligned}$$

So cd must be a perfect square such that its square root is equal to xy

b: Prove or provide counterexample: if cd is a perfect square and c $\neq$ d, then c & d are perfect squares

False, counterexample if cd = 36, c = 3, & d = 12. 36 is a perfect square but c & d are not

c: Prove or provide counterexample: if c & d are perfect squares such that $c > d$, $x^2 = c$, and $y^2 = d$, then $x > y$

$$\begin{gathered} c > d \\ \sqrt{c} > \sqrt{d} \\ x > y \end{gathered}$$

2.11: Critique the following "proof"

$$\begin{aligned} x &> y \\ x^2 &> y^2 \\ x^2 - y^2 &> 0 \\ (x+y)(x-y) &> 0 \\ x + y &> 0 \\ x &> -y \end{aligned}$$

There is no mathematical operation to go from step 4 to step 5

2.13: Write the following statements in terms of quantifiers and implications

a: Every positive real number has two distinct square roots

$\forall n, n \in \mathbb{R}, n > 0 \implies \exists a, b \in \mathbb{R} : a^2 = n \wedge b^2 = n \wedge a \neq b$

b: Every positive even number can be expressed as the sum of two prime numbers.

$\forall n, n \in \mathbb{N}, 2 \mid n \implies \exists p_1, p_2 \in \mathbb{N} : n = p_1 + p_2 \wedge 2, 3, ..., p_1 - 1 \nmid p_1 \wedge 2, 3, ..., p_2 - 1 \nmid p_2$

2.15: In a group of 6 people, there must be at least 3 whom 1 person, X, knows, or at least 3 whom X does not know

This statement strongly reflects the pigeonhole principal.
The principal states that when mapping set X on to set Y, the expression $\lceil \frac{|X|}{|Y|} \rceil$ can be used to find the minimum amount of X members that share members of Y.
There are 2 categories of people and 5 people who must be sorted.

$$\lceil \frac{|X|}{|Y|} \rceil$$
$$\lceil \frac{5}{2} \rceil$$
$$\lceil 2.5 \rceil$$
$$3$$

There must be atleast 3 people who are sorted into the categories, known or unknown