

**NEW**

**Semester - II**

**MATHEMATICS -II**



UNIT

2

Integral Calculus (समाकलन गणित)

# UNIT-II

## Integral Calculus

### UNIT - II: Integral Calculus

(12 periods)

Integration as inverse operation of differentiation. Simple integration by substitution, by parts and by partial fractions (for linear factors only). Introduction to definite integration. Use of formulae  $\int_0^{\frac{\pi}{2}} \sin^n x dx$ ,  $\int_0^{\frac{\pi}{2}} \cos^n x dx$ ,  $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$  for solving problems, where m and n are positive integers. Applications of integration for (i). Simple problems on evaluation of area bounded by a curve and axes. (ii). calculation of volume of a solid formed by revolution of an area about axes. (Simple problems).



### TOPICS

- ✓ 1. समाकलन की परिभाषा (Definition of Integration)
- ✓ 2. समाकलन के प्रकार (Types of Integration)
- ✓ 3. समाकलन से संबंधित सूत्र (Formula related to Integration)
- ✓ 4. प्रतिस्थापन द्वारा समाकलन (Integration by Substitution)
- ✓ 5. खण्डशः समाकलन (Integration by Parts)
- ⑥ 6. आंशिक भिन्नों द्वारा समाकलन (Integration by partial fractions)
- 7. गामा फलन द्वारा समाकलन (Integration Using Gama Function)
- 8. समाकलन के अनुप्रयोग (Applications of Integration)

Q.45:-  $\int \frac{x^2}{(x-1)(3x-1)(3x-2)} dx$  का समाकलन ज्ञात करो। (Find the integral).

आंशिक भिन्न द्वारा (by partial fraction)

$$\frac{x^2}{(x-1)(3x-1)(3x-2)} = \frac{A}{(x-1)} + \frac{B}{(3x-1)} + \frac{C}{(3x-2)}$$

$$\frac{x^2}{(x-1)(3x-1)(3x-2)} = \frac{A(3x-1)(3x-2) + B(x-1)(3x-2) + C(x-1)(3x-1)}{(x-1)(3x-1)(3x-2)}$$

$$x^2 = A(3x-1)(3x-2) + B(x-1)(3x-2) + C(x-1)(3x-1) \text{ ————— (1)}$$



समी ① में  $x = 1$  रखने पर

$$1^2 = A(3 \times 1 - 1)(3 \times 1 - 2) + B(1 - 1)(3 \times 1 - 2) + C(1 - 1)(3 \times 1 - 1)$$

$$1 = A(2)(1) + 0 + 0$$

$$A = \frac{1}{2}$$

समी ① में  $x = \frac{1}{3}$  रखने पर

$$\left(\frac{1}{3}\right)^2 = A\left(3 \times \frac{1}{3} - 1\right)\left(3 \times \frac{1}{3} - 2\right) + B\left(\frac{1}{3} - 1\right)\left(3 \times \frac{1}{3} - 2\right) + C\left(\frac{1}{3} - 1\right)\left(3 \times \frac{1}{3} - 1\right)$$

$$\frac{1}{9} = 0 + B\left(-\frac{2}{3}\right)(-1) + 0$$

$$\frac{1}{9} = \frac{2}{3} B$$

$$\frac{1}{9} \times \frac{3}{2} = B$$

$$B = \frac{1}{6}$$

सभी ① में  $x = \frac{2}{3}$  रखने पर

$$\left(\frac{2}{3}\right)^2 = A\left(3 \times \frac{2}{3} - 1\right)\left(3 \times \frac{2}{3} - 2\right) + B\left(\frac{2}{3} - 1\right)\left(3 \times \frac{2}{3} - 2\right) + C\left(\frac{2}{3} - 1\right)\left(3 \times \frac{2}{3} - 1\right)$$

$$\frac{4}{9} = 0 + 0 + C\left(-\frac{1}{3}\right)(1)$$

$$\frac{4}{9} = -\frac{1}{3} \cdot C$$

$$\frac{4}{9} \times \left(\frac{3}{1}\right) = -C$$

$$C = -\frac{4}{3}$$

$$\text{अतः } \frac{x^2}{(x-1)(3x-1)(3x-2)} = \frac{\frac{1}{2}}{(x-1)} + \frac{\frac{1}{6}}{(3x-1)} + \frac{-\frac{4}{3}}{(3x-2)}$$

Integration (समाकलन)

$$= \frac{1}{2} \int \frac{1}{(x-1)} dx + \frac{1}{6} \int \frac{1}{(3x-1)} dx - \frac{4}{3} \int \frac{1}{(3x-2)} dx$$

$$= \frac{1}{2} \log_e(x-1) + \frac{1}{6} \log_e(3x-1) - \frac{4}{3} \log_e(3x-2)$$

$$+ C$$

$$= \frac{1}{2} \log_e(x-1) + \frac{1}{6} \log_e(3x-1) - \frac{4}{3} \log_e(3x-2) + C$$

Ans

Q.46:-  $\int \frac{1}{(1+x-x^2-x^3)} dx$  का समाकलन ज्ञात करो। (Find the integral).

Factor (गुणनखण्ड) करने पर

$x=1$  रखने

$$= 1 + 1 - (1)^2 - (1)^3$$

$$= 1 + 1 - 1 - 1$$

$$= 0$$

अतः  $(x-1)$  इसका एक गुणनखण्ड (फिक्टर) है।

$$\begin{array}{r}
 x-1 \overline{) -x^3 - x^2 + x + 1} \quad (-x^2 - 2x - 1) \\
 \underline{-x^3 + x^2} \phantom{+ x + 1} \\
 -2x^2 + x + 1 \\
 \underline{-2x^2 + 2x} \phantom{+ 1} \\
 -x + 1 \\
 \underline{-x + 1} \\
 0
 \end{array}$$

दूसरा फिक्टर  $(-x^2 - 2x - 1)$  है।  $-\int \frac{1}{(x-1)(x+1)^2} dx$

$$-(x^2 + 2x + 1) = -(x+1)^2 \quad = \int \frac{1}{(1-x)(1+x)^2} dx$$



अथवा

Q.46:-  $\int \frac{1}{(1+x-x^2-x^3)} dx$  का समाकलन ज्ञात करो। (Find the integral).

$$= \int \frac{1}{1(1+x)-x^2(1+x)} dx$$

$$= \int \frac{1}{(1+x)(1-x^2)} dx$$

$$= \int \frac{1}{(1+x)(1+x)(1-x)} dx$$

$$= \int \frac{1}{(1-x)(1+x)^2} dx$$

By Partial fraction.

$$\frac{1}{(1-x)(1+x)^2} = \frac{A}{(1-x)} + \frac{B}{(1+x)} + \frac{C}{(1+x)^2}$$

Home work!

Q.47:-  $\int \frac{1+4x}{x(x^2-4)} dx$  का समाकलन ज्ञात करो। (Find the integral).

Home work.

$$= \int \frac{1+4x}{x(x^2-2^2)} dx$$

$$= \int \frac{1+4x}{x(x+2)(x-2)} dx$$

By Partial Fraction.

$$\frac{1+4x}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$$



Q.48:-  $\int \frac{1}{(x^3+1)} dx$  का समाकलन ज्ञात करो। (Find the integral).

$$a^3+b^3 = (a+b)(a^2+b^2-ab)$$

Home work.

$$= \int \frac{1}{(x+1)(x^2+x+1)} dx$$

By Partial Fraction

$$\frac{1}{(x+1)(x^2+x+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+x+1)}$$

*out of syllabus*