NEW

Semester - II

MATHEMATICS –II

UNIT

2 Integral Calculus (समाकलन गणित)

TOPICS

1. समाकलन की परिभाषा (Definition of Integration)
2. समाकलन के प्रकार (Types of Integration)
3. समाकलन से संबंधित सूत्र (Formula related to Integration)
4. प्रतिस्थापन द्वारा समाकलन (Integration by Substitution)
5. खण्डश: समाकलन (Integration by Parts)
6. आंशिक भिन्नों द्वारा समाकलन (Integration by partial fractions)
7. गामा फलन द्वारा समाकलन (Integration Using Gama Function)
8. समाकलन के अनुप्रयोग (Applications of Integration)

ILATE

Q.27:- $\int x \cos x \, dx$ का समाकलन ज्ञात करो। (H.W.)

Find the integral.

$$\Rightarrow \int \underset{I}{x} \cdot \underset{II}{\cos x} \cdot dx = x \cdot \int \cos x \cdot dx - \int \left[\frac{dx}{dx} \cdot \int \cos x \cdot dx \right] dx$$

$$= x \cdot \sin x - \int 1 \cdot \sin x \cdot dx$$

$$= x \cdot \sin x + \cos x + C \quad \underline{\text{Ans}}$$

ILATE

$$\int I \cdot II\, dx = I \int II\, dx - \int \left[\frac{dI}{dx} \cdot \int II \cdot dx\right] dx$$

Q.28:- $\int x\, e^{ax}\, dx$ **का समाकलन ज्ञात करो।**

Find the integral.

$$\int x \cdot e^{ax} \cdot dx = x \cdot \int e^{ax} \cdot dx - \int \left[\frac{dx}{dx} \cdot \int e^{ax} \cdot dx\right] dx$$

$$= x \cdot \frac{e^{ax}}{a} - \int 1 \cdot \frac{e^{ax}}{a} \cdot dx$$

$$= \frac{x \cdot e^{ax}}{a} - \frac{1}{a} \int e^{ax} \cdot dx$$

$$= \frac{x \cdot e^{ax}}{a} - \frac{1}{a} \cdot \frac{e^{ax}}{a} + C = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right) + C \quad \text{Ans}$$

ILATE

Q.29:- $\int x^2 e^x \, dx$ का समाकलन ज्ञात करो।

Find the integral.

$$\int \underset{I}{x^2} \cdot \underset{II}{e^x} \, dx = x^2 \int e^x \, dx - \int \left[\frac{dx^2}{dx} \cdot \int e^x \, dx \right] dx$$

$$= x^2 e^x - \int 2x \cdot e^x \, dx$$

$$= x^2 e^x - 2 \int \underset{I}{x} \cdot \underset{II}{e^x} \, dx$$

$$= x^2 e^x - 2 \left[x \int e^x \, dx - \int \left[\frac{dx}{dx} \cdot \int e^x \, dx \right] dx \right]$$

$$= x^2 e^x - 2 \left[x \cdot e^x - \int 1 \cdot e^x \, dx \right]$$

$$= x^2 e^x - 2x \cdot e^x + 2 \cdot e^x + C$$

$$= e^x \left(x^2 - 2x + 2 \right) + C$$ Ans

ILATE

Q.30:- $\int \underset{I}{x^2}\,\underset{II}{\sin x}$ का समाकलन ज्ञात करो। (H.W.)

Find the integral.

$$\int \underset{I}{x^2}\cdot\underset{II}{\sin x}\,dx = x^2\int \sin x - \int\left[\frac{dx^2}{dx}\cdot\int \sin x\,dx\right]dx$$

$$= -x^2\cdot\cos x - \int\left[2x\cdot(-\cos x)\right]dx$$

$$= -x^2\cos x + 2\int \underset{I}{x}\cdot\underset{II}{\cos x}\cdot dx$$

$$= -x^2\cos x + 2\left[x\cdot\int\cos x\,dx - \int\left[\frac{dx}{dx}\cdot\int\cos x\,dx\right]dx\right]$$

$$= -x^2\cos x + 2\left[x\cdot\sin x - \int 1\cdot\sin x\,dx\right]$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

Ans

ILATE

Q.31:- $\int \log_e x \, dx$ का समाकलन ज्ञात करो।

Find the integral.

$$\int \underset{\text{I}}{\log_e x} \cdot \underset{\text{II}}{1} \, dx = \log_e x \cdot \int 1 \cdot dx - \int \left[\frac{d \log_e x}{dx} \cdot \int 1 \, dx \right] dx$$

$$= \log_e x \cdot x - \int \frac{1}{x} \times x \cdot dx$$

$$= x \cdot \log_e x - x + C$$

$$= x(\log_e x - 1) + C \quad \underline{\text{Ans}}$$

ILATE

Q.32:- $\int \tan^{-1}x\,dx$ का समाकलन ज्ञात करो। (H.W.)

Find the integral.

$$\int \underset{I}{\tan^{-1}x} \cdot \underset{II}{1} = \tan^{-1}x \int 1\,dx - \int \left[\frac{d}{dx}\tan^{-1}x \cdot \int 1\,dx\right] dx$$

$$= \tan^{-1}x \cdot x - \int \left[\frac{1}{1+x^2} \cdot x\right] dx$$

$$= x\cdot\tan^{-1}x - \int \frac{x}{1+x^2}\,dx$$

माना $1+x^2 = t$

d.w.r. to x, $2x\cdot dx = dt$

$x\cdot dx = \frac{dt}{2}$

$$x\cdot\tan^{-1}x - \int \frac{1}{t}\,\frac{dt}{2}$$

$$x\tan^{-1}x - \frac{1}{2}\int \frac{1}{t}\cdot dt$$

$$x\cdot\tan^{-1}x - \frac{1}{2}\log_e t + C$$

$$x\tan^{-1}x - \frac{1}{2}\log_e\left(1+x^2\right) + C$$

Ans

ILATE

Q.33:- $\int (\log_e x)^2 \, dx$ **का समाकलन ज्ञात करो।**

Find the integral.

$$\int \underset{\text{I}}{(\log_e x)^2} \cdot \underset{\text{II}}{1} \, dx = (\log_e x)^2 \int 1 \, dx - \int \left[\frac{d(\log_e x)^2}{dx} \cdot \int 1 \, dx \right] dx$$

$$= (\log_e x)^2 \cdot x - \int \left[2 \cdot \log_e x \times \frac{1}{x} \times x \right] dx$$

$$= x \cdot (\log_e x)^2 - 2 \int \log_e x \cdot dx$$

$$= x (\log_e x)^2 - 2 \cdot \int \log_e x \cdot 1 \cdot dx$$

$$= x\cdot(\log_e x)^2 - 2\left[\log_e x\cdot\int 1\,dx - \int\left(\frac{d}{dx}(\log_e x)\cdot\left(\int 1\cdot dx\right)dx\right]\right.$$

$$= x\cdot(\log_e x)^2 - 2\left[\log_e x\cdot x - \int \frac{1}{x}\times x\cdot dx\right]$$

$$= x(\log_e x)^2 - 2x\cdot\log_e x + 2x + C \quad \underline{\text{Ans}}$$

Q.34 :- $\int \frac{\sin(\tan^{-1}x)}{1+x^2}\,dx$

माना $\tan^{-1}x = t$

d.w.r.to x

$\frac{1}{1+x^2}\cdot dx = dt$

$= \int \sin t \cdot dt$

$= -\cos t + C$

$= -\cos(\tan^{-1}x) + C$ Ans

Q.35 :- $\int x \cdot \cos^2 x \cdot dx$ का समाकलन ज्ञात कीजिए ।

$\because \quad \cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

$\frac{\cos 2x + 1}{2} = \cos^2 x$

$$= \int x \cdot \left(\frac{\cos 2x + 1}{2}\right) dx$$

$$= \frac{1}{2}\int (x \cdot \cos 2x + x)\, dx$$

$$= \frac{1}{2}\int \underset{\text{I}}{x} \cdot \underset{\text{II}}{\cos 2x}\, dx + \frac{1}{2}\int x \cdot dx$$

$$= \frac{1}{2}\left[x \cdot \int \cos 2x \cdot dx - \int \left[\frac{dx}{dx} \cdot \int \cos 2x\, dx\right] dx\right] + \frac{1}{2} \times \frac{x^2}{2}$$

$$= \frac{1}{2}\left[x \cdot \frac{\sin 2x}{2} - \int 1 \cdot \frac{\sin 2x}{2} dx\right] + \frac{x^2}{4}$$

$$= \frac{x \cdot \sin 2x}{4} + \frac{1}{4} \times \frac{\cos 2x}{2} + \frac{x^2}{4} + C$$

$$= \frac{x \cdot \sin 2x}{4} + \frac{\cos 2x}{8} + \frac{x^2}{4} + C$$

Ans

Q.36 $\int x \cdot \sin 3x \cdot dx$

$-\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} + C$ Ans

$$\int \underset{\text{I}}{x} \cdot \underset{\text{II}}{\sin 3x}\, dx = x \int \sin 3x\, dx - \int \left[\frac{dx}{dx} \cdot \int \sin 3x\, dx \right] dx$$

$$= x \cdot \frac{(-\cos 3x)}{3} - \int 1 \cdot \left(-\frac{\cos 3x}{3}\right) dx$$

$$= -\frac{x \cdot \cos 3x}{3} + \frac{1}{3} \int \cos 3x \cdot dx$$

$$= -\frac{x\cos 3x}{3} + \frac{1}{3} \cdot \frac{\sin 3x}{3} + C$$

$$= -\frac{x\cos 3x}{3} + \frac{\sin 3x}{9} + C$$ Ans