

NEW

Semester - II

MATHEMATICS -II

UNIT

2

Integral Calculus (समाकलन गणित)

TOPICS

- ✓ 1. समाकलन की परिभाषा (Definition of Integration)
- ✓ 2. समाकलन के प्रकार (Types of Integration)
- ✓ 3. समाकलन से संबंधित सूत्र (Formula related to Integration)
- ✓ 4. प्रतिस्थापन द्वारा समाकलन (Integration by Substitution)
- ✓ 5. खण्डशः समाकलन (Integration by Parts)
- 6. आंशिक भिन्नों द्वारा समाकलन (Integration by partial fractions)
- 7. गामा फलन द्वारा समाकलन (Integration Using Gama Function)
- 8. समाकलन के अनुप्रयोग (Applications of Integration)

Q.37:- $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) dx$ का समाकलन ज्ञात करो।
Find the integral.

$$\text{माना } x = \tan \theta \Rightarrow \theta = \tan^{-1} x$$

$$= \int \cos^{-1}\left(\frac{1-\tan^2 \theta}{1+\tan^2 \theta}\right) \cdot dx$$

$$= \int \cos^{-1}(\cos 2\theta) \cdot dx$$

$$= \int 2\theta \cdot dx$$

$$= 2 \int \tan^{-1} x \cdot dx$$

$$= 2 \int \tan^{-1} x \cdot 1 dx$$

$$= 2 \left[\tan^{-1} x \int 1 dx - \int \left(\frac{\tan^{-1} x}{dx} \cdot \int 1 dx \right) dx \right]$$

$$= 2 \left[\tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x \cdot dx \right]$$

$$= 2x \cdot \tan^{-1} x - 2 \int \frac{x}{1+x^2} dx$$

$$\text{माना } 1+x^2 = t$$

d.w.r.t x

$$2x \cdot dx = dt$$

$$x \cdot dx = \frac{dt}{2}$$

$$= 2x \cdot \tan^{-1} x - 2 \int \frac{1}{t} \cdot \frac{dt}{2}$$

$$= 2x \cdot \tan^{-1} x - \frac{2}{2} \int \frac{1}{t} \cdot dt$$

$$= 2x \cdot \tan^{-1} x - \log_e t + C$$

$$= 2x \cdot \tan^{-1} x - \log_e (1+x^2) + C \quad \underline{\underline{\text{Ans}}}$$

Q.38:- $\int \frac{x \tan^1 x}{(1+x^2)^{3/2}} dx$ का समाकलन ज्ञात करो।
Find the integral.

माना $x = \tan \theta \Rightarrow dx = \sec^2 \theta \cdot d\theta$

$$= \int \frac{\tan \theta \cdot \tan^{-1}(\tan \theta)}{(1 + \tan^2 \theta)^{3/2}} \cdot \sec^2 \theta \cdot d\theta$$

$$= \int \frac{\tan \theta \cdot \theta}{(\sec^2 \theta)^{3/2}} \cdot \sec^2 \theta \cdot d\theta$$

$$= \int \frac{\theta \cdot \tan \theta}{\sec^3 \theta} \cdot \sec^2 \theta \cdot d\theta$$

$$= \int \frac{\theta \cdot \tan \theta}{\sec \theta} \cdot d\theta$$

$$= \int \frac{\theta \cdot \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta}} \cdot d\theta$$

$$= \int \theta \cdot \sin \theta \cdot d\theta$$

I II

$$= \theta \cdot \int \sin \theta \cdot d\theta - \int \left[\frac{d\theta}{d\theta} \cdot \int \sin \theta \cdot d\theta \right] d\theta$$

$$= -\theta \cdot \cos \theta - \int 1 \cdot (-\cos \theta) d\theta$$

$$= -\theta \cdot \cos \theta + \int \cos \theta \cdot d\theta$$

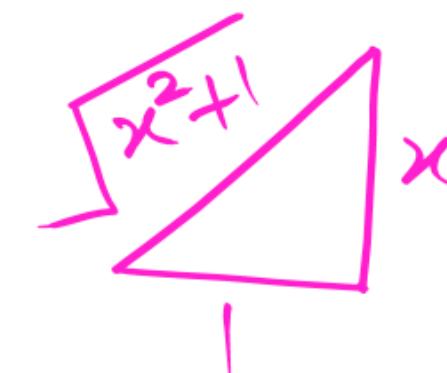
$$= -\theta \cdot \cos \theta + \sin \theta + C$$

$$= -\tan^{-1}x \cdot \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + C$$

$$= -\frac{\tan^{-1}x + x}{\sqrt{1+x^2}} + C \underline{\text{Ans}}$$

$$\therefore x = \tan \theta$$

$$\tan \theta = \frac{x}{1} = \frac{L}{A}$$



$$\sin \theta = \frac{x}{\sqrt{x^2 + 1}}$$

$$\cos \theta = \frac{1}{\sqrt{x^2 + 1}}$$

Q.39:- $\int \frac{x^2 \tan^{-1}x}{(1+x^2)} dx$ का समाकलन ज्ञात करो।
Find the integral.

माना $\tan^{-1}x = t \Rightarrow x = \tan t$
d.w.r.t x

$$\frac{1}{1+x^2} \cdot dx = dt$$

$$= \int x^2 \cdot t \cdot dt$$

$$= \int \tan^2 t \cdot t \cdot dt$$

$$\because 1 + \tan^2 t = \sec^2 t$$

$$\tan^2 t = \sec^2 t - 1$$

$$= \int (\sec^2 t - 1) \cdot t \cdot dt$$

$$= \int (t \cdot \sec^2 t - t) dt$$

$$= \int_{\text{I}} t \cdot \sec^2 t dt - \int_{\text{II}} t \cdot dt$$

$$= t \cdot \int \sec^2 t \cdot dt - \int \left[\frac{dt}{dt} \cdot \int \sec^2 t \cdot dt \right] dt - \frac{t^2}{2}$$

$$= t \cdot \tan t - \int \tan t \cdot dt - \frac{t^2}{2}$$

$$= t \cdot \tan t - \log \sec t - \frac{t^2}{2} + C$$

$$= \tan^{-1} x \cdot x - \log(\sqrt{1+x^2}) - \frac{(\tan^{-1} x)^2}{2} + C$$

$$= x \cdot \tan^{-1} x - \log(\sqrt{1+x^2}) - \frac{(\tan^{-1} x)^2}{2} + C \quad \underline{\underline{\text{Ans}}}$$

$$\therefore \tan^{-1} x = t$$

$$\& x = \tan t$$

$$\therefore 1 + \tan^2 t = \sec^2 t$$

$$1 + x^2 = \sec^2 t$$

$$\sqrt{1+x^2} = \sec t$$

Q.40:- $\int \frac{xe^x}{(1+x)^2} dx$ का समाकलन ज्ञात करो।
Find the integral.

$$= \int \frac{(x+1-1)e^x}{(1+x)^2} dx$$

$$= \int \frac{(x+1)e^x - e^x}{(1+x)^2} dx$$

$$= \int \left[\frac{(x+1)e^x}{(1+x)^2} - \frac{e^x}{(1+x)^2} \right] dx$$

$$= \int \frac{1}{(1+x)} \cdot e^x dx - \int \frac{e^x}{(1+x)^2} dx$$

I II

$$= \frac{1}{(1+x)} \int e^x dx - \int \left[\frac{d}{dx} \left(\frac{1}{1+x} \right) \cdot \int e^x dx \right] dx - \int \frac{e^x}{(1+x)^2} dx$$

$$= \frac{1}{1+x} \cdot e^x - \int \frac{-1}{(1+x)^2} \cdot e^x \cdot dx - \int \frac{e^x}{(1+x)^2} dx$$

$$= \frac{e^x}{1+x} + \int \frac{e^x}{(1+x)^2} dx - \int \frac{e^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C \quad \underline{\text{Ans}}$$

Q.41:- $\int \frac{e^x(x^2 + 1)}{(1 + x)^2} dx$

dx का समाकलन ज्ञात करो।
Find the integral.

Q.42:- $\int e^x \sin x \, dx$ का समाकलन ज्ञात करो।

Find the integral.

Q.43:- $\int e^x (\sin x + \cos x) dx$ का समाकलन ज्ञात करो।

Find the integral.