NEW

Semester - II

MATHEMATICS –II

UNIT

2 Integral Calculus (समाकलन गणित)

UNIT-II
Integral Calculus

UNIT - II: Integral Calculus **(12 periods)**

Integration as inverse operation of differentiation. Simple integration by substitution, by parts and by partial fractions (for linear factors only). Introduction to definite integration. Use of formulae $\int_0^{\frac{\pi}{2}} \sin^n x dx, \int_0^{\frac{\pi}{2}} \cos^n x dx, \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$ for solving problems ,where m and n are positive integers. Applications of integration for (i). Simple problems on evaluation of area bounded by a curve and axes. (ii). calculation of volume of a solid formed by revolution of an area about axes. (Simple problems).

TOPICS

1. समाकलन की परिभाषा (Definition of Integration)
2. समाकलन के प्रकार (Types of Integration)
3. समाकलन से संबंधित सूत्र (Formula related to Integration)
4. प्रतिस्थापन द्वारा समाकलन (Integration by Substitution)
5. खण्डश: समाकलन (Integration by Parts)
6. आंशिक भिन्नों द्वारा समाकलन (Integration by partial fractions)
7. गामा फलन द्वारा समाकलन (Integration Using Gama Function)
8. समाकलन के अनुप्रयोग (Applications of Integration)

Q.48:- $\int \frac{1}{(x^3+1)}$ dx का समाकलन ज्ञात करो। **(Find the integral).**

$a^3+b^3 = (a+b)(a^2+b^2-ab)$

$$= \int \frac{1}{(x+1)(x^2+1-x)}\,dx$$

By Partial Fraction

out of Syllabus

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-x+1)}$$

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A(x^2-x+1)+(Bx+C)(x+1)}{(x+1)(x^2-x+1)}$$

$$1 = A(x^2-x+1)+(Bx+C)(x+1) \quad \text{——①}$$

समी ① में $x=-1$ रखने पर

$$1 = A((-1)^2-(-1)+1)+(B(-1)+C)(-1+1)$$

$$1 = A(1+1+1)+0$$

$$1 = 3A$$

$$\boxed{A = \frac{1}{3}}$$

from Eq①

$$1 = Ax^2-Ax+A+Bx^2+Bx+Cx+C$$

$$1 = (A+B)x^2+(-A+B+C)x+A+C \quad \text{——①}$$

comparing (तुलना करने पर)

$$A+B=0 \quad \text{——(i)}$$

$$\frac{1}{3}+B=0 \Rightarrow \boxed{B=-\frac{1}{3}}$$

$$-A+B+C=0 \quad \text{——(ii)}$$

$$-\frac{1}{3}+\left(-\frac{1}{3}\right)+C=0$$

$$\frac{-1-1}{3}+C=0$$

$$-\frac{2}{3}+C=0$$

$$\boxed{C=\frac{2}{3}}$$

A, B, C का मान रखने पर

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{\frac{1}{3}}{(x+1)} + \frac{-\frac{1}{3}x+\frac{2}{3}}{(x^2-x+1)}$$

Integrating (समाकलन)

$$= \frac{1}{3}\int \frac{1}{(x+1)}\,dx - \int \frac{\frac{-x+2}{3}}{x^2-x+1}\,dx$$

$$= \frac{1}{3}\cdot \log(x+1) - \frac{1}{3}\int \frac{2-x}{x^2-x+1}\cdot dx$$

→ Hold

①

$$= \int \frac{x-2}{x^2-x+1}\,dx$$

माना $x-2 = \frac{1}{2}(2x-1) + \frac{1}{2} - 2$

$$= \int \frac{\frac{1}{2}(2x-1) + \frac{1}{2} - 2}{(x^2-x+1)}\,dx$$

$$= \frac{1}{2}\int \frac{(2x-1)}{(x^2-x+1)}\,dx - \frac{3}{2}\int \frac{1}{(x^2-x+1)}$$

माना $x^2-x+1 = t$

$(2x-1)dx = dt$

$$= \frac{1}{2}\int \frac{1}{t}\cdot dt - \frac{3}{2}\int \frac{1}{x^2 - 2\cdot x\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \frac{1}{2}\log t - \frac{3}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}}\,dx$$

$$= \frac{1}{2}\log\left(x^2-x+1\right) - \frac{3}{2}\int \frac{1}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$

$$\int \frac{1}{x^2+a^2}\cdot dx = \frac{1}{a}\cdot\tan^{-1}\left(\frac{x}{a}\right)$$

$$= \frac{1}{2}\log\left(x^2-x+1\right) - \frac{3}{2}\times\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\sqrt{3}/2}\right)$$

$$= \frac{1}{2}\log\left(x^2-x+1\right) - \sqrt{3}\cdot\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$$

↳ सभी ① में रखने पर

Q.50:- $\int \frac{x^4}{(x^2+1)}$ dx का समाकलन ज्ञात करो। **(Find the integral).**

माना $x^2 = t$

$$= \int \frac{t^2}{(t+1)}\,dx$$

$$= \int \frac{t^2-1+1}{(t+1)}\,dx$$

$$= \int \frac{(t^2-1)}{(t+1)}\,dx + \int \frac{1}{(t+1)}\,dx$$

$$= \int \frac{(t+1)(t-1)}{(t+1)}\,dx + \int \frac{1}{(t+1)}\,dx$$

$$= \int (t-1)\,dx + \int \frac{1}{(t+1)}\,dx$$

$t = x^2$ रखने पर

$$= \int (x^2-1)\,dx + \int \frac{1}{x^2+1}\,dx$$

$$= \frac{x^3}{3} - x + \tan^{-1}x + C$$ Ans

Q.51:- $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}$ dx का समाकलन ज्ञात करो। **(Find the integral).**

माना $\tan x = t$

d.w.r. to x

$\sec^2 x \cdot dx = dt$

$$\int \frac{1}{(1+t)(2+t)} dt$$

by Partial fraction

$$\frac{1}{(1+t)(2+t)} = \frac{A}{(1+t)} + \frac{B}{(2+t)}$$

$1 = A(2+t) + B\cdot(1+t)$ ——— ①

$t = -1$ रखने पर

$1 = A(2-1) + B(1-1)$

$1 = A + 0$

$\boxed{A = 1}$

$t = -2$ रखने पर

$1 = A(2-2) + B(1-2)$

$1 = 0 + B(-1)$

$\boxed{B = -1}$

A, B का मान रखने पर

$$\frac{1}{(1+t)(2+t)} = \frac{1}{(1+t)} + \frac{-1}{(2+t)}$$

Integrating

$$= \int \frac{1}{1+t}\,dt - \int \frac{1}{(2+t)}\,dt$$

$$= \log(1+t) - \log(2+t) + C \quad \because t = \tan x$$

$$= \log(1+\tan x) - \log(2+\tan x) + C \quad \underline{\text{Ans}}$$

Q.52:- $\int \frac{1-x}{1+x}\,dx$ का समाकलन ज्ञात करो। **(Find the integral).**

$$= \int \frac{1}{1+x}\,dx - \int \frac{x}{1+x}\,dx$$

$$= \log(1+x) - \int \frac{x+1-1}{(1+x)}\,dx$$

$$= \log(1+x) - \int \frac{1+x}{1+x}\,dx + \int \frac{1}{(1+x)}\,dx$$

$$= \log(1+x) - x + \log(1+x) + C = 2\log(1+x) - x + C$$ Ans

Q.53:- $\int \frac{1}{x(x^4+1)}\,dx$ का समाकलन ज्ञात करो। **(Find the integral).**

अंश व हर में x^3 से गुणा करने पर

$$\int \frac{x^3}{x^4(x^4+1)}\,dx$$

माना $x^4 = t$

$$4.x^3.dx = dt$$

$$x^3 dx = \frac{dt}{4}$$

$$= \int \frac{1}{t(t+1)} \cdot \frac{dt}{4}$$

$$= \frac{1}{4}\int \frac{1}{t(t+1)}\,dt$$

By Partial Fraction.

$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$$

$$1 = A(t+1) + B.t \quad \text{—— (1)}$$

$t = 0$ रखने पर

$$1 = A(0+1) + B \times 0$$

$$1 = A(1) + 0$$

$$\boxed{A = 1}$$

$t = -1$ रखने पर

$$1 = A(-1+1) + B(-1)$$

$$1 = 0 - B$$

$$\boxed{B = -1}$$

A, B का मान रखने पर

$$\frac{1}{4}\cdot\frac{1}{t(t+1)} = \frac{1}{4}\left[\frac{1}{t} + \frac{-1}{(t+1)}\right]$$

Integration

$$= \frac{1}{4}\int \frac{1}{t}\,dt - \frac{1}{4}\int \frac{1}{t+1}\,dt$$

$$= \frac{1}{4}\log t - \frac{1}{4}\log(t+1) + C \qquad \because t = x^4$$

$$= \frac{1}{4}\log x^4 - \frac{1}{4}\log(x^4+1) + C \quad \underline{\text{Ans}}$$

Q.54:- $\int \frac{\sin x}{\cos^2 x - 5\cos x + 4}\, dx$ का समाकलन ज्ञात करो। **(Find the integral).**

माना $\cos x = t$

d. w. r. to x

$-\sin x \cdot dx = dt$

$\sin x \cdot dx = -dt$

$$= \int \frac{-dt}{t^2 - 5t + 4}$$

$$= -\int \frac{1}{t^2 - 4t - t + 4}\, dt$$

$$= -\int \frac{1}{(t-1)(t-4)}\, dt$$

by Partial Fraction

$$\frac{1}{(t-1)(t-4)} = \frac{A}{(t-1)} + \frac{B}{(t-4)}$$

$$1 = A(t-4) + B(t-1) \quad \text{——} (1)$$

$t = 1$ रखने पर

$1 = A(1-4) + B(1-1) \Rightarrow 1 = -3A + 0$

$\boxed{A = -\frac{1}{3}}$

समी ① में $t=4$ रखने पर

$$1 = A(4-4) + B(4-1)$$

$$1 = 0 + B(3)$$

$$\boxed{B = \frac{1}{3}}$$

A, B का मान रखने पर

$$-\frac{1}{(t-1)(t-4)} = -\left[\frac{-\frac{1}{3}}{(t-1)} + \frac{\frac{1}{3}}{(t-4)}\right]$$

Integrating

$$= \frac{1}{3}\int \frac{1}{(t-1)}dt - \frac{1}{3}\int \frac{1}{(t-4)}dt$$

$$= \frac{1}{3}\log(t-1) - \frac{1}{3}\log(t-4) + C$$

$$= \frac{1}{3}\log(\cos x - 1) - \frac{1}{3}\log(\cos x - 4) + C$$

$$= \frac{1}{3}\log\left(\frac{\cos x - 1}{\cos x - 4}\right) + C$$ Ans