NEW

Semester - II

MATHEMATICS −II

UNIT

2 Integral Calculus (समाकलन गणित)

Type-II $\displaystyle\int \frac{f'(x)}{f(x)} \cdot dx$ के रूप के समाकलन

जहाँ $f(x)$ is a function (फलन)

& $f'(x) \to f(x)$ का अवकलन (differential)

Rule:- जिसका Differential दिया होता है, उसे t मानकर समाकलन करते है।

यहां पर, माना $f(x) = t$

$$d.w.r.to\ x$$

$$f'(x) = \frac{dt}{dx}$$

$$f'(x) \cdot dx = dt$$

अतः $\to \displaystyle\int \frac{1}{t} dt = \log_e t + C = \log_e f(x) + C$ **Ans**

Q.13:- $\int \sin^3 x \, dx$ का समाकलन ज्ञात करो।

Find the integral.

Method-I

$$= \int \frac{3\sin x - \sin 3x}{4} \, dx$$

$$\because \ \sin 3x = 3\sin x - 4\sin^3 x$$

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$$

$$= \frac{1}{4} \int \left(3\sin x - \sin 3x \right) dx$$

$$= \frac{1}{4} \left[3 \cdot (-\cos x) - \left(-\frac{\cos 3x}{3} \right) \right] + C$$

$$= \frac{-3\cos x}{4} + \frac{\cos 3x}{12} + C \ \underline{Ans}$$

Q.13:- $\int \sin^3 x\ dx$ का समाकलन ज्ञात करो।

Find the integral.

Method–II

$$\int \sin x \cdot \sin^2 x\ dx \qquad \because \sin^2 x = 1 - \cos^2 x$$

$$\int \sin x \left(1 - \cos^2 x\right) \cdot dx$$

माना $\cos x = t$

d.w.r to x

$$-\sin x \cdot dx = dt$$

$$\sin x \cdot dx = -dt$$

$$\int (1 - t^2) \cdot (-dt)$$

$$\int (t^2 - 1) \cdot dt$$

$$\frac{t^3}{3} - t + C \Rightarrow \frac{\cos^3 x}{3} - \cos x + C \quad \underline{Ans}$$

Q.14:- $\displaystyle\int \left(\frac{e^{m\tan^{-1}x}}{1+x^2}\right)\cdot dx$ का समाकलन ज्ञात करो।

Find the integral.

माना $\tan^{-1}x = t$

d. w. r. to x

$$\frac{1}{1+x^2} = \frac{dt}{dx}$$

$$\frac{dx}{1+x^2} = dt$$

$$= \int e^{m\cdot t}\, dt$$

$$= \frac{e^{m\cdot t}}{m} + C$$

$$= \frac{e^{m\cdot\tan^{-1}x}}{m} + C \quad \underline{\text{Ans}}$$

Q.15:- $\displaystyle\int \frac{(e^x - e^{-x})}{(e^x + e^{-x})} \, dx$ का समाकलन ज्ञात करो। Find the integral.

$$\text{माना} \quad e^x + e^{-x} = t$$

$$d.w.r. \text{ to } x$$

$$e^x + (-1 \cdot e^{-x}) = \frac{dt}{dx}$$

$$(e^x - e^{-x}) \, dx = dt$$

$$\int \frac{dt}{t} = \int \frac{1}{t} \cdot dt = \log_e t + C$$

$$= \log_e (e^x + e^{-x}) + C \quad \underline{\underline{Ans}}$$

Q.16:- $\displaystyle\int \frac{\cot x}{\sqrt{\sin x}}\ dx$ का समाकलन ज्ञात करो।

Find the integral.

$$\because \cot x = \frac{\cos x}{\sin x}$$

$$= \int \frac{\cos x}{\sin x \cdot (\sin x)^{1/2}}\ dx$$

$$= \int \frac{\cos x}{(\sin x)^{3/2}}\ dx$$

माना $\sin x = t$

d. w. r to x

$\cos x\ dx = dt.$

$$= \int \frac{1}{t^{3/2}}\ dt$$

$$= \int t^{-\frac{3}{2}}\ dt$$

$$= \frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c$$

$$\frac{(\sin x)^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$-2 \cdot \frac{1}{(\sin x)^{1/2}} + c$$

$$\frac{-2}{\sqrt{\sin x}} + c \quad \underline{\underline{Ans}}$$

Q.17:- $\displaystyle\int \frac{\sec^2 x}{3+4\tan x}\, dx$ का समाकलन ज्ञात करो।

Find the integral.

माना $\quad 3 + 4\tan x = t$

$\qquad$ d.w.r. to x

$\qquad 0 + 4\cdot \sec^2 x = \dfrac{dt}{dx}$

$\qquad \sec^2 x \cdot dx = \dfrac{dt}{4}$

$\displaystyle\int \frac{1}{t}\cdot \frac{dt}{4}$

$= \dfrac{1}{4}\displaystyle\int \frac{1}{t}\cdot dt$

$= \dfrac{1}{4}\log_e t + c$

$= \dfrac{1}{4}\log_e\left(3 + 4\tan x\right) + c \quad \underline{\underline{Ans}}$

Q.18:- $\displaystyle\int \frac{e^x(1+x)}{\cos^2(xe^x)}\, dx$ का समाकलन ज्ञात करो।

Find the integral.

माना $\underset{\text{I}}{x} \cdot \underset{\text{II}}{e^x} = t$

d.w.r. to x

$$x \cdot e^x + e^x \cdot 1 = \frac{dt}{dx}$$

$$e^x(x+1) = \frac{dt}{dx}$$

$$e^x(1+x)\, dx = dt$$

$$\int \frac{1}{\cos^2 t} \cdot dt$$

$$= \int \sec^2 t \cdot dt$$

$$= \tan t + c$$

$$= \tan(x \cdot e^x) + c \quad \underline{\underline{Ans}}$$

Q.19:- $\displaystyle\int \frac{x \tan^{-1} x^2}{1 + x^4} \, dx$ का समाकलन ज्ञात करो।

Find the integral.

माना $\tan^{-1} x^2 = t$

d.w.r.to x

$$\frac{1}{1 + (x^2)^2} \cdot (2x) = \frac{dt}{dx}$$

$$\frac{2x}{1 + x^4} = \frac{dt}{dx}$$

$$\frac{x}{1 + x^4} \cdot dx = \frac{dt}{2}$$

$$= \int t \cdot \frac{dt}{2}$$

$$= \frac{1}{2} \int t \cdot dt$$

$$= \frac{1}{2} \cdot \frac{t^2}{2} + C$$

$$= \frac{t^2}{4} + C$$

$$= \frac{\left(\tan^{-1} x^2\right)^2}{4} + C \quad \underline{Ans}$$

Q.20:- $\int \dfrac{1}{\sqrt{x}} \cos \sqrt{x} \cdot dx$ का समाकलन ज्ञात करो। Find the integral.

$$\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

माना $\sqrt{x} = t$

$\quad$ d. w. r. to x

$$\frac{1}{2\sqrt{x}} \cdot dx = dt$$

$$\frac{dx}{\sqrt{x}} = 2 \cdot dt$$

$$= \int \cos t \cdot (2 \cdot dt)$$

$$= 2 \int \cos t \cdot dt$$

$$= 2 \cdot \sin t + C$$

$$= 2 \cdot \sin \sqrt{x} + C \quad \text{Ans}$$

Q.21:- $\displaystyle\int \frac{x^2+1}{(1+x)^2}\,dx$ का समाकलन ज्ञात करो।

Find the integral.

$$= \int \frac{x^2+1+2x-2x}{(1+x)^2}$$

$$= \int \frac{(x+1)^2-2x}{(1+x)^2}$$

$$= \int \left[\frac{(x+1)^2}{(x+1)^2} - \frac{2x}{(1+x)^2}\right] dx$$

$$= \int 1\cdot dx \;-\; 2\int \frac{x}{(1+x)^2}\cdot dx$$

(H.W.)

Q.22:- $\displaystyle\int \frac{1 + \sin 3x}{(3x - \cos 3x)}$ dx का समाकलन ज्ञात करो।

Find the integral.

माना $\quad 3x - \cos 3x = t$

d.w.r.to x

$$3 + \sin 3x \,(3) = \frac{dt}{dx}$$

$$3 + 3\sin 3x = \frac{dt}{dx}$$

$$3(1 + \sin 3x)\, dx = dt$$

$$(1 + \sin 3x)\, dx = \frac{dt}{3}$$

$$= \int \frac{1}{t} \cdot \frac{dt}{3}$$

$$= \frac{1}{3} \cdot \log_e t + C$$

$$= \frac{1}{3} \log_e (3x - \cos 3x) + C \quad \underline{Ans}$$