Homework 1 – due 11:59 pm Jan 28 (submit on icollege)

1 (Question 1.2 in the textbook) Consider a signal-plus-noise model of the general form $x_t = s_t + w_t$, where w_t is Gaussian white noise with $\sigma_w^2 = 1$. Simulate and plot $n = 200$ observations from each of the following two models:

a. $x_t = s_t + w_t$, for $t = 1, \ldots, 200$, where

$$s_t = \begin{cases} 0 & t = 1, \ldots, 100 \\ 10 \exp -\frac{(t-100)}{20} cos(2\pi t/4) & t = 101, \ldots, 200. \end{cases}$$

- Hint: (You can use the following R code)

1. s = c(rep(0,100), 10*exp(-(1:100)/20)*cos(2*pi*1:100/4));
2. x = ts(s + rnorm(200, 0, 1));
3. plot(x)

b. $x_t = s_t + w_t$, for $t = 1, \ldots, 200$, where

$$s_t = \begin{cases} 0 & t = 1, \ldots, 100 \\ 10 \exp -\frac{(t-100)}{200} cos(2\pi t/4) & t = 101, \ldots, 200. \end{cases}$$

c. Compare the general appearance of the series (a) and (b) with the earth-quake series and the explosion series shown in Figure 1.7 (see below).

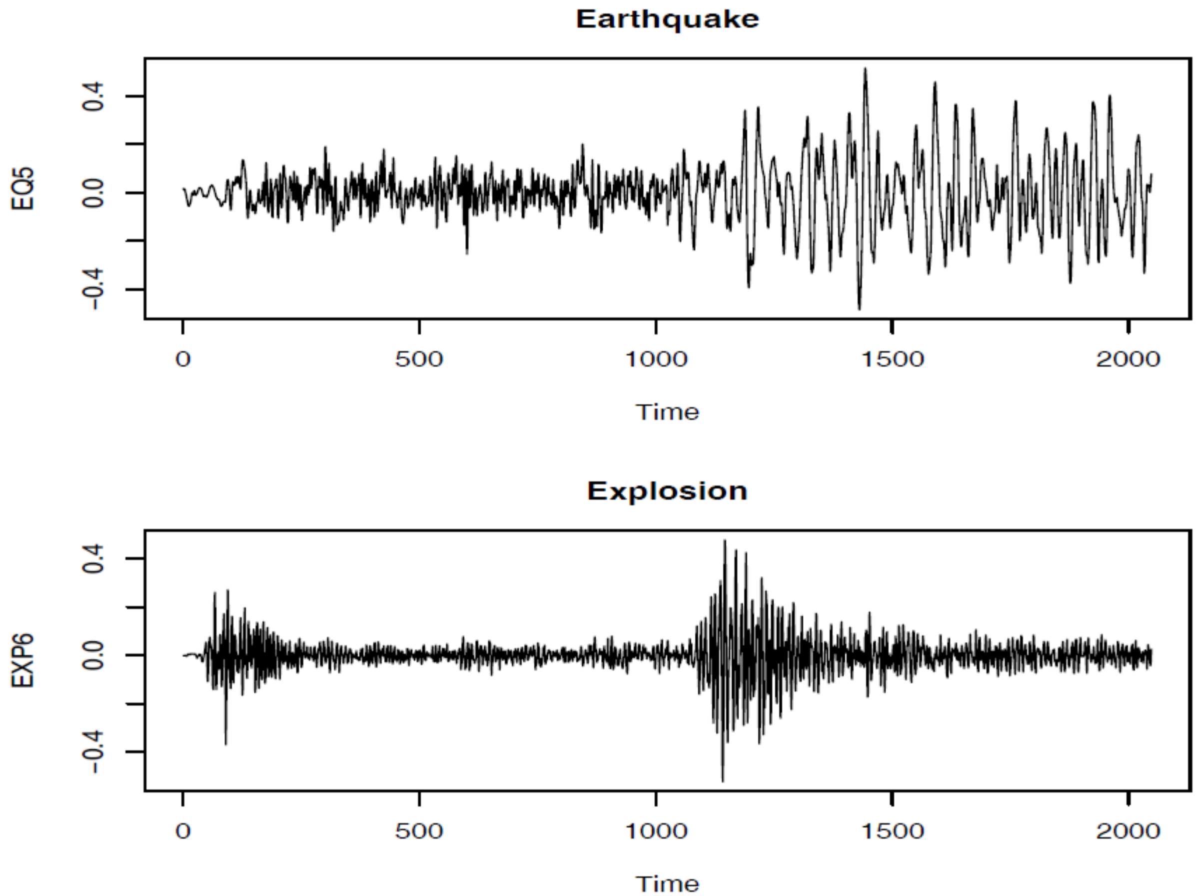

Fig. 1.7. Arrival phases from an earthquake (top) and explosion (bottom) at 40 points per second.

2. Consider a process consisting of a linear trend with an additive noise term consisting of independent random variables wt with zero means and variances σ_w^2 , that is, $x_t = \beta_0 + \beta_1 t + w_t$; where β_0 and β_1 are fixed constants.

   a. Show x_t is nonstationary.

   b. Show that the first difference series $\nabla x_t = x_t - x_{t-1}$ is stationary by finding its mean and autocovariance function.

   c. Repeat part (b) if w_t is replaced by a general stationary process, say y_t, with mean function y and autocovariance function $y(h)$.

3. Consider the two weekly time series oil and gas (included in the astsa package.). The oil series is in dollars per barrel, while the gas series is in cents per gallon. Use R codes "?oil" and "?gas" to get the help file on the two data sets.

a. Plot the data on the same graph. Do you believe the series are stationary (explain your answer)?

b. In economics, we often use the percentage change in price (termed growth rate or return), rather than the absolute price change. Use a transformation of the form $y_t = \nabla \log x_t = \log x_t - \log x_{t-1}$ to the data, where x_t is the oil or gas price series. Plot the data on the same graph, look at the sample ACFs of the transformed data, and comment.

4. (Question 1.7 in the textbook) For a moving average process of the form $x_t = w_{t-1} + 2w_t + w_{t+1}$, where w_t are independent with zero means and variance σ_w^2, determine the autocovariance and autocorrelation functions as a function of lag $h = s - t$ and draw the ACF as a function of h.