

Perception

Single Layer

Multi Layer

AND Gate
OR Gate

XOR Gate

Multi Layer *more neurons ↑*

Single Layer

AND

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

$1 \rightarrow 1$

OR

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$↑$

XOR

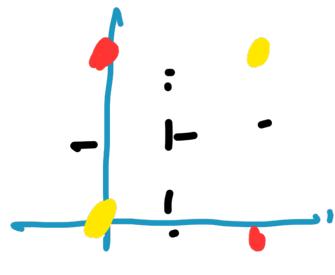
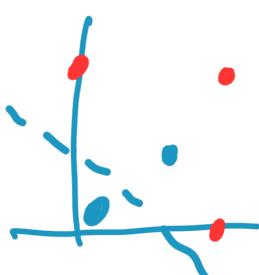
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$x_1 \bar{x}_2 + \bar{x}_1 x_2 \uparrow$

Decision Surface

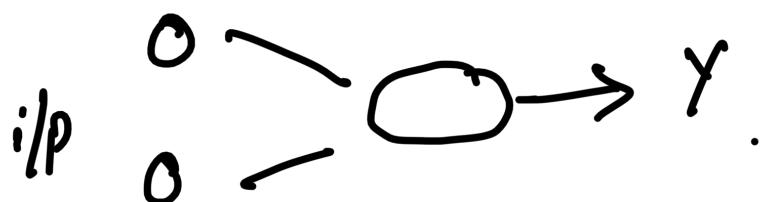
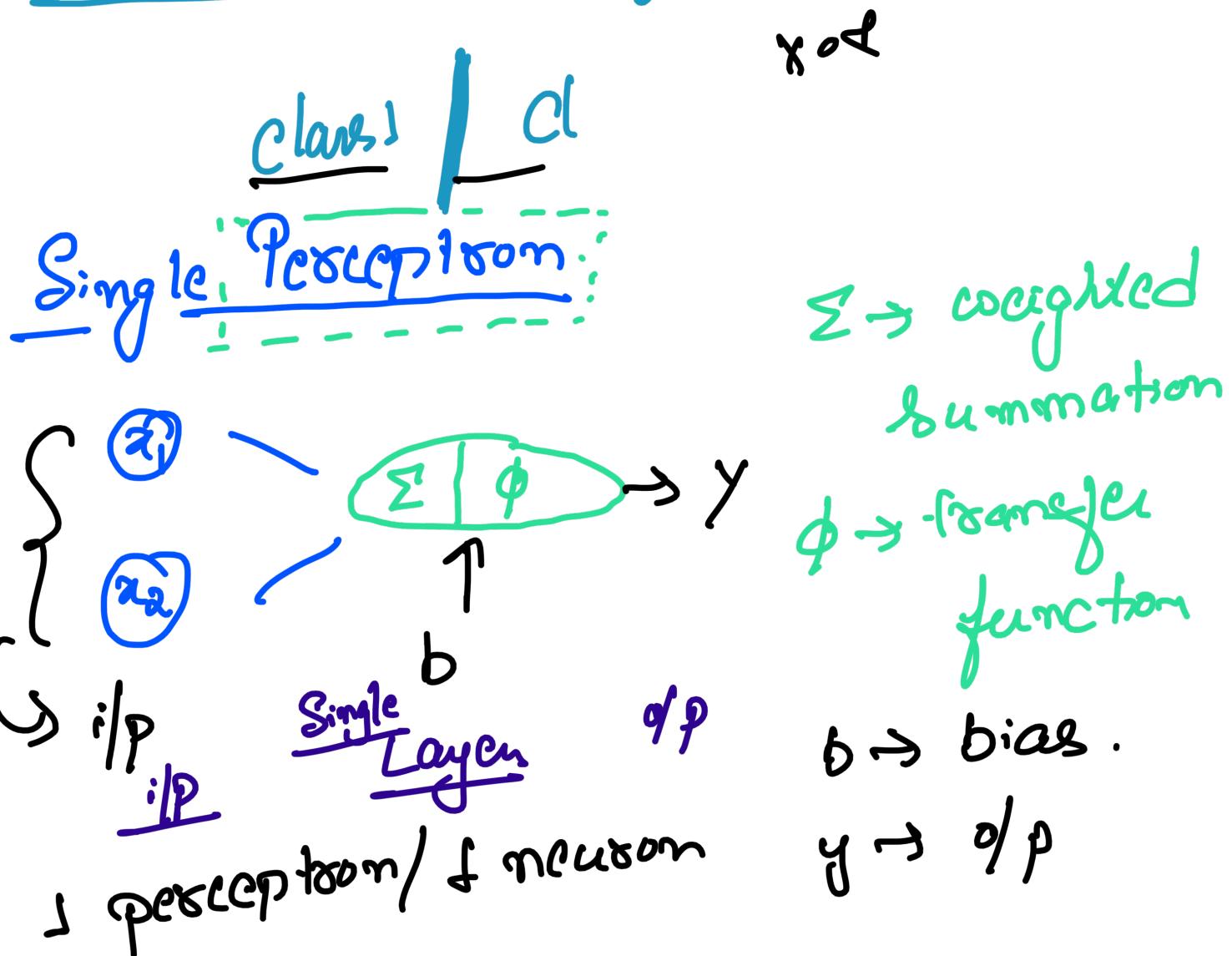


Linearly Separable

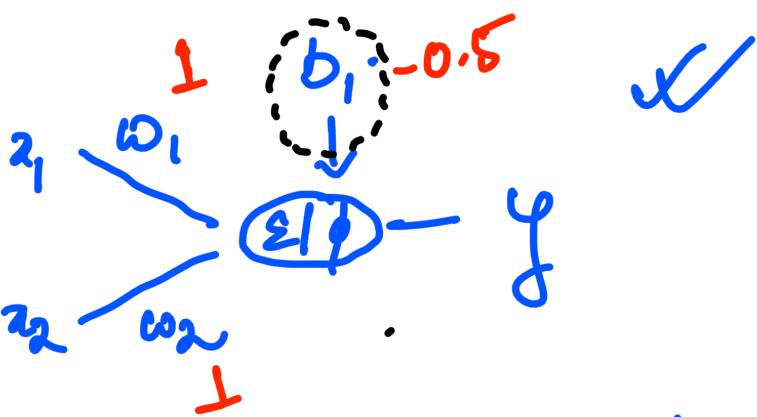


Can we have
a 'linear'
decision
surface?

Linear Decision Surface



Perception OR gate



x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\sum = (x_1 w_1 + x_2 w_2 + b) \checkmark$$

$$y = \phi(\sum)$$

$$x_1 = 0 \quad x_2 = 0$$

$$\begin{aligned} & x_1 w_1 + x_2 w_2 + b \\ & 0 \times 1 + 0 \times 1 + -0.5 \\ & = -0.5 \end{aligned}$$

$$y = \phi(-0.5)$$

$$= 0 \quad \checkmark$$

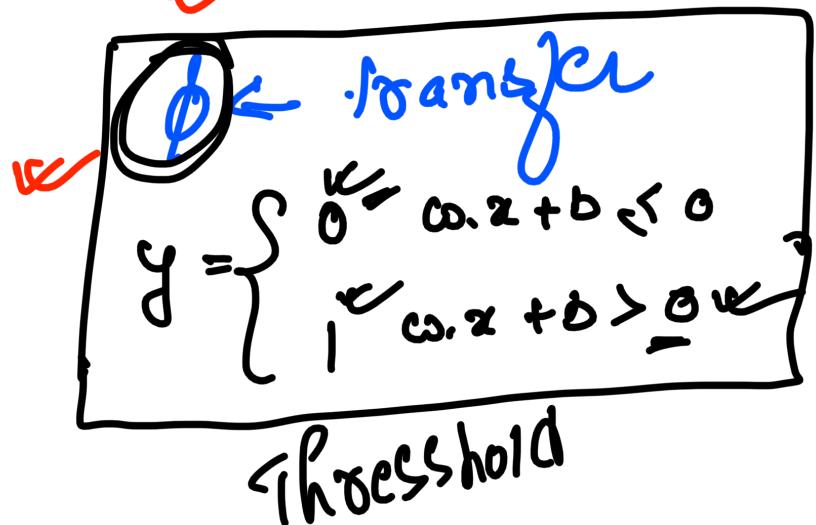
$$x_1 = 0 \quad x_2 = 1$$

$$\begin{aligned} & x_1 w_1 + x_2 w_2 + b \\ & 0 \times 1 + 1 \times 1 + -0.5 \\ & = 0.5 \end{aligned}$$

Supervised (y)

4 samples
2 attributes/
dimension

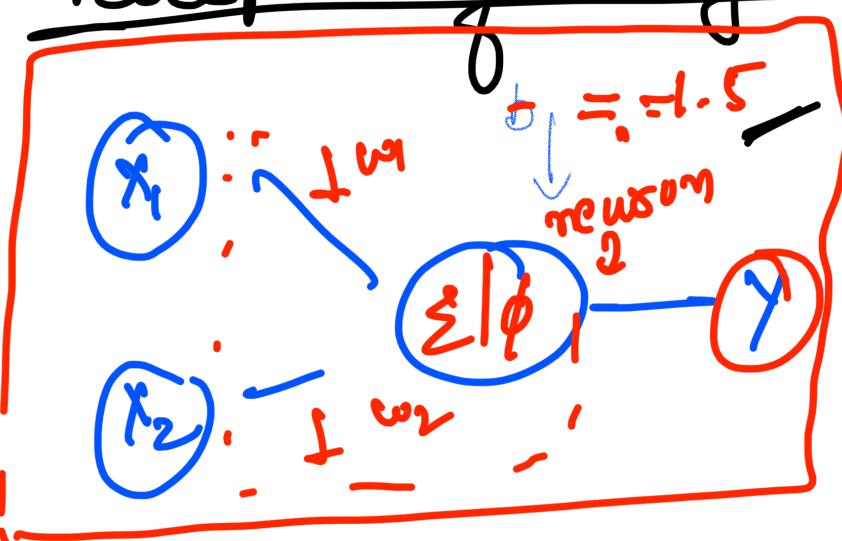
$$\left\{ \begin{array}{l} w_1 = 1 \quad b_1 = -0.5 \\ w_2 = 1 \end{array} \right.$$



$$y = \phi(0.5)$$

$$= 1$$

Perception of AND gate



$$x_1 = 0 \quad x_2 = 0$$

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

$$\begin{aligned} w_1 &= 1 & b_1 &= -1.5 \\ w_2 &= 1 \end{aligned}$$

$$\begin{aligned} x_1 w_1 + x_2 w_2 + b_1 \\ 0 \times 1 + 0 \times 1 + -1.5 \\ = -1.5 \end{aligned}$$

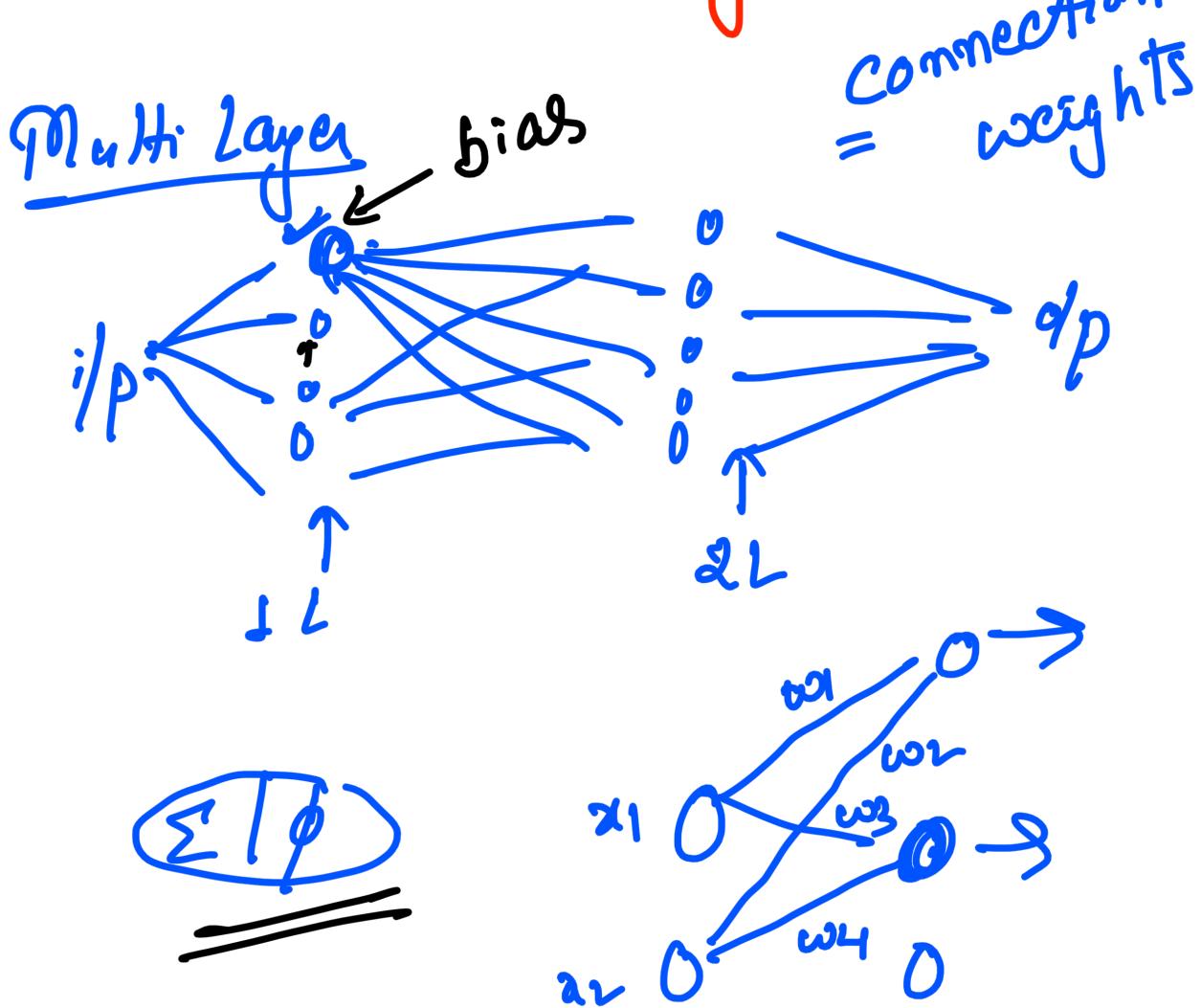
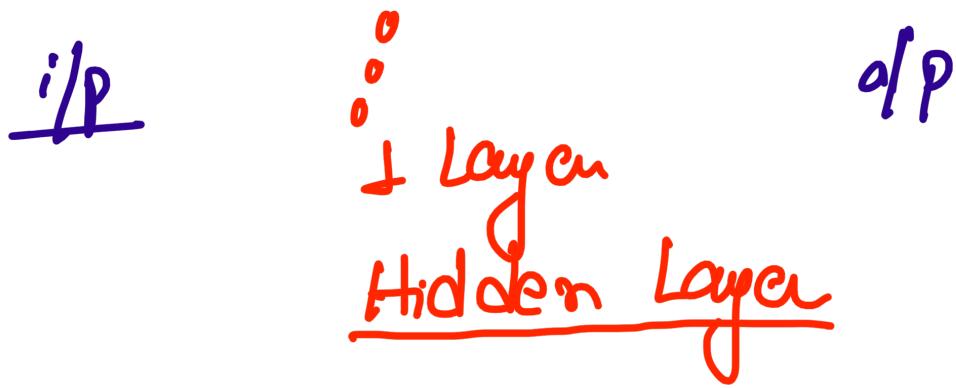
$$\begin{aligned} y &= \phi(-1.5) \\ &= 0 \end{aligned}$$

$$x_1 = 1 \quad x_2 = 1$$

$$\begin{aligned} x_1 w_1 + x_2 w_2 + b_1 \\ = 1 \times 1 + 1 \times 1 + (-1.5) \\ = 0.5 \end{aligned}$$

$$\begin{aligned} y &= \phi(0.5) \\ &= 1 \end{aligned}$$

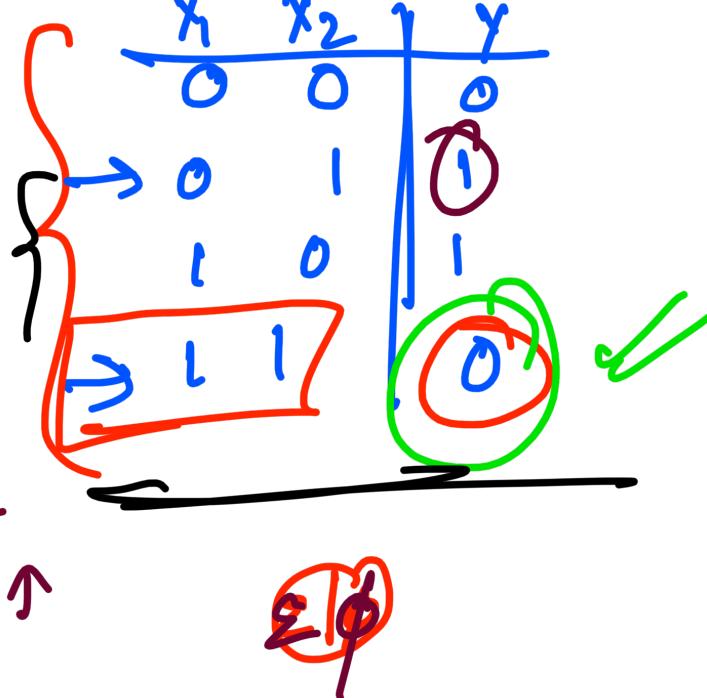
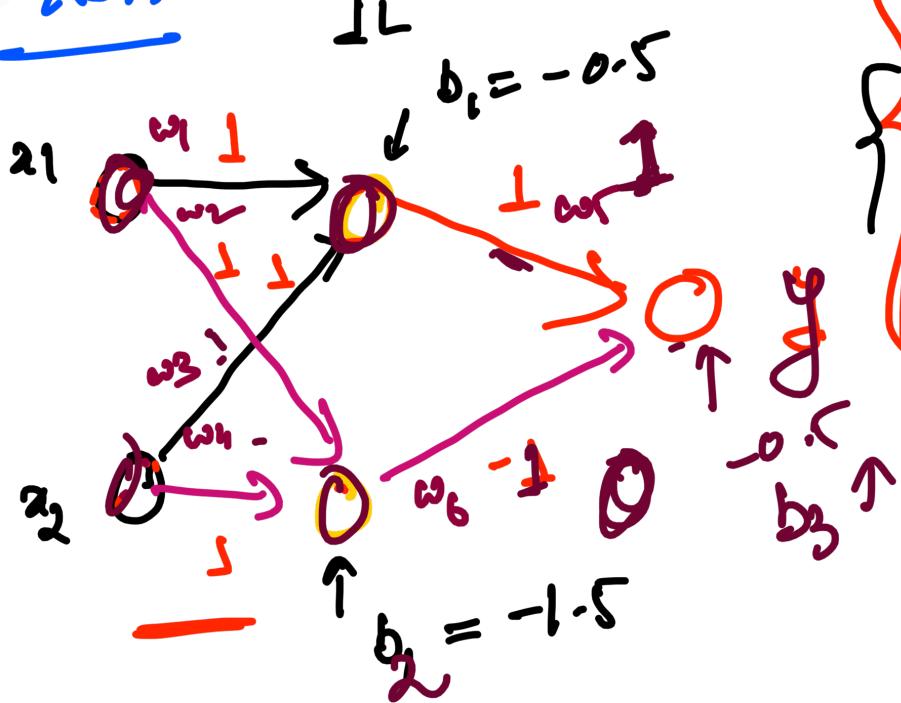
Single Layer



$$\phi \left(x_1 w_1 + x_2 w_2 \right)$$

$$\phi \left(x_1 w_3 + x_2 w_4 \right)$$

XOR



$$y = \begin{cases} 0 & \omega \cdot x + b \leq 0 \\ 1 & \omega \cdot x + b > 0 \end{cases}$$

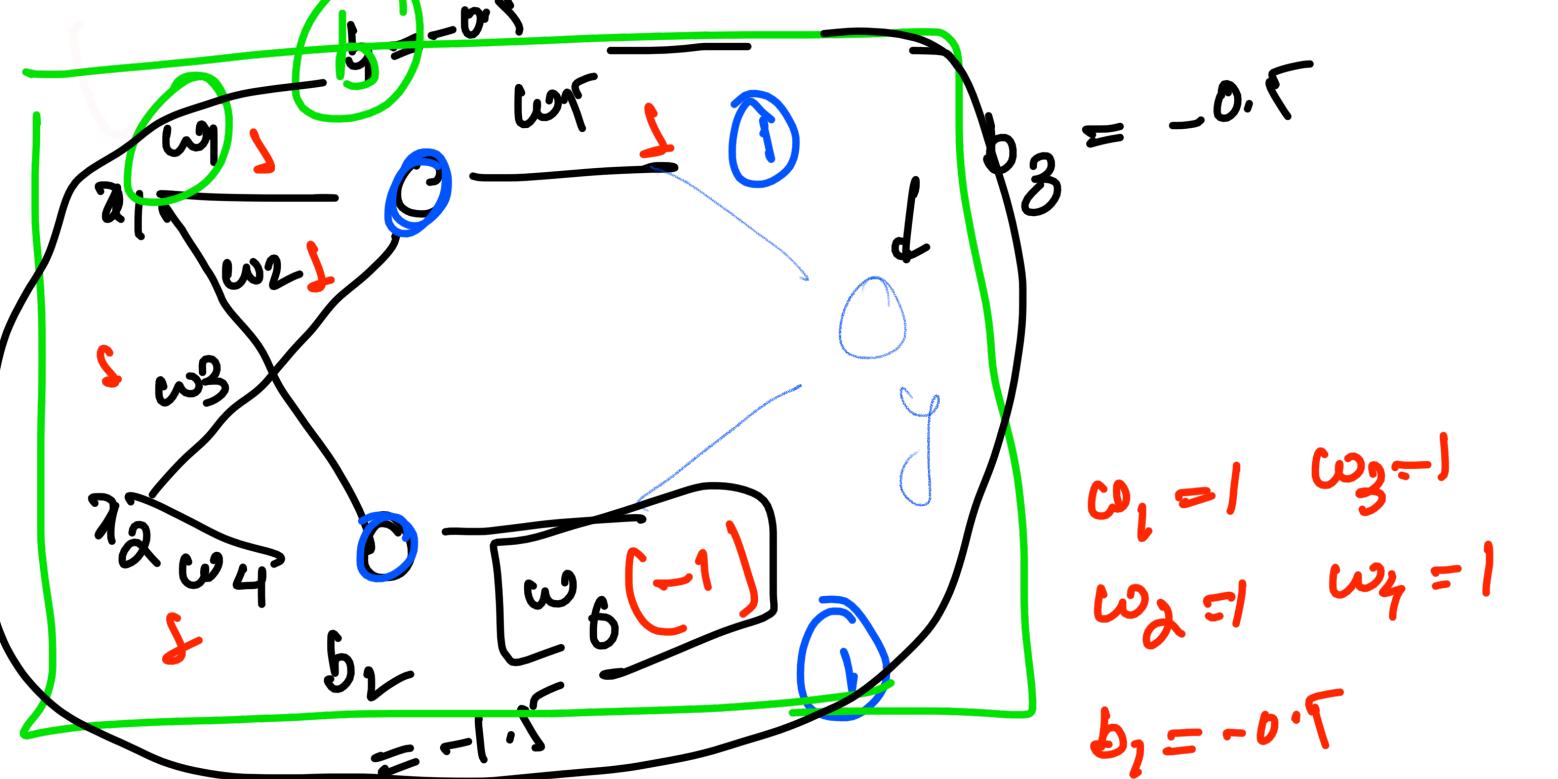
$$x_1 = 0 \quad x_2 = 1$$

$$\begin{aligned} & \phi(w_1x_1 + w_2x_2 + b_1) \\ &= \phi(0 \cdot 0 + 1 \cdot 1 - 0.5) \\ &= \phi(0.5) \stackrel{=} 1 \end{aligned}$$

$$\begin{aligned} & x_1 = 0 \quad x_2 = 1 \\ & \phi(w_3x_1 + w_4x_2 + b_2) \\ &= \phi(0 \cdot 0 + 1 \cdot 1 - 1.5) \\ &= \phi(-0.5) \\ & \stackrel{=} 0 \end{aligned}$$

$$\begin{aligned} & \phi(w_1x_1 + w_2x_2 + b_1) \\ &= 1 \cdot 0 + 0 \cdot 1 + (-0.5) \\ &= -0.5 \end{aligned}$$

$$\phi(-0.5) \stackrel{=} 1$$



$$z_1 = 1 \quad z_2 = 1$$

$$\begin{aligned} w_1 &= 1 & w_3 &= 1 \\ w_2 &= 1 & w_4 &= 1 \end{aligned}$$

$$b_1 = -1.5$$

$$b_2 = -1.5$$

$$w_5 = 1$$

$$b_3 = 0.5$$

$$w_6 = -1$$

$$(z_1 w_1 + z_2 w_2 + b_1) \phi$$

$$1((1 \times 1 + 1 \times 1) + -1.5) \phi (z_3 w_3 + z_2 w_4 + b_2)$$

$$= \phi(-1.5)$$

$$\begin{aligned} &= \phi(1 \times 1 + 1 \times 1 + -1.5) \\ &= \phi(0.5) \end{aligned}$$

$$= 1$$

✓

$$\begin{aligned} &\phi(1 \times w_5 + 1 \times w_6 + b_3) \\ &= \phi(1 \times 1 + 1 \times (-1) + (-1.5)) \\ &= \end{aligned}$$

$$\begin{aligned} &= \phi(-0.1) \\ &= 0 \end{aligned}$$

weights = learnable
parameters

random numbers

