NEW

Semester - II

MATHEMATICS –II

UNIT

1 Determinants and Matrices (सारणिक तथा आव्यूह)

UNIT - I
Determinants and Matrices

Elementary properties of determinants upto 3rd order, consistency of equations, Crammer's rule.

Algebra of matrices, inverse of a matrix, matrix inverse method to solve a system of linear equations in three variables.

TOPICS

1. Determinant (सारणिक)
2. Rows and columns of a determinants (सारणिक की पंक्तियां तथा स्तम्भ)
3. Order of a determinant (सारणिक का क्रम)
4. Value of Determinant (सारणिक का मान)
5. Minor (उपसारणिक या लघुघटक)
6. Co-factor (सहखण्ड)
7. Properties of Determinant (सारणिक के गुणधर्म) Imp
8. Multiplication of two determinants (दो सारणिको का गुणनफल)
9. Crammer's rule (क्रैमर का नियम) Imp
10. Condition for Consistency (सुसंगत के प्रतिबन्ध) Imp
11. Condition of Collinearity of three points (तीन बिन्दुओं के संरेख होने का प्रतिवन्ध)

Qus:- In the determinant $\begin{vmatrix} 8 & 7 & 5 \\ 12 & 4 & 6 \\ 2 & 9 & 3 \end{vmatrix}$, find minors of 7 and 3 and cofactors of 5 and 9.

(Home work)

7 का minor $= \begin{vmatrix} 12 & 6 \\ 2 & 3 \end{vmatrix} = 36 - 12 = \underline{\underline{24}}$ Ans

3 का minor $= \begin{vmatrix} 8 & 7 \\ 12 & 4 \end{vmatrix} = 32 - 84 = \underline{\underline{-52}}$ Ans

5 का cofactor $= (-1)^{1+3} \begin{vmatrix} 12 & 4 \\ 2 & 9 \end{vmatrix} = +(108 - 8) = 100$ Ans

9 का cofactor $= (-1)^{3+2} \begin{vmatrix} 8 & 5 \\ 12 & 6 \end{vmatrix} = -(48 - 60) = -(-12) = \underline{\underline{12}}$ Ans

Qus:- Prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$ (U.P. Diploma 1984)

$R_1 \longrightarrow R_1 - R_2$

$$\begin{vmatrix} 0 & a-b & a^2-b^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_2 \longrightarrow R_2 - R_3$ $\begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$

R_1 में से $(a-b)$ तथा R_2 में से $(b-c)$ common लेने पर

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

C_1 के अनुदिश प्रसार

$$= (a-b)(b-c)\left[1\left(\cancel{b}+c-a-\cancel{b}\right)\right]$$

$$= (a-b)(b-c)(c-a) \quad \text{Proved}$$

Qus:- Prove that the determinant

(U.P. Diploma 2013)

$$\begin{array}{c c} R_1 \rightarrow & \\ R_2 \rightarrow & \\ R_3 \rightarrow & \end{array}\!\!\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

$R_1 \rightarrow R_1 - R_2$

$$\begin{vmatrix} x-y & x^2-y^2 & yz-zx \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$$

$R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} (x-y) & (x^2-y^2) & yz-zx \\ y-z & y^2-z^2 & zx-xy \\ z & z^2 & xy \end{vmatrix}$$

R_1 में से $(x-y)$ & R_2 में से $(y-z)$ common लेने पर

$$(x-y)(y-z)\begin{vmatrix} 1 & (x+y) & -z \\ 1 & (y+z) & -x \\ z & z^2 & xy \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$= (x-y)(y-z)\begin{vmatrix} 0 & x+y-y-z & -z+x \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$$

$$= (x-y)(y-z)\begin{vmatrix} 0 & x-z & x-z \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$$

$C_2 \rightarrow C_2 - C_3$

$$= (x-y)(y-z)\begin{vmatrix} 0 & 0 & x-z \\ 1 & y+z+x & -x \\ z & z^2-xy & xy \end{vmatrix}$$

R_1 के अनुदिश प्रसार

$$= (x-y)(y-z)\left[0-0+(x-z)\left(\cancel{z^2}-xy-yz-\cancel{z^2}-xz\right)\right]$$

$$=(x-y)(y-z)\left[(x-z)(-xy-yz-xz)\right]$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx)$$

Proved!

Qus:- Without expanding, prove that

(U.P. Diploma 2007)

$$\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$= \begin{vmatrix} 0 & x-y & y+z-z-x \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$$

$R_2 - R_3$ $$= \begin{vmatrix} 0 & x-y & y-x \\ 0 & y-z & z+x-x-y \\ 1 & z & x+y \end{vmatrix}$$

$$= \begin{vmatrix} 0 & x-y & -(x-y) \\ 0 & y-z & -(y-z) \\ 1 & z & x+y \end{vmatrix}$$

R_1 में से $(x-y)$ & R_2 में से $(y-z)$ common

$$= (x-y)(y-z) \begin{vmatrix} 0 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & z & x+y \end{vmatrix}$$

C_1 के अनुदिश प्रसार

$$= (x-y)(y-z)\left[0-0+1\left(-1+1\right)\right]$$

$$= (x-y)(y-z)(0)$$

$$= 0$$ Proved!

Qus:- Find the roots of the equation (H.W.)

$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$$

$x = 2, -1$ Ans

Qus:- Solve the equation $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} (x+a+b+c) & b & c \\ (x+a+b+c) & x+b & c \\ (x+a+b+c) & b & x+c \end{vmatrix} = 0$$

$$= (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

$R_1 \to R_1 - R_3$

$$(x+a+b+c) \begin{vmatrix} 0 & 0 & \cancel{c}-x-\cancel{c} \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

R_1 के अनुदिश प्रसार

$$(x+a+b+c)\left[-x\left(\cancel{b}-x-\cancel{b}\right)\right] = 0$$

$$(x+a+b+c)\left(x^2\right) = 0$$

$\boxed{x=0}$ $\boxed{x=-(a+b+c)}$

Ans