

NEW

Semester - II

MATHEMATICS -II

UNIT

2

Integral Calculus (समाकलन गणित)

Note:- If $k = \text{constant}$

$$\int k \cdot f(x) dx = k \cdot \int f(x) dx + c$$

$$\int 1 \cdot dx = x + c$$

$$l = x^0$$

$$\int x^0 \cdot dx = \frac{x^{0+1}}{0+1} = \cancel{x}$$

Q.1:- $\int (2x^3 + 5x + 6) dx$ का x के सापेक्ष समाकलन ज्ञात करो।

Find the integral of $\int (2x^3 + 5x + 6) dx$ with respect to x.

$$\begin{aligned} & \int (2x^3 + 5x + 6) dx \\ &= \int 2x^3 dx + \int 5x \cdot dx + \int 6 dx \\ &= 2 \int x^3 dx + 5 \int x \cdot dx + 6 \int 1 \cdot dx \\ &= 2 \left(\frac{x^{3+1}}{3+1} \right) + 5 \left(\frac{x^{1+1}}{1+1} \right) + 6(x) + C \\ &= \frac{x^4}{2} + \frac{5 \cdot x^2}{2} + 6x + C \quad \underline{\text{Ans}} \end{aligned}$$

Q.2:- $\int \frac{(1-x^2)^3}{x^2} dx$ का x के सापेक्ष समाकलन ज्ञात करो।

Find the integral of with respect to x.

$$\begin{aligned} & \int \frac{(1-x^2)^3}{x^2} dx \\ &= \int \frac{1-x^6-3x^4x^2(1-x^2)}{x^2} dx \\ &= \int \left[\frac{1}{x^2} - \frac{x^6}{x^2} - \frac{3x^2(1-x^2)}{x^2} \right] dx \end{aligned}$$

$$\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$\begin{aligned} &= \int \left(\frac{1}{x^2} - x^4 - 3 + 3x^2 \right) dx \\ &= \int (x^{-2} - x^4 - 3 + 3x^2) dx \\ &= \frac{x^{-2+1}}{-2+1} - \frac{x^{4+1}}{4+1} - 3x + 3 \cdot \frac{x^{2+1}}{2+1} + C \\ &= -\frac{1}{x} - \frac{x^5}{5} - 3x + x^3 + C \quad \underline{\text{Ans}} \end{aligned}$$

Q.3:- $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$ का x के सापेक्ष समाकलन ज्ञात करो।

Find the integral of with respect to x.

$$= \int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$= \int \left[(\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 + 2 \cdot \sqrt{x} \cdot \frac{1}{\sqrt{x}} \right] dx$$

$$= \int \left[x + \frac{1}{x} + 2 \right] dx$$

$$= \frac{x^2}{2} + \log x + 2x + C \quad \underline{\text{Ans}}$$

Q.4:- $\int \frac{5}{\sin^2 x \cdot \cos^2 x} dx$

dx का x के सापेक्ष समाकलन ज्ञात करो।

Find the integral of with respect to x.

$$= \int \frac{4x \cdot 5}{4x \sin^2 x \cdot \cos^2 x} dx$$

$$= \int \frac{20}{(2\sin x \cdot \cos x)^2} dx$$

$$= 20 \int \frac{1}{(\sin 2x)^2} dx$$

$$= 20 \int \operatorname{cosec}^2 2x \cdot dx$$

$$= 20 \left(-\frac{\cot 2x}{2} \right) + C$$

$$= -10 \cdot \underline{\underline{\cot 2x}} + \underline{\underline{C}}$$

Ans

$$\int \operatorname{cosec}^2 x dx = -\cot x$$

Note:- समाकलन में
x के गुणांक से भ्रग
कर दी जाती है।

Method-II

$$\int \frac{5}{\sin^2 x \cdot \cos^2 x} dx$$

$$= 5 \int \frac{1}{\sin^2 x \cdot \cos^2 x} dx$$

$$= 5 \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cdot \cos^2 x} dx$$

$$= 5 \left[\frac{\cancel{\sin^2 x}}{\sin^2 x \cdot \cos^2 x} + \frac{\cancel{\cos^2 x}}{\sin^2 x \cdot \cos^2 x} \right] dx$$

$$= 5 \int (\sec^2 x + \operatorname{cosec}^2 x) dx$$

$$= 5 (\tan x - \cot x) + C \quad \underline{\text{Ans}}$$

Q.5:- $\int \left(\frac{1 - \tan x}{1 + \tan x} \right) dx$ का x के सापेक्ष समाकलन ज्ञात करो।
 Find the integral of with respect to x.

$$= \int \frac{\tan 45^\circ - \tan x}{1 + \tan 45^\circ \cdot \tan x} \cdot dx \quad ; \because \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} = \tan(A - B)$$

$$= \int \tan(45^\circ - x) dx \quad \int \tan x \cdot dx = \log \sec x.$$

$$= \frac{\log \sec(45^\circ - x)}{-1} + C = -\log \sec(45^\circ - x) + C \quad \underline{\text{Ans}}$$

चरघातांकी फलनों पर आधारित प्रश्न

Questions based on exponential functions

$$\text{Q.6:- } \int \frac{(e^{7\log_e x} - e^{6\log_e x})}{(e^{5\log_e x} - e^{4\log_e x})} dx$$

$$= \int \frac{e^{\log_e x^7} - e^{\log_e x^6}}{e^{\log_e x^5} - e^{\log_e x^4}} dx$$

$$= \int \frac{x^7 - x^6}{x^5 - x^4} dx$$

$$\textcircled{1} \quad \log m^n = n \cdot \log m$$

$$\textcircled{2} \quad e^{\log_e x} = x$$

$$\int \frac{x^6(x-1)}{x^4(x-1)} dx$$

$$\int x^3 \cdot dx = \frac{x^3}{3} + C \quad \underline{\text{Ans}}$$

Q.7:- यदि $f'(x) = 3x^2 - \frac{2}{x^3}$ तथा $f(1) = 0$ तो $f(x)$ का मान ज्ञात करो।
If $f(1) = 0$ then find the value of $f(x)$.

$$f'(x) = 3x^2 - \frac{2}{x^3}$$

दोनों ओर x के सापेक्ष समाकलन

$$\int f'(x) dx = \int \left(3x^2 - \frac{2}{x^3}\right) dx$$

$$f(x) = 3 \cdot \frac{x^3}{3} - 2 \cdot \frac{x^{-3+1}}{-3+1} + C$$

$$f(x) = x^3 + \frac{1}{x^2} + C \quad \text{--- } ①$$

$f'(x)$ का अर्थ $\rightarrow f(x)$ का अवकलन (differential)

$$\frac{d f(x)}{dx} = f'(x) \text{ तो } \int f'(x) dx = f(x)$$

समी ① में $x=1$ रखने पर

$$f(1) = 1 + \frac{1}{1} + C$$

$$0 = 2 + C$$

$C = -2$ Put in Eq ①

$$f(x) = x^3 + \frac{1}{x^2} - 2 \quad \text{Ans}$$

Q.8:- यदि $f'(x) = 4x^3 - \frac{3}{x^4}$ तथा $f(2) = 0$ तो $f(x)$ का मान ज्ञात करो।

If $f(2) = 0$ then find the value of $f(x)$.

(H.W.)