NEW

Semester - II

MATHEMATICS –II

UNIT

2 Integral Calculus (समाकलन गणित)

UNIT-II
Integral Calculus

UNIT - II: Integral Calculus **(12 periods)**

Integration as inverse operation of differentiation. Simple integration by substitution, by parts and by partial fractions (for linear factors only). Introduction to definite integration. Use of formulae $\int_0^{\frac{\pi}{2}} \sin^n x dx$, $\int_0^{\frac{\pi}{2}} \cos^n x dx$, $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$ for solving problems ,where m and n are positive integers. Applications of integration for (i). Simple problems on evaluation of area bounded by a curve and axes. (ii). calculation of volume of a solid formed by revolution of an area about axes. (Simple problems).

TOPICS

1. समाकलन की परिभाषा (Definition of Integration)
2. समाकलन के प्रकार (Types of Integration)
3. समाकलन से संबंधित सूत्र (Formula related to Integration)
4. प्रतिस्थापन द्वारा समाकलन (Integration by Substitution)
5. खण्डश: समाकलन (Integration by Parts)
6. आंशिक भिन्नों द्वारा समाकलन (Integration by partial fractions)
7. गामा फलन द्वारा समाकलन (Integration Using Gama Function)
8. समाकलन के अनुप्रयोग (Applications of Integration)

Imp.

$\int_0^{\pi/2}$ $\sin^m x \cos^n x\, dx$ के रूप में समाकल का मान:

(The value of the integral in the form).

where m & n are Positive integer

$$\int_0^{\frac{\pi}{2}} \sin^m x \cdot \cos^n x \cdot dx = \frac{\Gamma\frac{m+1}{2} \cdot \Gamma\frac{n+1}{2}}{2 \cdot \Gamma\frac{m+n+2}{2}}$$

जैसे - $$\int_0^{\frac{\pi}{2}} \sin^3 x \cdot \cos^5 x \cdot dx = \frac{\Gamma\frac{3+1}{2} \; \Gamma\frac{5+1}{2}}{2\,\Gamma\frac{3+5+2}{2}} = \frac{\Gamma 2 \times \Gamma 3}{2\,\Gamma 5}$$

$$= \frac{1! \times 2!}{2 \times 4!}$$

$$= \frac{1 \times \cancel{2} \times 1}{\cancel{2} \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{1}{24}$$ Ans

① $$\int_0^{\frac{\pi}{2}} \sin^m x \cdot \cos^n x \, dx = \frac{\Gamma\left(\frac{m+1}{2}\right) \cdot \Gamma\left(\frac{n+1}{2}\right)}{2\,\Gamma\left(\frac{m+n+2}{2}\right)}$$

② $$\int_0^{\frac{\pi}{2}} \sin^n x \cdot dx = \int_0^{\frac{\pi}{2}} \sin^n x \cdot (\cos x)^0 dx = \frac{\Gamma\left(\frac{n+1}{2}\right) \cdot \Gamma\left(\frac{0+1}{2}\right)}{2\,\Gamma\left(\frac{n+2}{2}\right)}$$

③ $$\int_0^{\frac{\pi}{2}} \cos^n x \cdot dx = \int_0^{\frac{\pi}{2}} (\sin x)^0 \cdot \cos^n x \cdot dx = \frac{\Gamma\left(\frac{0+1}{2}\right) \cdot \Gamma\left(\frac{n+1}{2}\right)}{2\,\Gamma\left(\frac{n+2}{2}\right)}$$

Q.60:- $\int_0^{\pi/2} \cos^5 x \sin^4 x \, dx$ का मान ज्ञात करें। **(Find the value.)**

यह $\int_0^{\frac{\pi}{2}} \cos^m x \cdot \sin^n x \cdot dx$ के रूप में है।

Formula $\int_0^{\frac{\pi}{2}} \cos^m x \cdot \sin^n x \, dx = \dfrac{\Gamma\frac{m+1}{2} \cdot \Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+2}{2}}$

$$\int_0^{\frac{\pi}{2}} \cos^5 x \cdot \sin^4 x \, dx = \frac{\Gamma\frac{5+1}{2} \cdot \Gamma\frac{4+1}{2}}{2\,\Gamma\frac{5+4+2}{2}} = \frac{\Gamma 3 \cdot \Gamma\frac{5}{2}}{2\,\Gamma\frac{11}{2}}$$

$$= \frac{\cancel{2!} \times \cancel{\frac{3}{2}} \times \cancel{\frac{1}{2}} \times \cancel{\sqrt{\pi}}}{\cancel{2} \times \frac{9}{2} \times \frac{7}{2} \times \frac{5}{2} \times \cancel{\frac{3}{2}} \times \cancel{\frac{1}{2}} \times \cancel{\sqrt{\pi}}}$$

$$= \frac{1}{\frac{315}{8}}$$

$$= \frac{8}{315} \text{ Ans}$$

Q.61:- दिखाइयें कि $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{(\sin x)^{1/2}}\, dx \times \int_0^{\frac{\pi}{2}} (\sin x)^{1/2} dx$$

$$= \int_0^{\frac{\pi}{2}} (\sin x)^{-\frac{1}{2}} \cdot (\cos x)^0 \cdot dx \times \int_0^{\frac{\pi}{2}} (\sin x)^{1/2} \cdot (\cos x)^0 dx$$

$$= \frac{\Gamma\frac{-\frac{1}{2}+1}{2} \cdot \Gamma\frac{0+1}{2}}{2\,\Gamma\frac{-\frac{1}{2}+0+2}{2}} \times \frac{\Gamma\frac{\frac{1}{2}+1}{2} \cdot \Gamma\frac{0+1}{2}}{2\,\Gamma\left(\frac{1}{2}+0+2\right)/2}$$

$$= \frac{\Gamma\frac{1}{4} \times \Gamma\frac{1}{2}}{2\,\cancel{\Gamma\frac{3}{4}}} \times \frac{\cancel{\Gamma\frac{3}{4}} \times \Gamma\frac{1}{2}}{2\,\Gamma\frac{5}{4}}$$

$$= \frac{\cancel{\Gamma\frac{1}{4}}\,\sqrt{\pi} \times \sqrt{\pi}}{\cancel{4} \times \frac{1}{\cancel{4}} \cdot \cancel{\Gamma\frac{1}{4}}}$$

$$= \pi \text{ Ans}$$

Q.62:- $\int_0^{\pi/2} \cos^8 x \, dx$

$$= \int_0^{\frac{\pi}{2}} \cos^8 x \cdot (\sin x)^0 \cdot dx \quad \text{(Gamma function)}$$

$$= \frac{\Gamma\frac{8+1}{2} \cdot \Gamma\frac{0+1}{2}}{2\Gamma\frac{8+0+2}{2}} = \frac{\Gamma\frac{9}{2} \cdot \Gamma\frac{1}{2}}{2\Gamma\frac{10}{2}}$$

$$\frac{\left(\frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \cdot \sqrt{\pi}\right) \times \sqrt{\pi}}{2 \times 4!}$$

$$\frac{\left(\frac{105}{16}\right)\pi}{2 \times 24}$$

$$\frac{105\pi}{768} \text{ Ans}$$

Q.63:- $\int_0^a \frac{x^4}{\sqrt{(a^2-x^2)}}\,dx$

माना $x = a\sin\theta$

d. w.r.to x

$dx = a\cdot\cos\theta\cdot d\theta$

Limit change

$x=0$ पर $\theta = 0$

$x=a$ पर $\theta = \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \frac{a^4\sin^4\theta}{\sqrt{a^2-a^2\sin^2\theta}}\cdot a\cos\theta\cdot d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{a^4\sin^4\theta}{\cancel{a\cos\theta}}\,\cancel{a\cos\theta}\cdot d\theta$$

$$= a^4\int_0^{\frac{\pi}{2}} \sin^4\theta\cdot(\cos\theta)^0\,d\theta$$

$$= a^4\cdot\frac{\Gamma\frac{4+1}{2}\cdot\Gamma\frac{0+1}{2}}{2\,\Gamma\frac{4+0+2}{2}}$$

$$= \frac{a^4\cdot\Gamma\frac{5}{2}\cdot\Gamma\frac{1}{2}}{2\,\Gamma 3}$$

$$= \frac{a^4 \cdot \left(\frac{3}{2} \times \frac{1}{2} \cdot \sqrt{\pi}\right) \sqrt{\pi}}{2 \times 2!}$$

$$= \frac{a^4 \cdot \left(\frac{3}{4}\right) \cdot \pi}{4}$$

$$= \frac{3}{16} \pi a^4 \quad \underline{\underline{\text{Ans}}}$$

Q.64:- $\int_0^{\pi/2} \sin^6 x \cos^3 x \, dx$

$$\int_0^{\pi/2} \sin^m x \cdot \cos^n x \cdot dx = \frac{\Gamma\frac{m+1}{2} \cdot \Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+2}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sin^6 x \cdot \cos^3 x \cdot dx = \frac{\Gamma\frac{6+1}{2} \cdot \Gamma\frac{3+1}{2}}{2\,\Gamma\frac{6+3+2}{2}}$$

$$= \frac{\Gamma\frac{7}{2} \cdot \Gamma 2}{2\,\Gamma\frac{11}{2}}$$

$$= \frac{\left(\cancel{\frac{5}{2}} \times \cancel{\frac{3}{2}} \times \cancel{\frac{1}{2}} \cdot \cancel{\sqrt{\pi}}\right) \times 1!}{\cancel{2} \times \left(\frac{9}{\cancel{2}} \times \frac{7}{2} \times \cancel{\frac{5}{2}} \times \cancel{\frac{3}{2}} \times \cancel{\frac{1}{2}} \cancel{\sqrt{\pi}}\right)}$$

$$= \frac{1}{\frac{63}{2}}$$

$$= \frac{2}{63} \text{ Ans}$$

Q.65:- $\int_0^{\pi/2} \sin^8 x \cos^2 x \, dx$

$$= \frac{\Gamma\left(\frac{8+1}{2}\right) \Gamma\left(\frac{2+1}{2}\right)}{2\,\Gamma\left(\frac{8+2+2}{2}\right)}$$

$$= \frac{\Gamma\left(\frac{9}{2}\right) \cdot \Gamma\left(\frac{3}{2}\right)}{2\,\Gamma(6)}$$

$$= \frac{\left(\frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2}\sqrt{\pi}\right) \times \left(\frac{1}{2}\sqrt{\pi}\right)}{2 \times 5!}$$

$$\frac{\left(\frac{\overset{7}{\cancel{\overset{21}{\cancel{105}}}}}{32}\right)\pi}{2 \times \cancel{5} \times 4 \times \cancel{3} \times 2 \times 1}$$

$$= \frac{7\pi}{32 \times 16}$$

$$= \frac{7\pi}{512} \quad \text{Ans}$$

Q.66:- $\int_{0}^{\pi/2} \cos^5 x \sin^4 x \, dx$ (H.W.)