What are we weighting for?

A mechanistic model for probability weighting

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Full paper at https://bit.ly/lml-pw-r1

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Evolution of Decision Criteria : Ensemble Average Maintenance

Work in progress

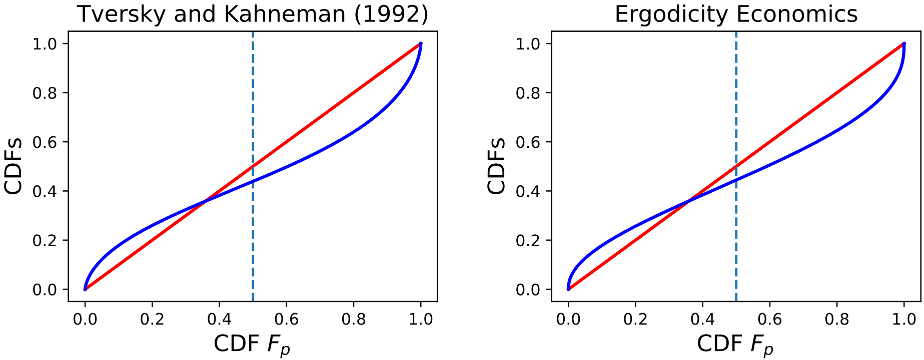
Expectation Value Theory: $$\mathrm{E}[\Delta X] = \sum_{n=1}^{\infty} \underbrace{\Delta x_n}_{\text{absolute payout}} \cdot \underbrace{p_n}_{\text{objective probability}}$$

Expected Utility Theory: $$\mathrm{E}\left[\Delta u\left(X\right)\right] = \sum_{n=1}^{\infty} \underbrace{\Delta u\left(x_n\right)}_{\text{function of Wealth}} \cdot \underbrace{p_n}_{\text{objective probability}}$$

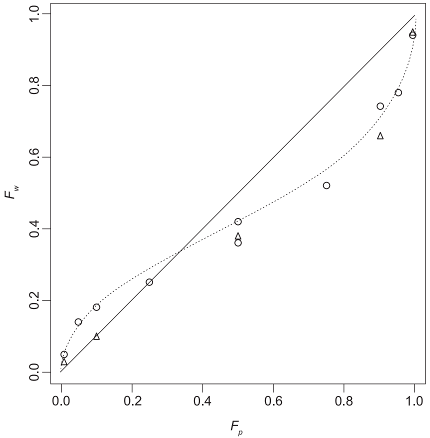
Cumulative Prospect Theory: $$\mathfrak{V}\left(f_X\right) = \sum_{n=1}^{\infty} \underbrace{\mathfrak{v}\left(\Delta x_n\right)}_{\text{function of the payout}} \cdot \underbrace{\pi\left(p_n\right)}_{\text{function of obj. probability}}$$

x wealth, Δx payout, p probability, u utility function, $\mathfrak{V}$ value/prospect,
$\mathfrak{v}$ value function, π probability weighting, f_X lottery/probability mass function of a random variable

Main results



Definition of Probability Weighting (PW)



(Tversky and Kahneman 1992, p. 310, Fig. 1, relabelled axes)

- empirical pattern: inverse-S shape
- Cumulative Prospect Theory (CPT)

Classical interpretation of PW:

- maladaptive irrational cognitive bias

In search of a mechanism

↪ How does this pattern emerge?

↪ Can we derive a functional form (rather than fit a function)?

Setup

Task: model payout, x, of a gamble as a random variable.

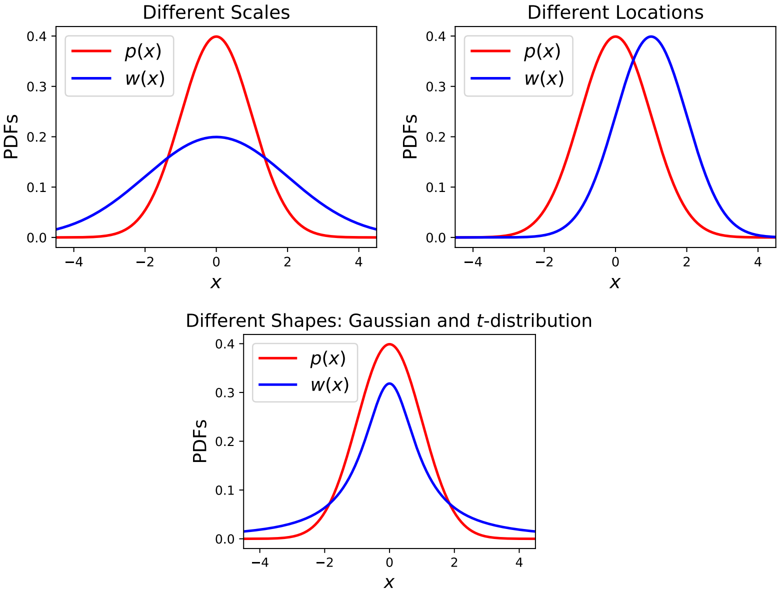
Disinterested Observer (DO)

DO assigns PDF $p(x)$
$\hookrightarrow$ CDF $F_p(x)$

Decision Maker (DM)

DM assigns different PDF $w(x)$
$\hookrightarrow$ CDF $F_w(x)$

Scales, Locations, Shapes



Thought Experiment: DM assumes greater scale

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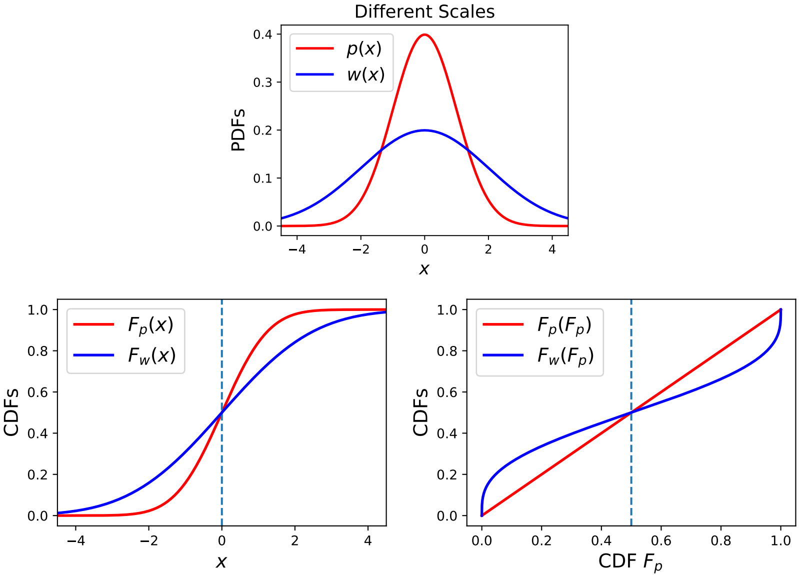


Functional form of the weighting function

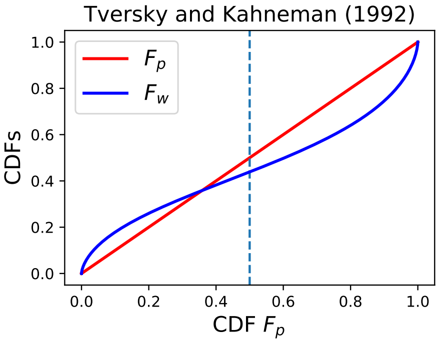
Gaussian case with different scale:

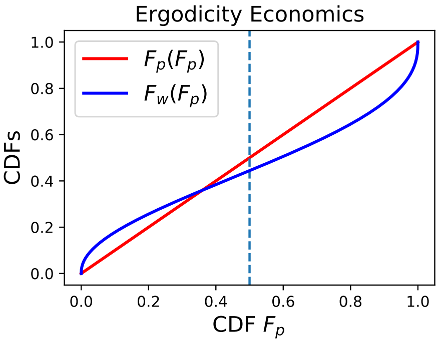
$$w(p) = p^{\frac{1}{\alpha^2}} \underbrace{\frac{\left(2\pi\sigma^2\right)^{\frac{1-\alpha^2}{2\alpha^2}}}{\alpha}}_{\text{normalisation factor}}, \tag{1}$$

where

- DO's scale is σ
- DM's scale is $\alpha\sigma$

Interim conclusion





- DM's greater scale gives inverse-S shape (unimodal distributions)
- difference in locations gives asymmetry
- reproduces observations of probability weighting

Job done. Thank you for your attention ;)

The Ergodicity Question

Typical DO concern

What happens on average to the ensemble of subjects?

$\neq$

Typical DM concern

What happens to me on average over time?

Why DM's greater scale?

- DM has no control over experiment
- experiment may be unclear to DM
- DM may not trust DO
- ...

Experiencing probabilities

- probabilities are not observable
- probabilities encountered as
  - known frequencies in ensemble of experiments (DO)
  - frequencies estimated over time (DM)

$\hookrightarrow$ **estimates have uncertainties – cautious DM accounts for these**

Estimating probabilities

Rare Event

- $p(x) = 0.001$
- 100 observations
- $\sim 99.5\%$ get 0 or 1 events
- $\hat{p}(x) = 0$ or $\hat{p}(x) = 0.01$

$\hookrightarrow$ $\hat{p}(x)$ off by 1000%

Common Event

- $p(x) = 0.5$
- 100 observations
- $\sim 99.5\%$ get between 35 and 65 events,
- $0.35 < \hat{p}(x) < 0.65$

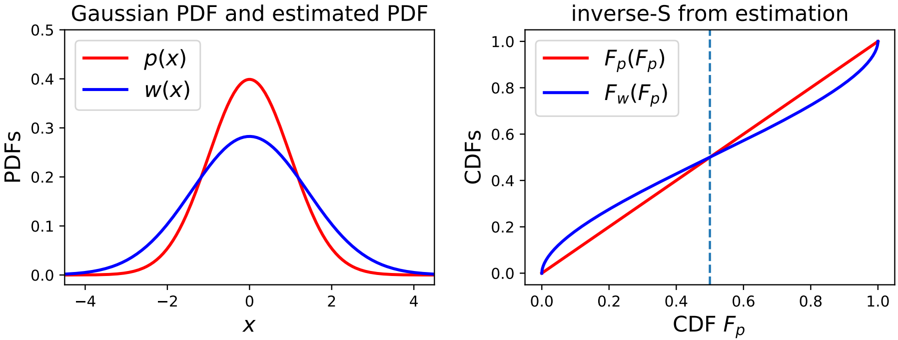
$\hookrightarrow$ $\hat{p}(x)$ off by 30%

$\hookrightarrow$ small $p(x)$, small count $\rightarrow$ big uncertainty

DMs don't like surprises

To avoid surprises, DMs add estimation uncertainty $\varepsilon\left[\hat{p}(x)\right]$ to every estimated probability, then normalize, s.t.

$$w(x) = \frac{\hat{p}(x)+\varepsilon\left[\hat{p}(x)\right]}{\int\left(\hat{p}(s)+\varepsilon\left[\hat{p}(s)\right]\right)ds} \tag{2}$$



Conclusion

Classical interpretation of PW

- overestimation of low probability events
- underestimation of high probability events

$\hookrightarrow$ maladaptive irrational cognitive bias

Ergodicity Economics and PW

- inverse-S shape: neutral indicator of different models of the world
- reported observations consistent with DM's extra uncertainty
- may arise from DM estimating probabilities over time

$\hookrightarrow$ Probability weighting is rational cautious behaviour under uncertainty over time

- testable prediction $\rightarrow$ Let's run an experiment!
- Manuscript at https://bit.ly/lml-pw-r1
- Interactive code at https://bit.ly/lml-pw-count-b

Thank you for your attention!

BACK UP

Probability Weighting as an Estimation Issue

"It is important to distinguish overweighting, which refers to a property of decision weights, from the overestimation that is commonly found in the assessment of the probability of rare events. [...] In many real-life situations, overestimation and overweighting may both operate to increase the impact of rare events." (Kahneman and Tversky 1979, p. 281)

$\hookrightarrow$ distinguish between

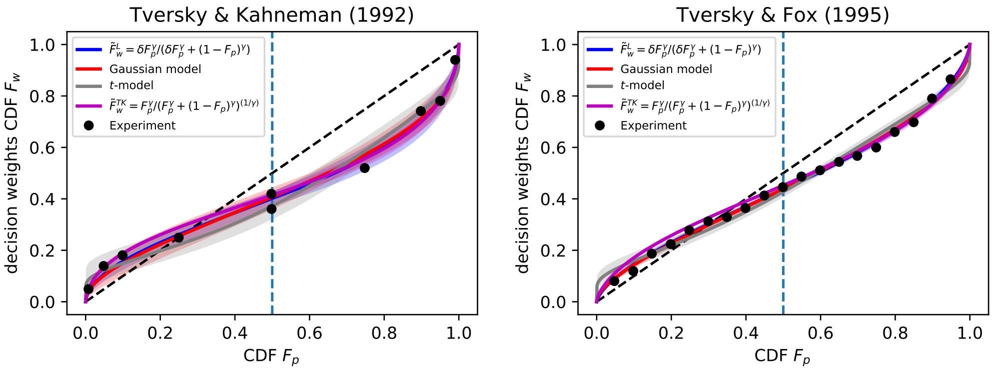
- uncertainty estimation and
- "weighting"

we analyse the former and find very good agreement with the empirical inverse-S pattern

$\hookrightarrow$ How big is the residual "probability weighting" after accounting for uncertainty estimation?

Main Results

Estimation Error Explains 99% of Probability Weighting



- similar fits of Gaussian & t-distributed model

↪ How big is the residual "probability weighting" after accounting for estimation errors?

Main Results

Functional Forms

◂ Gaussian

Tversky and Kahneman (1992, $\gamma = 0.68$)

$$\tilde{F}_w^{TK}\left(F_p;\gamma\right) = \left(F_p\right)^{\gamma} \frac{1}{\left[\left(F_p\right)^{\gamma} + \left(1 - F_p\right)^{\gamma}\right]^{1/\gamma}} \tag{3}$$

Lattimore, Baker, and Witte (1992)

$$\tilde{F}_w^{L}\left(F_p;\delta,\gamma\right) = \frac{\delta F_p^{\gamma}}{\delta F_p^{\gamma} + \left(1 - F_p\right)^{\gamma}} \tag{4}$$

Gaussian case with greater DM scale $\alpha\sigma$

$$w(p) = p^{\frac{1}{\alpha^2}} \frac{\left(2\pi\sigma^2\right)^{\frac{1-\alpha^2}{2\alpha^2}}}{\alpha}\,, \tag{5}$$

which is a power law in p with a pre-factor to ensure normalisation

References I

Kahneman, Daniel and Amos Tversky (1979). "Prospect Theory: An Analysis of Decision under Risk". *Econometrica* 47 (2), pp. 263–291. DOI:10.2307/1914185 (cit. on p. 17).

Lattimore, Pamela K., Joanna R. Baker, and A. Dryden Witte (1992). "Influence of Probability on Risky Choice: A Parametric Examination". *Journal of Economic Behavior and Organization* 17 (3), pp. 377–400. DOI:10.1016/S0167-2681(95)90015-2 (cit. on p. 19).

Tversky, Amos and Daniel Kahneman (1992). "Advances in Prospect Theory: Cumulative Representation of Uncertainty". *Journal of Risk and Uncertainty* 5 (4), pp. 297–323. DOI:10.1007/BF00122574 (cit. on pp. 4, 19).