NEW

Semester - II

MATHEMATICS –II

UNIT

2 Integral Calculus (समाकलन गणित)

UNIT-II
Integral Calculus

UNIT - II: Integral Calculus **(12 periods)**

Integration as inverse operation of differentiation. Simple integration by substitution, by parts and by partial fractions (for linear factors only). Introduction to definite integration. Use of formulae $\int_0^{\frac{\pi}{2}} \sin^n x dx$, $\int_0^{\frac{\pi}{2}} \cos^n x dx$, $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$ for solving problems ,where m and n are positive integers. Applications of integration for (i). Simple problems on evaluation of area bounded by a curve and axes. (ii). calculation of volume of a solid formed by revolution of an area about axes. (Simple problems).

TOPICS

1. समाकलन की परिभाषा (Definition of Integration)
2. समाकलन के प्रकार (Types of Integration)
3. समाकलन से संबंधित सूत्र (Formula related to Integration)
4. प्रतिस्थापन द्वारा समाकलन (Integration by Substitution)
5. खण्डश: समाकलन (Integration by Parts)
6. आंशिक भिन्नों द्वारा समाकलन (Integration by partial fractions)
7. गामा फलन द्वारा समाकलन (Integration Using Gama Function)
8. समाकलन के अनुप्रयोग (Applications of Integration)

5. गामा फलन (Gamma Function)

→ निश्चित समाकलन (Definite integration) $\int_0^\infty e^{-x} \cdot x^{n-1} \cdot dx$ को n का Gamma function कहते हैं।

→ n के Gamma function को $\overline{|n}$ से प्रदर्शित करते हैं।

$$\boxed{\overline{|n} = \int_0^\infty e^{-x} \cdot x^{n-1} \cdot dx}$$

where $n > 0$

Imp.

6. गामा फलन के गुण (Properties of Gamma Function)

① $\boxed{\Gamma n = (n-1)!}$

जैसे $\Gamma 5 = 4! = 4\times3\times2\times1 = \underline{\underline{24}}$

जब n Positive integer (धन पूर्णांक) हो ।

② $\boxed{\Gamma n = (n-1)\,\Gamma(n-1)}$

जैसे-

(i) $\Gamma\frac{5}{2} = \frac{3}{2}\Gamma\frac{3}{2}$

$= \frac{3}{2}\times\frac{1}{2}\times\Gamma\frac{1}{2}$

$= \frac{3}{4}\sqrt{\pi}$

(ii) $\Gamma\frac{9}{2} = \frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}$

$= \frac{105}{16}\sqrt{\pi}$

③ $\boxed{\Gamma\frac{1}{2} = \sqrt{\pi}}$

④ $\boxed{\Gamma 1 = 1}$

Factoriol :- ① $n! = n \times (n-1) \times (n-2) \times (n-3) \times \cdots \cdots 3 \times 2 \times 1$

जैसे- $5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

② $1! = 1$

③ $0! = 1$

Q.55:- $\Gamma 7$ तथा $\Gamma\left(\frac{9}{2}\right)$ का मान निकालें। (Find the value).

(i) $\Gamma 7 = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$

(ii) $\Gamma \frac{9}{2} = \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi}$

$= \frac{105}{16}\sqrt{\pi}$

Q.56:- $\Gamma\left(-\frac{1}{2}\right)$ तथा $\Gamma\left(-\frac{3}{2}\right)$ का मान निकालें। **(Find the value).**

① $\Gamma(n) = (n-1)\,\Gamma(n-1)$

या $\Gamma(n+1) = n \cdot \Gamma(n)$

या $\boxed{\Gamma(n) = \frac{\Gamma(n+1)}{n}}$

(i) $\Gamma\left(-\frac{1}{2}\right) = \frac{\Gamma\left(-\frac{1}{2}+1\right)}{-\frac{1}{2}}$

$= \frac{\Gamma\left(\frac{1}{2}\right)}{-\frac{1}{2}} = \frac{\sqrt{\pi}}{-\frac{1}{2}} = \underline{\underline{-2\sqrt{\pi}}}$

(ii) $\Gamma\left(-\frac{3}{2}\right) = \frac{\Gamma\left(-\frac{3}{2}+1\right)}{-\frac{3}{2}} = \frac{\Gamma\left(-\frac{1}{2}\right)}{-\frac{3}{2}} = \frac{-2\sqrt{\pi}}{-\frac{3}{2}} = \underline{\underline{\frac{4\sqrt{\pi}}{3}}}$

Q.57:- $\int_0^\infty x^5 e^{-x}$ का मान ज्ञात करें। **(Find the value).**

$\because$ Gamma function $\int_0^\infty e^{-x} \cdot x^{n-1} \cdot dx = \Gamma n$ (from definition)

$\int_0^\infty e^{-x} \cdot x^{6-1} \cdot dx = \Gamma 6$ ($\because$ तुलना करने पर $n = 6$)

$= 5!$

$= 5 \times 4 \times 3 \times 2 \times 1 = 120$ Ans

Q.58:- $\int_0^\infty \sqrt[4]{x}\cdot e^{-\sqrt{x}}\,dx$ का मान ज्ञात करें।

$$\int_0^\infty x^{1/4}\cdot e^{-\sqrt{x}}\cdot dx$$

माना $\sqrt{x} = t$

या $x = t^2$

d.w.r. to x

$1\cdot dx = 2t\cdot dt$

$$= \int_0^\infty (t^2)^{1/4}\cdot e^{-t}\cdot 2t\cdot dt$$

$$= 2\int_0^\infty t^{1/2}\cdot e^{-t}\cdot t\cdot dt$$

$$= 2\int_0^\infty t^{3/2}\cdot e^{-t}\cdot dt$$

from definition of Gamma function

$$\int_0^\infty e^{-x}\cdot x^{n-1}\cdot dx = \Gamma n$$

$$= 2\int_0^\infty e^{-t} \cdot t^{\frac{5}{2}-1} \cdot dt$$

$$= 2 \times \Gamma\left(\frac{5}{2}\right)$$

$$= 2 \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi}$$

$$= \frac{3\sqrt{\pi}}{2} \quad \underline{\underline{Ans}}$$

Q.59:- $\int_0^\infty x^{1/2} \cdot e^{-\sqrt[3]{x}}\, dx$ का मान ज्ञात करें।

$$\int_0^\infty x^{1/2} \cdot e^{-(x)^{1/3}} \cdot dx$$

माना $x^{1/3} = t$

या $x = t^3$

d. w. r. to x

$1 \cdot dx = 3 \cdot t^2 \cdot dt$

$$= \int_0^\infty (t^3)^{1/2} \cdot e^{-t} \cdot 3t^2 \cdot dt$$

$$= 3\int_0^\infty t^{7/2} \cdot e^{-t} \cdot dt$$

from definition of Gamma function

$$\int_0^\infty e^{-x} \cdot x^{n-1} \cdot dx = \Gamma n$$

$$= 3\int_0^\infty e^{-t} \cdot t^{9/2-1} \cdot dt$$

$$= 3 \times \Gamma\frac{9}{2}$$

$$= 3 \times \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi}$$

$$= \frac{315\sqrt{\pi}}{16}$$ Ans

$\int_0^{\pi/2} \sin^m x \cos^n x \, dx$ के रूप में समाकल का मान:

(The value of the integral in the form).