

In classical linear algebra,

$$X^T X \beta = X^T Y$$

$$\hat{\beta} = (X^T X)^{-1} X^T Y$$

$$\text{Let } (X^T X)^{-1} X^T = A$$

$$\hat{\beta} = AY$$

$$\text{Var}(\hat{\beta}) = \text{Var}(AY)$$

$$= A \text{Var}(Y) A^T$$

$$= A \text{Var}(\beta X + \varepsilon) A^T$$

β, X is fixed

$$= A \text{Var}(\varepsilon) A^T$$

$$\text{Let } \text{Var}(\varepsilon) = \sigma^2$$

$$= A \sigma^2 A^T$$

σ is a column vector

$$= A \cdot I \hat{\sigma}^2 A^T$$

Assume constant variance for error term. The estimate of std of error/residual is

$$= \hat{\sigma}^2 (A A^T)$$

$\hat{\sigma}$

$$A A^T = ((X^T X)^{-1} X^T) ((X^T X)^{-1} X^T)^T$$

$$= ((X^T X)^{-1} X^T) (X (X^T X)^{-1})$$

$$= (X^T X)^{-1}$$

$$\therefore \text{Var}(\hat{\beta}) = \hat{\sigma}^2 (X^T X)^{-1}$$

if X scale up to $100X$

$$\text{Var}(\hat{\beta})_{\text{new}} = \frac{1}{100^2} \hat{\sigma}^2 (X^T X)^{-1}$$

$$t = \frac{\hat{\beta}}{\text{std}(\hat{\beta})}$$

$$\therefore t_{\text{new}} = \frac{\hat{\beta}/100}{\frac{1}{100} \text{std}(\hat{\beta})} = \frac{\hat{\beta}}{\text{std}(\hat{\beta})}$$