NEW

Semester - II

MATHEMATICS –II

UNIT

2 Integral Calculus (समाकलन गणित)

UNIT-II
Integral Calculus

UNIT - II: Integral Calculus **(12 periods)**

Integration as inverse operation of differentiation. Simple integration by substitution, by parts and by partial fractions (for linear factors only). Introduction to definite integration. Use of formulae $\int_0^{\frac{\pi}{2}} \sin^n x dx, \int_0^{\frac{\pi}{2}} \cos^n x dx, \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$ for solving problems ,where m and n are positive integers. Applications of integration for (i). Simple problems on evaluation of area bounded by a curve and axes. (ii). calculation of volume of a solid formed by revolution of an area about axes. (Simple problems).

TOPICS

1. समाकलन की परिभाषा (Definition of Integration)
2. समाकलन के प्रकार (Types of Integration)
3. समाकलन से संबंधित सूत्र (Formula related to Integration)
4. प्रतिस्थापन द्वारा समाकलन (Integration by Substitution)
5. खण्डश: समाकलन (Integration by Parts)
6. आंशिक भिन्नों द्वारा समाकलन (Integration by partial fractions)
7. गामा फलन द्वारा समाकलन (Integration Using Gama Function)
8. समाकलन के अनुप्रयोग (Applications of Integration)

Integration (समाकलन) :- → जोड़ना (Sum) → S

Symbol (चिन्ह) :- S (Sum) की Elongated form (विकृत रूप) $\int$

समाकलन, अवकलन की विपरीत प्रक्रिया है।

(Reverse Process of differentiation is called integration)

यदि $F(x)$ function है तो

$$\frac{dF(x)}{dx} = f(x) \text{ तो } \int f(x).dx = F(x) + C$$

जैसे $\frac{d(\log_e x)}{dx} = \frac{1}{x}$

$$\int \frac{1}{x}.dx = \log_e x + C$$

Integral & Integration.

(समाकल) (समाकलन)

समाकल्य (Integrand)

$$\int f(x) \cdot dx = F(x) + c$$

समाकल्य (Integrand)

समाकल (Integral)

Process (प्रक्रिया) ⇒ समाकलन (integration)

समाकलन(Integration)

Definition:-

- समाकलन, अवकलन की विपरीत क्रिया है
(Integration is reverse process of differentiation)

if $F(x)$ is a function.

$$\frac{d\,F(x)}{dx} = f(x) \quad \text{तो} \quad \int f(x).dx = F(x)+c$$

c → समाकलन स्थिरांक (integration constant)

समाकल्य (Integrand)- जिस फलन का समाकलन करते हैं उसे समाकल्य (Integrand) कहते हैं।
(The function which is integrated is called integral.)

समाकल (Integral)- समाकलन की प्रक्रिया से प्राप्त फलन को समाकल (integral) कहते हैं।
(The function obtained from the process of integration is called integral.)

समाकलन (integration)-

समाकलन की प्रक्रिया से प्राप्त फलन को समाकल (integral) कहते हैं। जिस प्रक्रिया (process) द्वारा समाकल प्राप्त होता है उसे समाकलन (integration) कहते हैं।

The function obtained from the process of integration is called integral.
The process by which the integral is obtained is called integration.

समाकलन के प्रकार (Types of Integration)

समाकलन दो प्रकार के होते हैं-

(i) अनिश्चित समाकलन (Indefinite integration) $\rightarrow \int f(x)\cdot dx = F(x) + c$

(ii) निश्चित समाकलन (Definite integration) $\rightarrow \int_a^b f(x)\cdot dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$

Where a = Lower Limit (निम्न सीमा)
b = Upper Limit (उच्च सीमा)

जिसमें Limits का use करते हैं → निश्चित समाकलन (Definite integration)
जिसमें Limits का use नहीं करते → अनिश्चित समाकलन (Indefinite integration)

समाकलन से संबंधित सूत्र (Formula related to Integration)

① $\frac{dx^n}{dx} = n \cdot x^{n-1}$

② $\frac{d \log_e x}{dx} = \frac{1}{x}$

③ $\frac{de^x}{dx} = e^x$

④ $\frac{da^x}{dx} = a^x \cdot \log_e a$

① $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$

② $\int \frac{1}{x} \cdot dx = \log_e x + c$

③ $\int e^x \cdot dx = e^x + c$

④ $\int a^x \cdot dx = \frac{a^x}{\log_e a} + c$

⑤ $\frac{d\sin x}{dx} = \cos x$

⑥ $\frac{d\cos x}{dx} = -\sin x$

⑦ $\frac{d\tan x}{dx} = \sec^2 x$

⑧ $\frac{d\cot x}{dx} = -\operatorname{cosec}^2 x$

⑨ $\frac{d\sec x}{dx} = \sec x \cdot \tan x$

⑩ $\frac{d\operatorname{cosec} x}{dx} = -\operatorname{cosec} x \cdot \cot x$

⑤ $\int \sin x \cdot dx = -\cos x + c$

⑥ $\int \cos x \cdot dx = \sin x + c$

⑦ $\int \sec^2 x \cdot dx = \tan x + c$

⑧ $\int \operatorname{cosec}^2 x \cdot dx = -\cot x + c$

⑨ $\int \sec x \cdot \tan x \cdot dx = \sec x + c$

⑩ $\int \operatorname{cosec} x \cdot \cot x \cdot dx = -\operatorname{cosec} x + c$

(11) $\int \tan x \cdot dx = \log_e \sec x + c$

(12) $\int \cot x \cdot dx = \log_e \sin x + c$

(13) $\int \sec x \cdot dx = \log_e (\sec x + \tan x) + c$

(14) $\int \operatorname{cosec} x \cdot dx = \log_e (\operatorname{cosec} x - \cot x) + c$

(15) $\frac{d \sin^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}$

(16) $\frac{d \tan^{-1} x}{dx} = \frac{1}{1+x^2}$

(17) $\frac{d \sec^{-1} x}{dx} = \frac{1}{x\sqrt{x^2-1}}$

(15) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c = -\cos^{-1} x + c$

(16) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c = -\cot^{-1} x + c$

(17) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c = -\operatorname{cosec}^{-1} x + c$