Homework 2 – due at 11:59 Pm Feb 11th

1 (Question 3.2 in the textbook) Let $w_t : t = 0, 1, \ldots$ be a white noise process with variance σ_w^2 and let $|\phi| < 1$ be a constant. Consider the process $x_0 = w_0$ and

$$x_t = \phi x_{t-1} + w_t, t = 1, 2, \ldots$$

We might use this method to simulate an AR(1) process from simulated white noise. (30 pts)

a. Show that $x_t = \sum_{j=0}^{t} \phi^j w_{t-j}$ for any $t = 1, 2, \ldots$

b. Find the $E(x_t)$. (Hint: E(a+b) = E(a)+E(b))

c. Show that for $t = 0, 1, \ldots,$ (Hint: var(a+b) = var(a)+var(b) if a and b are independent and mean zero.)

$$var(x_t) = \frac{\sigma_w^2}{1-\phi^2}(1 - \phi^{2(t+1)})$$

d. Show that for $h \geq 0$, (Hint: cov(a+b,c+d) = cov(a,c)+cov(a,d)+ cov(b,c)+cov(b,d).)

$$cov(x_t, x_{t+h}) = \phi^h var(x_t)$$

e. Is x_t stationary?

f. Argue that, as $t \to \infty$, the process becomes stationary, so in a sense, x_t is "asymptotically stationary".

2. Let x_t represent the cardiovascular mortality series (cmort in astsa package). (20 pts)

a Plot the data and ACF, PACF. Comment on the stationarity and discuss which model you would use.

c Fit an AR(2) to x_t using the method of moment method (Yule-Walker Equation). You can use R as an calculator to calculate $\hat{\gamma}$ and matrix inverse.

C Provide the asymptotic covariance matrix for your estimator.

3. (Question 3.4 in the textbook) Identify the following models as ARMA(p,q) models (watch out for parameter redundancy), and determine whether they are causal and/or invertible: (10 pts)

a $x_t = 0.8x_{t-1} - 0.15x_{t-2} + w_t - 0.3w_{t-1}$.

b $x_t = x_{t-1} - 0.5x_{t-2} + w_t - w_{t-1}$

4. Use the attached data (Sales.xls) and perform time series analysis on MONTHLY sales. Make sure your analysis cover the following steps: 1) Data exploration to check whether the data is stationary and perform any transformation needed. 2) Model fitting 3) Model diagnosis. 4) Prediction. (40 pts)