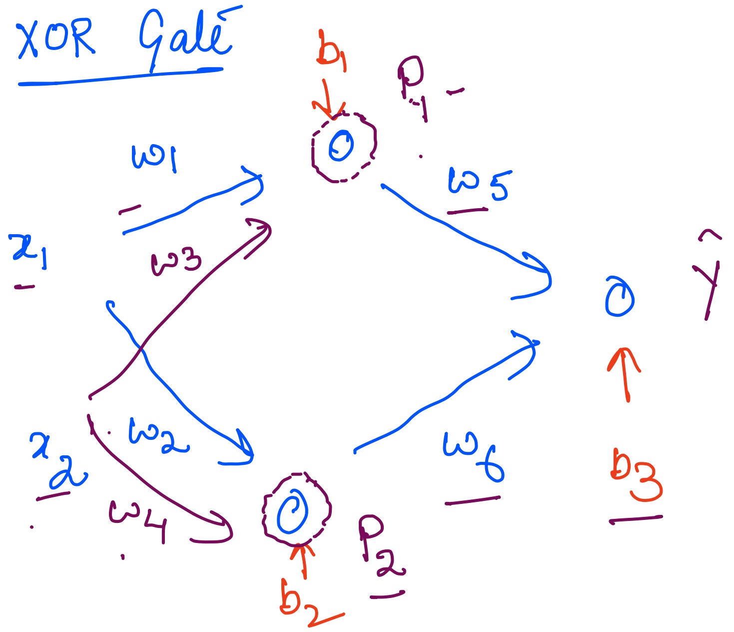
Perceptron 2

XOR Gate



x_1	x_2	y
0	0	0 ✓
0	1	1 ✓
1	0	1
1	1	0

Single Layer 2 neuron

↓

1 Hidden Layer with 2 neurons

$w_1 = w_2 = w_3 = w_4 = 1$ $\qquad w_5 = 1 \qquad w_6 = -1$

i/p,s $\quad b_1 = -0.5 \qquad b_2 = -1.5 \qquad b_3 = -0.5$

AF: $\phi = \begin{cases} 0 & w.x + b \leq 0 \\ 1 & w.x + b > 0 \end{cases}$

1st Case

$x_1 = 0 \qquad x_2 = 0$

$$P_1 = w_1 x_1 + w_2 x_2 + b_1$$
$$= 1 \times 0 + 1 \times 0 + (-0.5)$$
$$= 0 - 0.5$$
$$= -0.5$$
$$\phi(P_1) = 0$$

$$P_2 = x_1 w_2 + x_2 w_4 + b_2$$
$$= 0 \times 1 + 0 \times 1 + (-1.5)$$
$$= -1.5$$
$$\phi(P_2) = 0$$

$$\hat{y} = \phi(P_1) \times w_5 + \phi(P_2)\, w_6 + b_3$$

$$= 0 \times 1 + 0 \times -1 + (-0.5)$$

$$= \underline{-0.5}$$

$$\underline{\phi(\hat{y})} = \underline{0}$$

<u>2nd Case</u>

$x_1 = 0$ $\quad$ $x_2 = 1$

$$P_1 = x_1 w_1 + x_2 w_3 + b_1$$

$$= 0 \times 1 + 1 \times 1 + (-0.5)$$

$$= 0.5$$

$$\phi(P_1) = 1$$

$$P_2 = x_1 w_2 + x_2 w_4 + b_2$$

$$= 0 \times 1 + 1 \times 1 - 1.5$$

$$= -0.5$$

$$\phi(P_2) = 0$$

$$\hat{y} = \phi(P_1) \times w_5 + \phi(P_2) \times w_6 + b_3$$

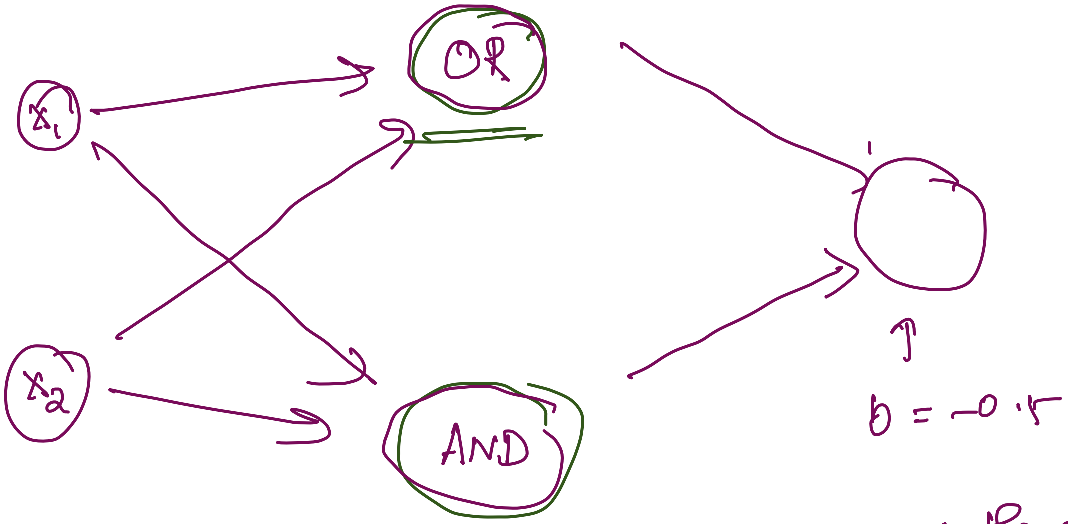
$$= 1 \times 1 + 0 \times (-1) - 0.5$$

$$= 1 - 0.5$$

$$= 0.5 \geq 0$$

$$\phi(\hat{y}) = 1 \checkmark$$

XOR



Is this the only way to solve the XOR ? **NO**

NOR

OR + NOT

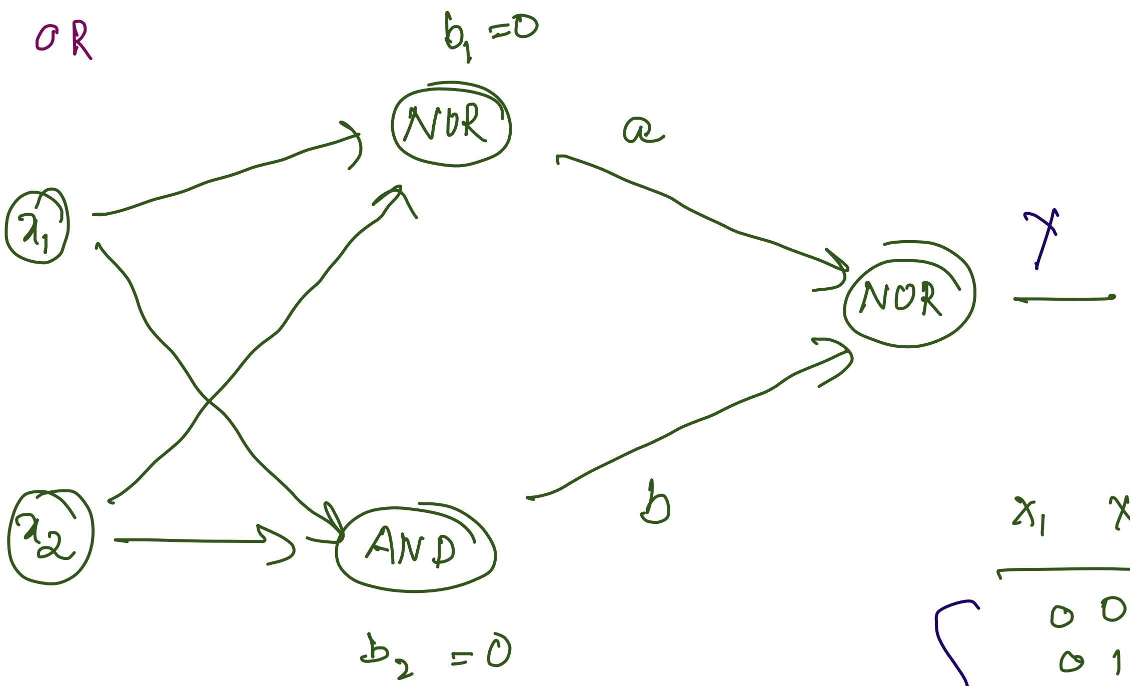
x_1	x_2	y ← Not
0	0	0
0	1	1
1	0	1
1	1	1

NOR ⟶

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

→ 1

OR



XOR

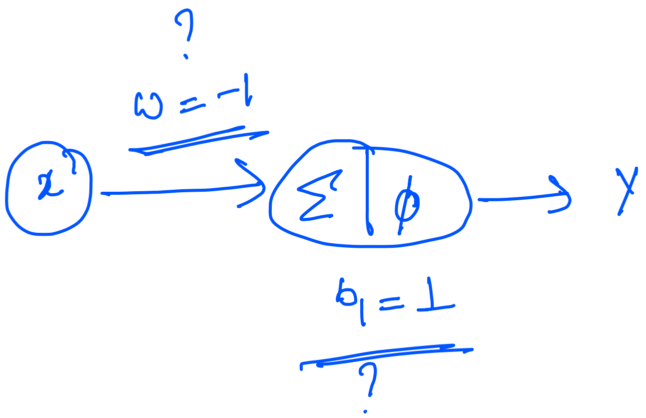
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

✓

x_1	x_2	NOR (a)	AND (b)	NOR (a, b)
0	0	1	0	0
0	1	0	0	1
1	0	0	0	1
1	1	0	1	0

—x—

NOT Gate



x	y
0	1 ✓
1	0 ✓

$$\phi = \begin{cases} 0 & wx + b \le 0 \\ 1 & wx + b > 0 \end{cases}$$

1st Case

$x = 0$

$\Sigma = xw + b$

$= 0 \times (-1) + 1$

$= 1$

$\phi(\Sigma) = 1$

2nd Case

$x = 1$

$\Sigma = 1 \times (-1) + 1$

$= -1 + 1$

$= 0 \le 0$

$\phi(\Sigma) = 0$ ✓

$w = 1 \qquad b = -0.5$

1st case $\qquad x = 0$

$$\Sigma = xw + b$$
$$= 0 \times (1) - 0.5$$
$$= -0.5$$
$$\phi(\Sigma) = 0 \quad \times$$

2nd case $\qquad x = 1$

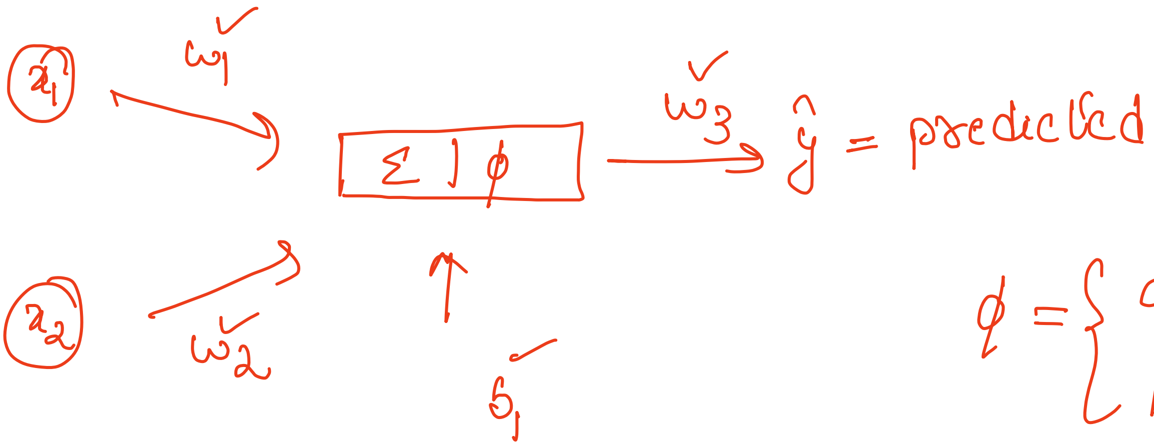
$$\Sigma = xw + b$$
$$= 1 \times 1 - 0.5$$
$$= 1 - 0.5$$
$$= 0.5 \quad \times$$
$$\phi(\Sigma) = 1$$

Perceptron Learning

Learning → weights & bias

↳ Algorithm can change itself

Hyperparameter ✗



$$\phi = \begin{cases} 0 & w.x+b<0 \\ 1 & w.x+b>0 \end{cases}$$

↳ Fixed

Learning

$$\boxed{\begin{aligned} w_i &= w_i + \Delta w \\ \Delta w_i &= \eta(y-\hat{y})\,x_i \end{aligned}}$$

hyper-parameter

η = learning rate

w_i = weights

Δw = amount of change required

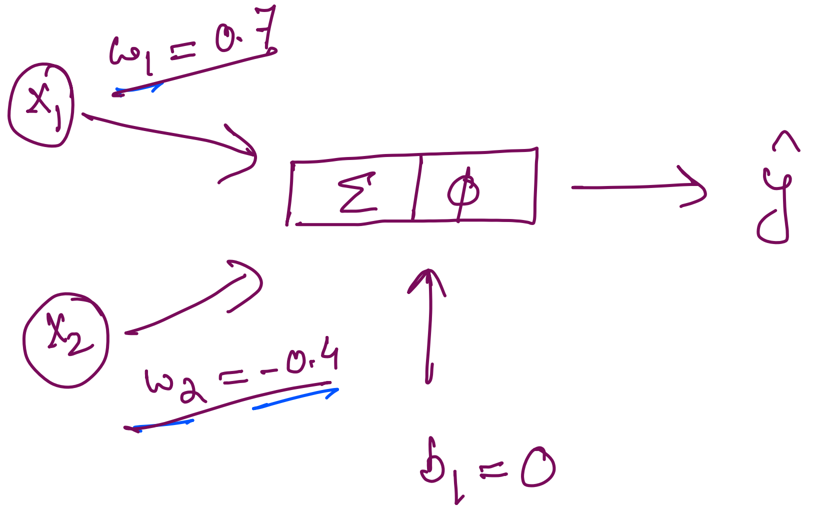
y = actual class

$\hat{y}$ = predicted class

x_i = input sample

Example

OR Gate



i/p

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$w_1 = 0.7 \quad w_2 = -0.4$

$\eta = 0.6$

$$\phi = \begin{cases} 0 & w \cdot x + b \le 0 \\ 1 & \text{else} \end{cases}$$

1st Case

$x_1 = 0 \qquad x_2 = 0$

$$\Sigma = x_1 w_1 + x_2 w_2 + b_1$$
$$= 0 \times 0.7 + 0 \times (-0.4) + 0$$
$$= 0$$
$$\phi(\Sigma) = 0$$

2nd Case

$x_1 = 0 \qquad x_2 = 1$

$$\Sigma = x_1 w_1 + x_2 w_2 + b_1$$
$$= 0 \times 0.7 + 1 \times (-0.4) + 0$$
$$= -0.4$$
$$\phi(\Sigma) = 0$$

$$w_i = w_i + \Delta w$$
$$w_1 = w_1 + \Delta w_1 \quad \text{—(1)}$$
$$w_2 = w_2 + \Delta w_2 \quad \text{—(2)}$$

(1)

$$\Delta w_1 = \eta (y - \hat{y}) x_1$$
$$= 0.6 (1 - 0)\, 0$$
$$= 0$$

$w_1 = 0.7$
$w_2 = -0.4$ | Initially

(1) $\Rightarrow$ $w_1 = w_1 + \underbrace{\Delta w_1}_{\to 0}$

$w_1 = w_1$

(2) $\Rightarrow$ $\Delta w_2 = \eta (y - \hat{y}) x_2$

$= 0.6 (1 - 0) \, 1$

$= 0.6$

New weight

$$\boxed{\begin{aligned} w_1 &= 0.7 \\ w_2 &= 0.2 \end{aligned}}$$

$w_2 = w_2 + \Delta w_2$

$= -0.4 + 0.6$

$= 0.2$

1st Case

$x_1 = 0 \quad x_2 = 0$

$\Sigma = 0$

$\phi(\Sigma) = 0$

2nd Case

$x_1 = 0 \quad x_2 = 1$

$\Sigma = x_1 w_1 + x_2 w_2 + b$

$= 0 + 1 \times (0.2) + 0$

$= 0.2 \geq 0$

$\phi(\Sigma) = 1$

3^{rd} case

$x_1 = 1 \quad x_2 = 0$

$$\Sigma = x_1 w_1 + x_2 w_2 + b_1$$
$$= 1 \times 0.7 + 0 + 0$$
$$= 0.7$$

$\phi(\Sigma) = 1$ ✓

$\hat{y} = y$

$$\Delta w_i = \eta \underbrace{(y - \hat{y})}_{0} x_i$$

4^{th}

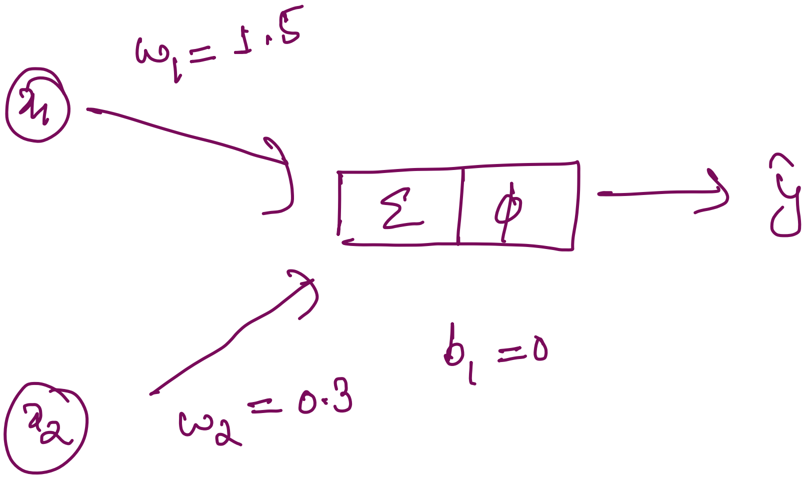
$x_1 = 1 \quad x_2 = 1$

$$\Sigma = x_1 w_1 + x_2 w_2 + b_1$$
$$= 0.7 + 0.2 + 0$$
$$= 0.9 \geq 0$$

$\phi(\Sigma) = 1$

$\hat{y} = y$

AND



x_1	x_2	y
0	0	0 ✓
0	1	0 ✓
1	0	0 ✓
1	1	1 ✓

$w_1 = 1.5 \quad w_2 = 0.3$

$\eta = 0.7$

$$\phi = \begin{cases} 0 & w.x + b < 1 \\ 1 & w.x + b \geq 1 \end{cases}$$

1st Case

$x_1 = 0 \qquad x_2 = 0$

$\Sigma = x_1 w_1 + x_2 w_2 + b$

$= 0 < 1$

$\phi(\Sigma) = 0$ ✓

2nd case

$x_1 = 0 \qquad x_1 = 1$

$\Sigma = x_1 w_1 + x_2 w_2 + b$

$= 0 + 1 \times 0.3 + 0$

$= 0.3 < 1$

$\phi(\Sigma) = 0$ ✓

Apply weight update

$w_1 = w_1 + \Delta w_1$

$\Delta w_1 = \eta (y - \hat{y}) x_1$

$= \eta (0 - 0) x_1$

$= 0$

$w_1 = w_1$

$w_2 = w_2 + \Delta w_2$

$\Delta w_2 = \eta (0 - 0) x_2$

$= 0$

$w_2 = w_2$

3rd case

$x_1 = 1 \qquad x_2 = 0$

$\Sigma = x_1 w_1 + x_2 w_2 + b$

$= 1 \times 1.5 + 0 + 0$

$= 1.5 \geq 1$

$\phi(\Sigma) = 1 \quad \hat{y}$

$w_1 = 1.5$
$w_2 = 0.3$ } Initial

$$\boxed{\begin{aligned} w_i &= w_i + \Delta w_i \\ \Delta w_i &= \eta (y - \hat{y}) x_i \end{aligned}}$$

$\eta = 0.7$

Apply weight update

$$\boxed{\begin{aligned} w_1 &= 0.8 \\ w_2 &= 0.3 \end{aligned}}$$ updated

Repeat 3rd case

$x_1 = 1 \qquad x_2 = 0$

$\Sigma = w_1 x_1 + x_2 w_2 + b$

$= 1 \times 0.8 + 0 + 0$

$= 0.8 < 1$

$\phi(\Sigma) = 0$

check for 1st & 2nd case!

4th case

$x_1 = 1 \qquad x_2 = 1$

$\Sigma = x_1 w_1 + x_2 w_2 + b$

$\phi(\Sigma) = \phi(0.8 + 0.3 + 0)$

$= \phi(1.1) \geq 1$

$= \underline{1}$ ✓

$(y - \hat{y})$ → error

LR is very very small

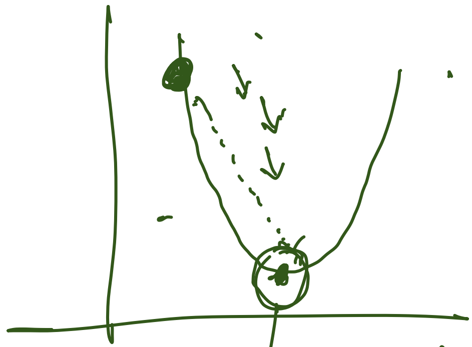
1×10^{-3}

1×10^{-2}

Real life problem

$$\Delta w_1 = \underbrace{\eta}_{\text{Adjust LR}} \underbrace{(y - \hat{y})}_{\text{Error}} x_1$$

global optimum.



Local minimum

SGD

Convex optimization