NEW

Semester - II

MATHEMATICS –II

UNIT

2 Integral Calculus (समाकलन गणित)

TOPICS

1. समाकलन की परिभाषा (Definition of Integration)
2. समाकलन के प्रकार (Types of Integration)
3. समाकलन से संबंधित सूत्र (Formula related to Integration)
4. प्रतिस्थापन द्वारा समाकलन (Integration by Substitution)
5. खण्डश: समाकलन (Integration by Parts)
6. आंशिक भिन्नों द्वारा समाकलन (Integration by partial fractions)
7. गामा फलन द्वारा समाकलन (Integration Using Gama Function)
8. समाकलन के अनुप्रयोग (Applications of Integration)

$\int \sqrt{\tan x}\, dx$

SUbstitute $\tan x = t^2$

$\Rightarrow sec^2 x dx = 2tdt$

$\Rightarrow (1+t^4)dx = 2tdt$

$$\Rightarrow dx = \frac{2tdt}{1+t^4}$$

So the integral becomes $\int \frac{t.2tdt}{1+t^4}$

$$= \int \frac{2t^2 dt}{1+t^4}$$

$$= \int \frac{t^2+1}{1+t^4}dt + \int \frac{t^2-1}{1+t^4}dt$$

Dividing noth numerators and denominators by 't^2', we get

$$\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt + \int \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt$$

$$= \int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)^2+2}dt + \int \frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}\right)^2-2}dt$$

Substitute $t - \frac{1}{t} = u$ in the first integral and

$t + \frac{1}{t} = v$ in the second integral

$$\Rightarrow \left(1+\frac{1}{t^2}\right)dt = du \text{ and } \left(1-\frac{1}{t^2}\right)dt = dv$$

Hence, the integral becomes

$$\int \frac{du}{u^2+2} + \int \frac{dv}{v^2-2}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + \frac{1}{2\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + C$$

Here $u = t - \frac{1}{t}$ and $v = t + \frac{1}{t}$

and $t = \sqrt{\tan x}$

Substituting back the values, we get the integral in original variable 'x'.

Type-3

$\int \frac{1}{x^{1/m} + x^{1/n}} \, dx$ के रूप में समाकलन

Rule :- इस प्रकार के Question में m तथा n का L.C.M. लेते हैं। (माना p है।)

$x = t^p$ मानकर समाकलन करते हैं।

Q.23:- $\int \frac{1}{(x^{1/2}+x^{1/3})}dx$ का समाकलन ज्ञात करो। **Find the integral.**

2 व 3 का L.C.M. = 6

माना $x = t^6$

d.w.r.to x

$dx = 6t^5 \cdot dt$

$= \int \frac{1}{(t^6)^{1/2} + (t^6)^{1/3}} 6 \cdot t^5 \cdot dt$

$= 6\int \frac{t^5}{t^3+t^2} dt$

$= 6\int \frac{t^5}{t^2(t+1)} dt$

$= 6\int \frac{t^3}{t+1} dt$

$= 6\int \frac{t^3+1-1}{t+1} dt$

$= 6\int \left[\frac{(t^3+1^3)}{t+1} - \frac{1}{t+1}\right] dt$

$\because a^3+b^3 = (a+b)(a^2+b^2-ab)$

$$= 6\int\left[\frac{\cancel{(t+1)}(t^2+1^2-t\cdot 1)}{\cancel{(t+1)}} - \frac{1}{(t+1)}\right]dt$$

$$= 6\int(t^2+1-t)\,dt - 6\int\frac{1}{(t+1)}\,dt$$

$$= 6\left(\frac{t^3}{3}+t-\frac{t^2}{2}\right) - 6\cdot\log_e(t+1) + C$$

$\because x = t^6$

$x^{1/6} = t$

$$= 2\cdot(x^{1/6})^3 + 6\cdot(x^{1/6}) - 3\cdot(x^{1/6})^2 - 6\cdot\log_e(x^{1/6}+1) + C$$

$$= 2\cdot x^{1/2} + 6x^{1/6} - 3\cdot x^{1/3} - 6\cdot\log_e(x^{1/6}+1) + C \quad \underline{\underline{\text{Ans}}}$$

Q.24:- $\int \frac{dx}{(1+x)^{1/2}+(1+x)^{1/3}}$ का समाकलन ज्ञात करो।

Find the integral.

2 & 3 का L.C.M = 6

माना $(1+x) = t^6$

d. w. r. to x

$(0+1).dx = 6t^5.dt$

$dx = 6t^5.dt$

$$= \int \frac{6t^5.dt}{(t^6)^{1/2}+(t^6)^{1/3}}$$

$$= 6\int \frac{t^5}{t^3+t^2}dt$$

$$= 6\int \frac{t^5}{t^2(t+1)}dt$$

$$= 6\int \frac{t^3}{t+1}dt$$

$$= 6\int \frac{t^3+1-1}{t+1}dt$$

$$= 6\int \left[\frac{(t^3+1^3)}{(t+1)} - \frac{1}{(t+1)}\right]dt$$

$\because a^3+b^3 = (a+b)(a^2+b^2-ab)$

$$= 6\int\left[\frac{\cancel{(t+1)}(t^2+1^2-t\cdot 1)}{\cancel{(t+1)}} - \frac{1}{(t+1)}\right]dt$$

$$= 6\int(t^2+1-t)\,dt - 6\int\frac{1}{(t+1)}\,dt$$

$$= 6\left(\frac{t^3}{3}+t-\frac{t^2}{2}\right) - 6\cdot\log_e(t+1) + c$$

$\because (1+x) = t^6$

$(1+x)^{1/6} = t$

$$= 2\cdot\left[(1+x)^{1/6}\right]^3 + 6\,(1+x)^{1/6} - 3\cdot\left[(1+x)^{1/6}\right]^2 - 6\log_e\left[(1+x)^{1/6}+1\right] + c$$

$$= 2\cdot(1+x)^{1/2} + 6\,(1+x)^{1/6} - 3\,(1+x)^{1/3} - 6\log_e\left((1+x)^{1/6}+1\right) + c$$ Ans

Type-4

$$\int \frac{f(x)}{\sqrt{x+a} \pm \sqrt{x+b}}\,dx$$ के रूप के समाकलन

Rule :– हर (Denominator) का परिमेयकरण करके समाकलन करते हैं।

Q.25:- $\int \frac{1}{(\sqrt{2x+3}-\sqrt{2x+1})}dx$ का समाकलन ज्ञात करो।

Find the integral.

हर का परिमेयकरण

$$= \int \frac{1}{(\sqrt{2x+3}-\sqrt{2x+1})} \times \frac{(\sqrt{2x+3}+\sqrt{2x+1})}{(\sqrt{2x+3}+\sqrt{2x+1})}dx$$

$\because (a-b)(a+b) = a^2-b^2$

$$= \int \frac{\sqrt{2x+3}+\sqrt{2x+1}}{(\sqrt{2x+3})^2-(\sqrt{2x+1})^2}dx$$

$$= \int \frac{\sqrt{2x+3}+\sqrt{2x+1}}{\cancel{2x}+3-\cancel{2x}-1} = \int \frac{\sqrt{2x+3}+\sqrt{2x+1}}{2}dx$$

$$= \frac{1}{2}\int\left[(2x+3)^{1/2} + (2x+1)^{1/2}\right]dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$= \frac{1}{2}\left[\frac{(2x+3)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\times 2} + \frac{(2x+1)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\times 2}\right] + c$$

$$= \frac{1}{2}\left[\frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{\cancel{2}}\times\cancel{2}} + \frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{\cancel{2}}\times\cancel{2}}\right] + c$$

$$= \frac{1}{6}\left[(2x+3)^{3/2} + (2x+1)^{3/2}\right] + c$$

Ans