Lista 8

2) $\ln(x) \sim N(\mu, \sigma^2)$ então $e^{\ln(x)} = X \sim$ log normal

Então teremos que

$$E(e^{t\ln(x)}) = \exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$$

$t=1$

$$E(X) = \exp\left\{\mu + \frac{\sigma^2}{2}\right\}$$

Com $t=2$ teremos

$$E(e^{2\ln(x)}) = \exp\{2\mu + 2\sigma^2\}$$

$$E(X^2) = \exp\{2(\mu+\sigma^2)\}$$

Sendo assim teremos:

$$Var(X) = E(X^2) - E(X)^2 = \exp\{2(\mu+\sigma^2)\} - \exp\left\{2\left(\mu+\frac{\sigma^2}{2}\right)\right\}$$

$$= e^{2\mu} \cdot \left(e^{2\sigma^2} - e^{\sigma^2}\right)$$

$$E(X) = \exp\left\{\mu + \sigma^2/2\right\}$$

8) Temos que a estatística de ordem é dada por

$$f_{(k)}(x) = f(x) \cdot \frac{n!}{(n-k)!(k-1)!} \left[1 - F(x)\right]^{n-k} F(x)^{k-1}$$

Em nosso caso temos que $f(x) = 1$ e $F(x) = x$, assim teremos

$$f_{(k)}(x) = \frac{n!}{(n-k)!(k-1)!} (1-x)^{n-k} x^{k-1}$$

$$= \frac{\Gamma(n+1)}{\Gamma(n+1)\Gamma(k)} (1-x)^{n-k} x^{k-1}$$

$$\therefore X_k \sim Beta(k, n+1-k)$$