What are we weighting for?

A mechanistic model for probability weighting

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Full paper at https://bit.ly/lml-pw-r1

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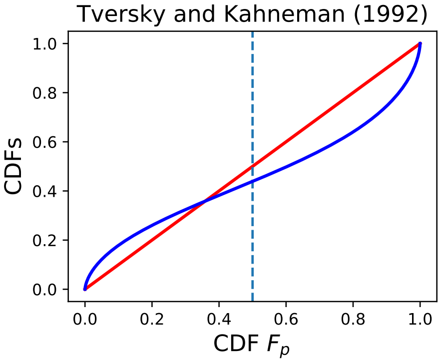
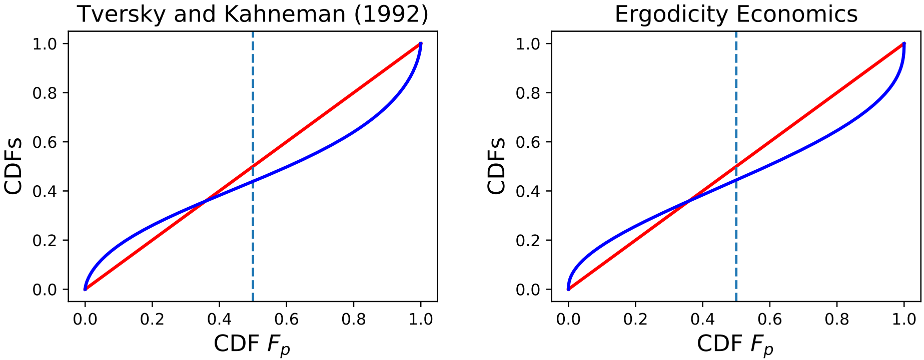
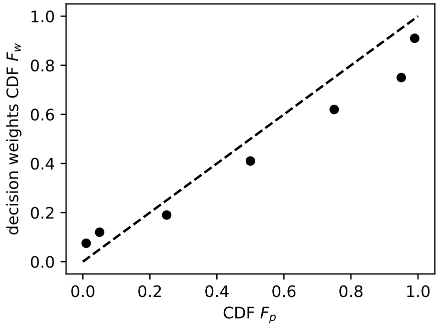
Main results



Main results



Definition of Probability Weighting (PW)



(Preston and Baratta 1948, p. 188, Fig. 1, relabelled axes)

- empirical pattern: inverse-S shape
- Cumulative Prospect Theory (CPT)

Classical interpretation of PW:

- maladaptive irrational cognitive bias

↪ How does this pattern emerge?

↪ Can we derive a functional form (rather than fit a function)?

Related Literature

- TTBOOK no motivation of the functional form, all functions (and parameters therein) used in the literature have no meaning besides producing a fit
- no stable mapping from $p \to w$ (Stewart et al. 2015)
- Ungemach et al. (2009) and Wulff et al. (2018) meta-analysis find all possible weighting curves
- Description-experience gap, experimental design is a key confounder, *i.e.* less overweighting in decisions-from-experience (Hertwig et al. 2004; Hertwig and Erev 2009)

Statistical explanations

-
- PW as a Bayesian heuristic (Martins 2006)

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Setup

Task: model payout, x, of a gamble as a random variable.

Disinterested Observer (DO)

DO assigns PDF $p(x)$
$\hookrightarrow$ CDF $F_p(x)$

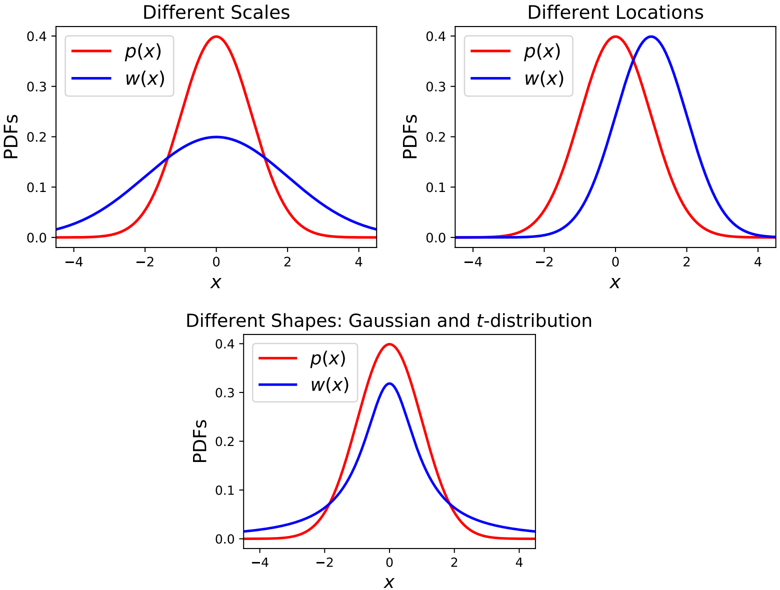
$$F_p(x) = \int_{-\infty}^{x} p(s)\,\mathrm{d}s$$

Decision Maker (DM)

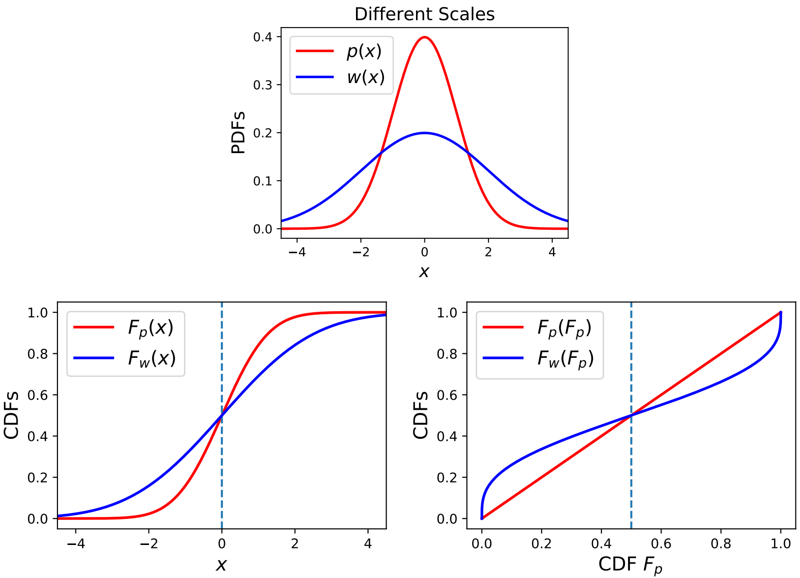
DM assigns different PDF $w(x)$
$\hookrightarrow$ CDF $F_w(x)$

$$F_w(x) = \int_{-\infty}^{x} w(s)\,\mathrm{d}s$$

Scales, Locations, Shapes



Thought Experiment: DM assumes greater scale



Functional form of the weighting function

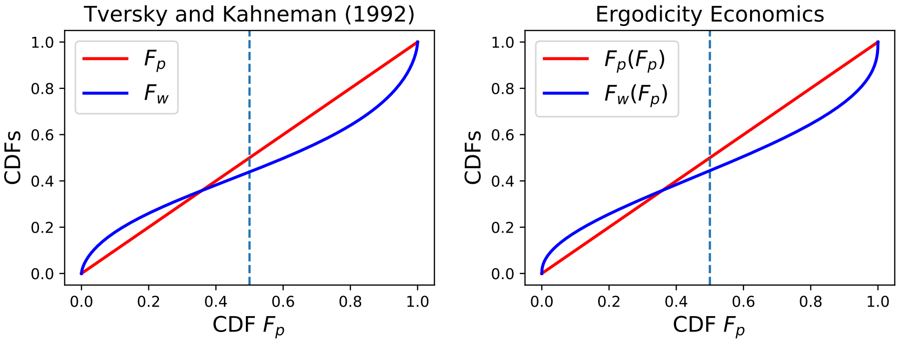
Gaussian case with different scale:

$$w(p) = p^{\frac{1}{\alpha^2}} \underbrace{\frac{\left(2\pi\sigma^2\right)^{\frac{1-\alpha^2}{2\alpha^2}}}{\alpha}}_{\text{normalisation factor}}, \tag{1}$$

where

- DO's scale is σ
- DM's scale is $\alpha\sigma$

Interim conclusion



- DM's greater scale gives inverse-S shape (unimodal distributions)
- difference in locations gives asymmetry
- reproduces observations of probability weighting

Job done. Thank you for your attention ;)

The Ergodicity Question

Typical DO concern

What happens on average to the ensemble of subjects?

$\neq$

Typical DM concern

What happens to me on average over time?

Why DM's greater scale?

- DM has no control over experiment
- experiment may be unclear to DM
- DM may not trust DO
- ...

Experiencing probabilities

- probabilities are not observable
- probabilities encountered as
  - known frequencies in ensemble of experiments (DO)
  - frequencies estimated over time (DM)

$\hookrightarrow$ **estimates have uncertainties – cautious DM accounts for these**

Estimating probabilities

Rare Event

- $p(x) = 0.001$
- 100 observations
- $\sim$ 99.5% get 0 or 1 events
- $\hat{p}(x) = 0$ or $\hat{p}(x) = 0.01$

$\hookrightarrow$ $\hat{p}(x)$ off by 1000%

Common Event

- $p(x) = 0.5$
- 100 observations
- $\sim$ 99.5% get between 35 and 65 events,
- $0.35 < \hat{p}(x) < 0.65$

$\hookrightarrow$ $\hat{p}(x)$ off by 30%

$\hookrightarrow$ small $p(x)$, small count $\rightarrow$ big uncertainty

DMs don't like surprises

To avoid surprises, DMs add estimation uncertainty $\varepsilon\left[\hat{p}(x)\right]$ to every estimated probability, then normalize, s.t.

$$w(x) = \frac{\hat{p}(x)+\varepsilon\left[\hat{p}(x)\right]}{\int\left(\hat{p}(s)+\varepsilon\left[\hat{p}(s)\right]\right)ds} \tag{2}$$

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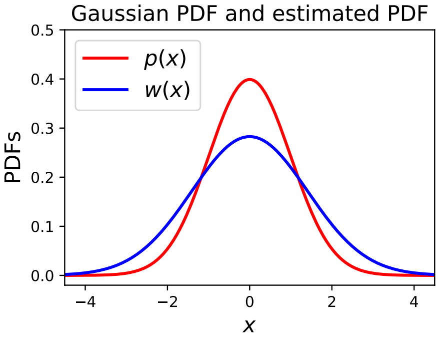

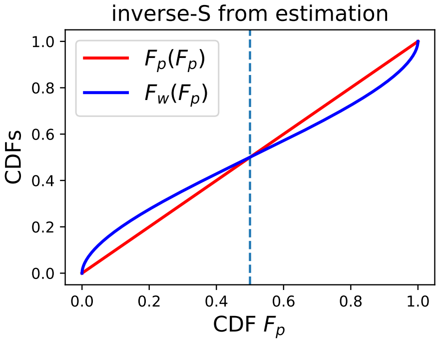

Conclusion

Classical interpretation of PW

- overestimation of low probability events
- underestimation of high probability events

$\hookrightarrow$ maladaptive irrational cognitive bias

Ergodicity Economics and PW

- inverse-S shape: neutral indicator of different models of the world
- reported observations consistent with DM's extra uncertainty
- may arise from DM estimating probabilities over time

$\hookrightarrow$ Probability weighting is rational cautious behaviour under uncertainty over time

- testable prediction $\rightarrow$ Let's run an experiment!
- Manuscript at https://bit.ly/lml-pw-r1
- Interactive code at https://bit.ly/lml-pw-count-b

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BACK UP

Probability Weighting as an Estimation Issue

"It is important to distinguish overweighting, which refers to a property of decision weights, from the overestimation that is commonly found in the assessment of the probability of rare events. [...] In many real-life situations, overestimation and overweighting may both operate to increase the impact of rare events." (Kahneman and Tversky 1979, p. 281)

- ↪ distinguish between
  - uncertainty estimation and
  - "weighting"

  we analyse the former and find very good agreement with the empirical inverse-S pattern
- ↪ How big is the residual "probability weighting" after accounting for uncertainty estimation?

◂ Main Results

Estimation Error Explains 99% of Probability Weighting

- similar fits of Gaussian & t-distributed model

$\hookrightarrow$ How big is the residual “probability weighting” after accounting for estimation errors?

◂ Main Results

Functional Forms

Gaussian

Tversky and Kahneman (1992, $\gamma = 0.68$)

$$\tilde{F}_w^{TK}\left(F_p;\gamma\right) = \left(F_p\right)^{\gamma} \frac{1}{\left[\left(F_p\right)^{\gamma} + \left(1 - F_p\right)^{\gamma}\right]^{1/\gamma}} \tag{3}$$

Lattimore et al. (1992)

$$\tilde{F}_w^{L}\left(F_p;\delta,\gamma\right) = \frac{\delta F_p^{\gamma}}{\delta F_p^{\gamma} + \left(1 - F_p\right)^{\gamma}} \tag{4}$$

Gaussian case with greater DM scale $\alpha\sigma$

$$w(p) = p^{\frac{1}{\alpha^2}} \frac{\left(2\pi\sigma^2\right)^{\frac{1-\alpha^2}{2\alpha^2}}}{\alpha}, \tag{5}$$

which is a power law in p with a pre-factor to ensure normalisation

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Back Up

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