James Hippler (HipplerJ)
Oregon State University
CS 325-400 (Spring 2018)
Analysis of Algorithms

Homework 6

Due Date: Sunday, May 20, 2018

1. Shortest Paths using LP: **(7 points)**

Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- We can compute the shortest path from s to t in a weighted directed graph by solving.

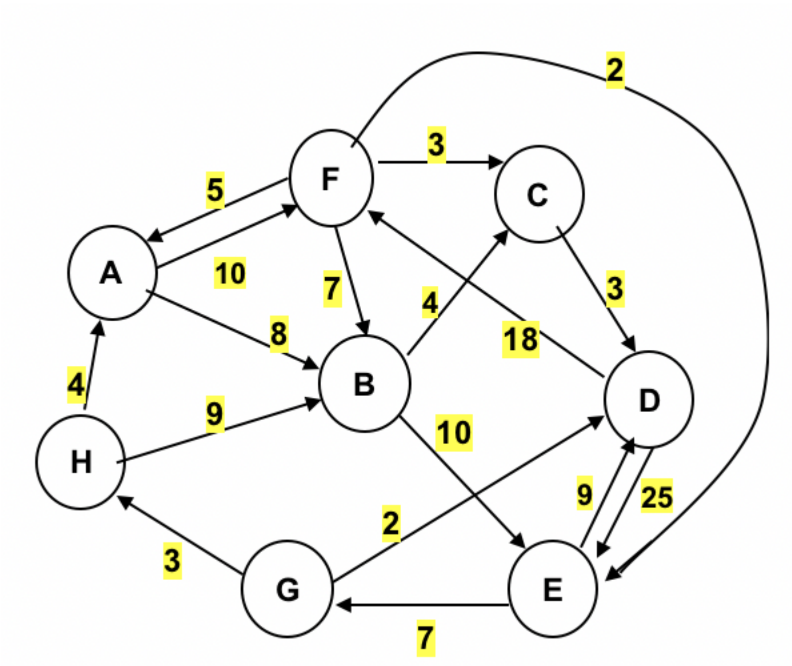
  max dt
  subject to
  ds = 0
  dv – du ≤ w(u,v) for all (u,v)∈E

- We can compute the single-source by changing the objective function to

  max $\sum_{v \in V} dv$

Use linear programming to answer the questions below. Submit a copy of the LP code and output.

a) Find the distance of the shortest path from G to C in the graph below.
b) Find the distances of the shortest paths from G to all other vertices.



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ANSWERS:

a) **The distance of the shortest path from G to C in the graph above is 16.**
Shortest Path Distance: 16 ($G \rightarrow H \rightarrow B \rightarrow C$)

LINDO Code:

```
max c
ST
          g = 0
          d - g <= 2
          h - g <= 3
          a - h <= 4
          b - h <= 9
          f - d <= 18
          e - d <= 25
          c - b <= 4
          e - b <= 10
          b - a <= 8
          f - a <= 10
          c - f <= 3
          a - f <= 5
          b - f <= 7
          e - f <= 2
          g - e <= 7
          d - e <= 9
          d - c <= 3
END
```

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Results:

```
LP OPTIMUM FOUND AT STEP      0

      OBJECTIVE FUNCTION VALUE

      1)      16.00000

 VARIABLE        VALUE          REDUCED COST
        C        16.000000          0.000000
        G         0.000000          0.000000
        D         2.000000          0.000000
        H         3.000000          0.000000
        A         7.000000          0.000000
        B        12.000000          0.000000
        F        17.000000          0.000000
        E        19.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         0.000000          0.000000
       4)         0.000000          1.000000
       5)         0.000000          0.000000
       6)         0.000000          1.000000
       7)         3.000000          0.000000
       8)         8.000000          0.000000
       9)         0.000000          1.000000
      10)         3.000000          0.000000
      11)         3.000000          0.000000
      12)         0.000000          0.000000
      13)         4.000000          0.000000
      14)        15.000000          0.000000
      15)        12.000000          0.000000
      16)         0.000000          0.000000
      17)        26.000000          0.000000
      18)        26.000000          0.000000
      19)        17.000000          0.000000

NO. ITERATIONS=       0
```

b) The distance of the shortest paths from G to all other vertices are listed below

- $\boldsymbol{G \rightarrow A = 7\ (3 + 4)}$
- $\boldsymbol{G \rightarrow B = 12\ (3 + 9)}$
- $\boldsymbol{G \rightarrow C = 16\ (3 + 9 + 4)}$
- $\boldsymbol{G \rightarrow D = 2\ (2)}$
- $\boldsymbol{G \rightarrow E = 19\ (3 + 4 + 10 + 2)}$
- $\boldsymbol{G \rightarrow F = 17\ (3 + 4 + 10)}$
- $\boldsymbol{G \rightarrow H = 3\ (3)}$

LINDO Code:

```
max a + b + c + d + e + f + h
ST
        g = 0
        d - g <= 2
        h - g <= 3
        a - h <= 4
        b - h <= 9
        f - d <= 18
        e - d <= 25
        c - b <= 4
        e - b <= 10
        b - a <= 8
        f - a <= 10
        c - f <= 3
        a - f <= 5
        b - f <= 7
        e - f <= 2
        g - e <= 7
        d - e <= 9
        d - c <= 3
END
```

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Oregon State University

Results:

```
LP OPTIMUM FOUND AT STEP       0

       OBJECTIVE FUNCTION VALUE

       1)      76.00000

 VARIABLE        VALUE          REDUCED COST
        A         7.000000          0.000000
        B        12.000000          0.000000
        C        16.000000          0.000000
        D         2.000000          0.000000
        E        19.000000          0.000000
        F        17.000000          0.000000
        H         3.000000          0.000000
        G         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          7.000000
       3)         0.000000          1.000000
       4)         0.000000          6.000000
       5)         0.000000          3.000000
       6)         0.000000          2.000000
       7)         3.000000          0.000000
       8)         8.000000          0.000000
       9)         0.000000          1.000000
      10)         3.000000          0.000000
      11)         3.000000          0.000000
      12)         0.000000          2.000000
      13)         4.000000          0.000000
      14)        15.000000          0.000000
      15)        12.000000          0.000000
      16)         0.000000          1.000000
      17)        26.000000          0.000000
      18)        26.000000          0.000000
      19)        17.000000          0.000000

NO. ITERATIONS=       0
```

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Oregon State University

2. Product Mix: **(7 points)**
Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	**Cost per yard**	**Yards available per month**
Silk	$20	1,000
Polyester	$6	2,000
Cotton	$9	1,250

Product Information	**Type of Tie**			
	Silk = s	**Poly = p**	**Blend1 = b**	**Blend2 = c**
Selling Price per tie	$6.70	$3.55	$4.31	$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	**Type of Tie**			
	Silk	**Polyester**	**Blend 1 (50/50)**	**Blend 2 (30/70)**
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal numbers of ties of each type to maximize profit?

ANSWER:

Maximum Profit: $120,196.00

Optimal Numbers of Ties of Each Type

- **Silk Ties: 7,000**
- **Polyester Ties: 13,625**
- **Blend 1 (50/50) Ties: 13,100**
- **Blend 2 (30/70) Ties: 8,500**

LINDO Code:

```
max
3.45 s + 2.32 p + 2.81 b1 + 3.25 b2
ST
        0.125 s <= 1000
        0.08 p + 0.05 b1 + 0.03 b2 <= 2000
        0.05 b1 + 0.07 b2 <= 1250

        s >= 6000
        s <= 7000

        p >= 10000
        p <= 14000

        b1 >= 13000
        b1 <= 16000

        b2 >= 6000
        b2 <= 8500
END
```

Results:

```
LP OPTIMUM FOUND AT STEP      0

      OBJECTIVE FUNCTION VALUE

      1)       120196.0

 VARIABLE        VALUE          REDUCED COST
        S      7000.000000          0.000000
        P     13625.000000          0.000000
       B1     13100.000000          0.000000
       B2      8500.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)       125.000000          0.000000
       3)         0.000000         29.000000
       4)         0.000000         27.200001
       5)      1000.000000          0.000000
       6)         0.000000          3.450000
       7)      3625.000000          0.000000
       8)       375.000000          0.000000
       9)       100.000000          0.000000
      10)      2900.000000          0.000000
      11)      2500.000000          0.000000
      12)         0.000000          0.476000

NO. ITERATIONS=       0
```

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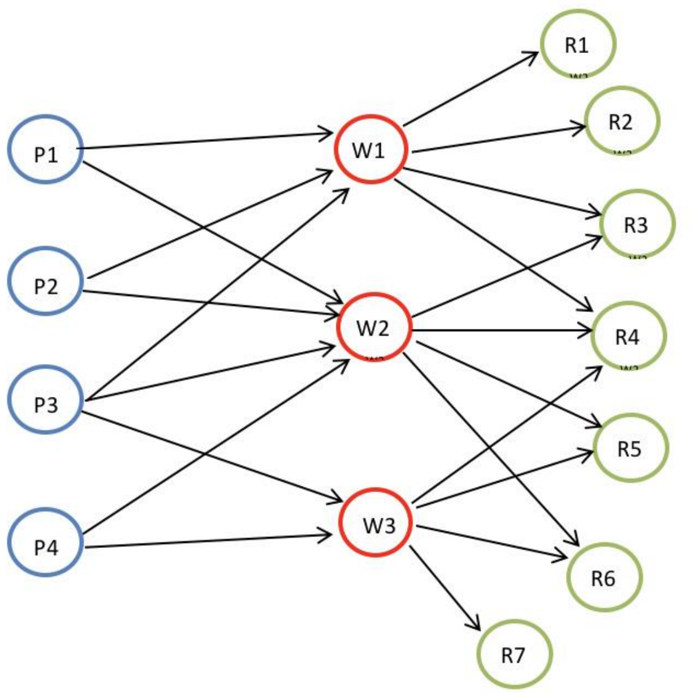
Oregon State University

3. Transshipment Model **(10 points)**

This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (p_i) must be shipped to a Warehouse (w_j) before being shipped to the Retailer (r_k). Each Plant will have an associated supply (s_i) and each Retailer will have a demand (d_k). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i,j) from plant (p_i)to warehouse (w_j) have costs associated denoted cp(i,j). The edges (j,k) from a warehouse (w_j)to a retailer (r_k) have costs associated denoted cw(j,k).

The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



Below are the costs of shipping from a plant to a warehouse and then a warehouse to a retailer. If it is impossible to ship between the two locations an X is placed in the table.

cost	W1	W2	W3
P1	$10	$15	X
P2	$11	$8	X
P3	$13	$8	$9
P4	X	$14	$8

cost	R1	R2	R3	R4	R5	R6	R7
W1	$5	$6	$7	$10	X	X	X
W2	X	X	$12	$8	$10	$14	X
W3	X	X	X	$14	$12	$12	$6

The tables below give the capacity of each plant (supply) and the demand for each retailer (per week).

	P1	P2	P3	P4
Supply	150	450	250	150

	R1	R2	R3	R4	R5	R6	R7
Demand	100	150	100	200	200	150	100

Part A: Determine the number of refrigerators to be shipped from the plants to the warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost?

Part B: Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes. What is the optimal solution for this modified model? Is it feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not?

Part C: Instead of closing Warehouse 2 management has decide to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so what is the optimal solution when warehouse 2 is limited to 100 refrigerators?

<u>Note:</u> *Include a copy of the code for all parts of the problem.*

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ANSWERS:

PART A

Minimum Cost: $17,100.00

Optimal Shipping Routes

$\boldsymbol{P1 \rightarrow W1 = 150}$
$\boldsymbol{P2 \rightarrow W1 = 200}$
$\boldsymbol{P2 \rightarrow W2 = 250}$
$\boldsymbol{P3 \rightarrow W2 = 150}$
$\boldsymbol{P3 \rightarrow W3 = 100}$
$\boldsymbol{P4 \rightarrow W3 = 150}$
$\boldsymbol{W1 \rightarrow R1 = 100}$
$\boldsymbol{W1 \rightarrow R2 = 150}$
$\boldsymbol{W1 \rightarrow R3 = 100}$
$\boldsymbol{W2 \rightarrow R4 = 200}$
$\boldsymbol{W2 \rightarrow R5 = 200}$

LINDO Code:

```
min
10p1w1 + 15p1w2 +
11p2w1 + 8p2w2 +
13p3w1 + 8p3w2 + 9p3w3 +
14p4w2 + 8p4w3 +
5w1r1 + 6w1r2 + 7w1r3 + 10w1r4 +
12w2r3 + 8w2r4 + 10w2r5 + 14w2r6 +
14w3r4 + 12w3r5 +  12w3r6 + 6w3r7

ST
        p1w1 + p1w2 <= 150
        p2w1 + p2w2 <= 450
        p3w1 + p3w2 + p3w3 <= 250
        p4w2 + p4w3 <= 150

        w1r1 >= 100
        w1r2 >= 150
        w1r3 + w2r3 >= 100
        w1r4 + w2r4 + w3r4 >= 200
        w2r5 + w3r5 >= 200
        w2r6 + w3r6 >= 150
        w3r7 >= 100

        w1r1 + w1r2 + w1r3 + w1r4 - p1w1 - p2w1 - p3w1 = 0
        w2r3 + w2r4 + w2r5 + w2r6 - p1w2 - p2w2 - p3w2 - p4w2 = 0
        w3r4 + w3r5 + w3r6 + w3r7 - p3w3 - p4w3 = 0

END
```

James Hippler (HipplerJ)
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Results:

```
LP OPTIMUM FOUND AT STEP      13

       OBJECTIVE FUNCTION VALUE

       1)      17100.00

 VARIABLE        VALUE          REDUCED COST
     P1W1       150.000000          0.000000
     P1W2         0.000000          8.000000
     P2W1       200.000000          0.000000
     P2W2       250.000000          0.000000
     P3W1         0.000000          2.000000
     P3W2       150.000000          0.000000
     P3W3       100.000000          0.000000
     P4W2         0.000000          7.000000
     P4W3       150.000000          0.000000
     W1R1       100.000000          0.000000
     W1R2       150.000000          0.000000
     W1R3       100.000000          0.000000
     W1R4         0.000000          5.000000
     W2R3         0.000000          2.000000
     W2R4       200.000000          0.000000
     W2R5       200.000000          0.000000
     W2R6         0.000000          1.000000
     W3R4         0.000000          7.000000
     W3R5         0.000000          3.000000
     W3R6       150.000000          0.000000
     W3R7       100.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         0.000000          0.000000
       4)         0.000000          0.000000
       5)         0.000000          1.000000
       6)         0.000000        -16.000000
       7)         0.000000        -17.000000
       8)         0.000000        -18.000000
       9)         0.000000        -16.000000
      10)         0.000000        -18.000000
      11)         0.000000        -21.000000
      12)         0.000000        -15.000000
      13)         0.000000         11.000000
      14)         0.000000          8.000000
      15)         0.000000          9.000000

NO. ITERATIONS=      13
```

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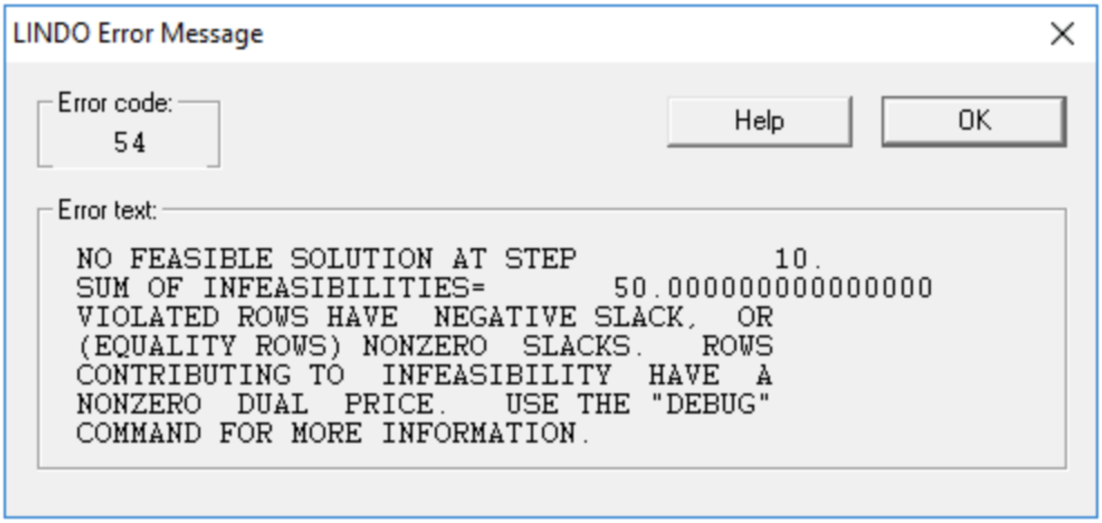
PART B

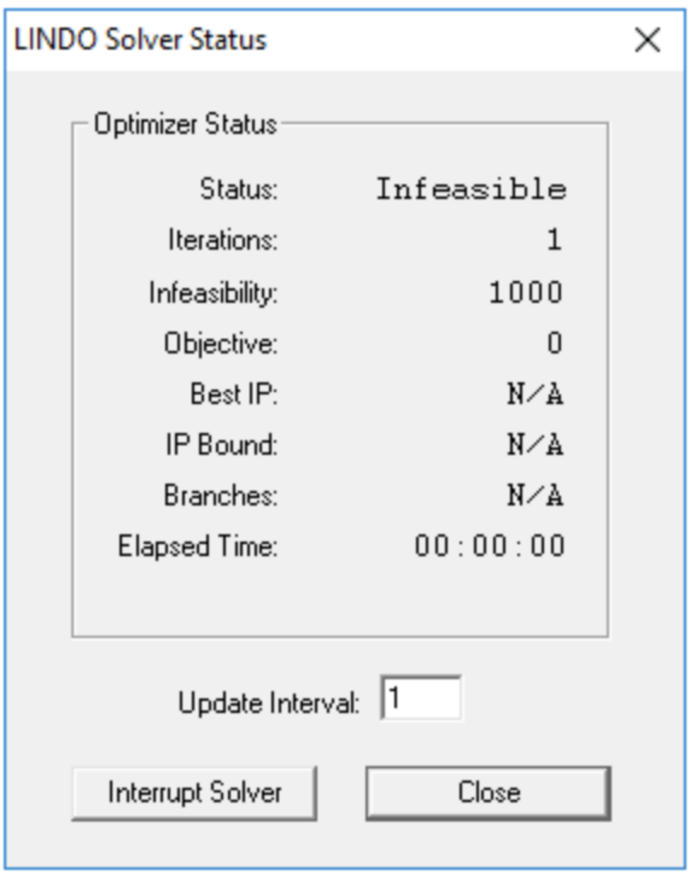
It is NOT feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2. Closing Warehouse 2 and eliminating all of the associated routes generates a scenario where the constraints do not allow for an applicable solution. This is attributed to the requirement that a minimum of 450 refrigerators be shipped to retailers r4, r5, r6, and r7 from warehouse 3. Unfortunately, the only manufacturing plants with connections to warehouse 3 (p3 and p4) can only produce a maximum of 400 refrigerators. This results in a deficit of 50 refrigerators from warehouse 3 to its associated retailers. When executing the code below in LINDO, we're greeted with an error message that says "No Feasible solution…Violated rows have negative slack"

LINDO Code:

```
min
10p1w1 +
11p2w1 +
13p3w1 + 9p3w3 +
8p4w3 +
5w1r1 + 6w1r2 + 7w1r3 + 10w1r4 +
14w3r4 + 12w3r5 + 12w3r6 + 6w3r7

ST
        p1w1 <= 150
        p2w1 <= 450
        p3w1 + p3w3 <= 250
        p4w3 <= 150

        w1r1 >= 100
        w1r2 >= 150
        w1r3 >= 100
        w1r4 + w3r4 >= 200
        w3r5 >= 200
        w3r6 >= 150
        w3r7 >= 100

        w1r1 + w1r2 + w1r3 + w1r4 - p1w1 - p2w1 - p3w1 = 0
        w3r4 + w3r5 + w3r6 + w3r7 - p3w3 - p4w3 = 0
END
```

Results:


LINDO Error Message
Error code:
54
Help
OK
Error text:
NO FEASIBLE SOLUTION AT STEP 10.
SUM OF INFEASIBILITIES= 50.000000000000000
VIOLATED ROWS HAVE NEGATIVE SLACK, OR
(EQUALITY ROWS) NONZERO SLACKS. ROWS
CONTRIBUTING TO INFEASIBILITY HAVE A
NONZERO DUAL PRICE. USE THE "DEBUG"
COMMAND FOR MORE INFORMATION.


LINDO Solver Status
Optimizer Status
Status: Infeasible
Iterations: 1
Infeasibility: 1000
Objective: 0
Best IP: N/A
IP Bound: N/A
Branches: N/A
Elapsed Time: 00:00:00
Update Interval: 1
Interrupt Solver
Close

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PART C

If we limit the plant shipments to warehouse 2 to a maximum of 100 refrigerator the optimal minimum cost is $18,300.00. LINDO Code and corresponding results have been included below.

LINDO Code:

```
min
10p1w1 + 15p1w2 +
11p2w1 + 8p2w2 +
13p3w1 + 8p3w2 + 9p3w3 +
14p4w2 + 8p4w3 +
5w1r1 + 6w1r2 + 7w1r3 + 10w1r4 +
12w2r3 + 8w2r4 + 10w2r5 + 14w2r6 +
14w3r4 + 12w3r5 +  12w3r6 + 6w3r7

ST
        p1w1 + p1w2 <= 150
        p2w1 + p2w2 <= 450
        p3w1 + p3w2 + p3w3 <= 250
        p4w2 + p4w3 <= 150

        p1w2 + p2w2 + p3w2 + p4w2 <= 100

        w1r1 >= 100
        w1r2 >= 150
        w1r3 + w2r3 >= 100
        w1r4 + w2r4 + w3r4 >= 200
        w2r5 + w3r5 >= 200
        w2r6 + w3r6 >= 150
        w3r7 >= 100

        w1r1 + w1r2 + w1r3 + w1r4 - p1w1 - p2w1 - p3w1 = 0
        w2r3 + w2r4 + w2r5 + w2r6 - p1w2 - p2w2 - p3w2 - p4w2 = 0
        w3r4 + w3r5 + w3r6 + w3r7 - p3w3 - p4w3 = 0
END
```

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Oregon State University

Results:

```
LP OPTIMUM FOUND AT STEP     15

       OBJECTIVE FUNCTION VALUE

       1)      18300.00

 VARIABLE        VALUE          REDUCED COST
     P1W1       150.000000          0.000000
     P1W2         0.000000          8.000000
     P2W1       350.000000          0.000000
     P2W2       100.000000          0.000000
     P3W1         0.000000          4.000000
     P3W2         0.000000          2.000000
     P3W3       250.000000          0.000000
     P4W2         0.000000          9.000000
     P4W3       150.000000          0.000000
     W1R1       100.000000          0.000000
     W1R2       150.000000          0.000000
     W1R3       100.000000          0.000000
     W1R4       150.000000          0.000000
     W2R3         0.000000          7.000000
     W2R4        50.000000          0.000000
     W2R5        50.000000          0.000000
     W2R6         0.000000          4.000000
     W3R4         0.000000          4.000000
     W3R5       150.000000          0.000000
     W3R6       150.000000          0.000000
     W3R7       100.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         0.000000          0.000000
       4)         0.000000          2.000000
       5)         0.000000          3.000000
       6)         0.000000          5.000000
       7)         0.000000        -16.000000
       8)         0.000000        -17.000000
       9)         0.000000        -18.000000
      10)         0.000000        -21.000000
      11)         0.000000        -23.000000
      12)         0.000000        -23.000000
      13)         0.000000        -17.000000
      14)         0.000000         11.000000
      15)         0.000000         13.000000
      16)         0.000000         11.000000

NO. ITERATIONS=      15
```

4. Making Change Revisited **(6 points)**

Given coins of denominations (value) $1 = v_1 < v_2 < \ldots < v_n$, we wish to make change for an amount A using as few coins as possible. Assume that v_i's and A are integers. Since $v_1 = 1$ there will always be a solution. Solve the coin change problem from HW 3 using integer programming. For each the following denomination sets and amounts formulate the problem as an integer program with an objective function and constraints, determine the optimal solution using LINDO, MATLAB or Excel. What is the minimum number of coins used in each case and how many of each coin is used? Include a copy of your code.

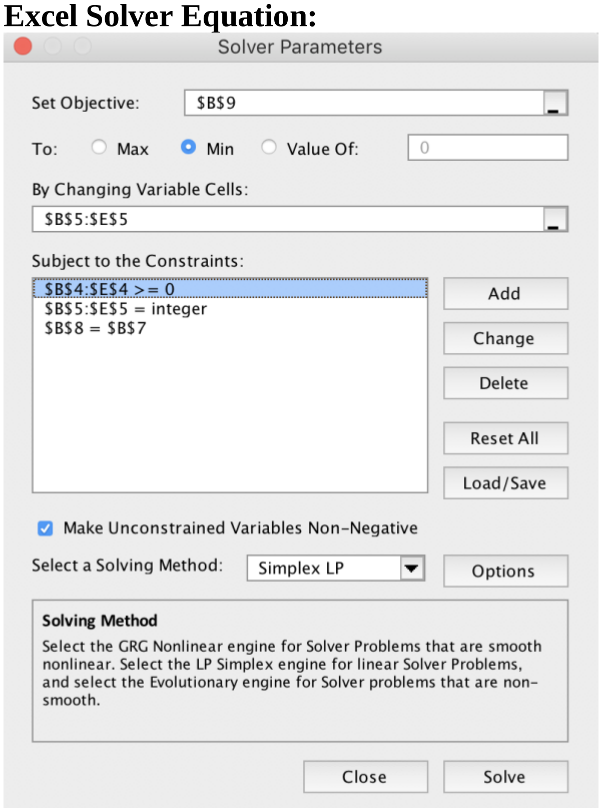
a) V = [1, 5, 10, 25] and A = 202.
b) V = [1, 3, 7, 12, 27] and A = 293

ANSWERS:

I used Excel rather than LINDO for these problems as this was the example provided in the "Coin Change Problem Revisited" Document. The Linear Programming functionality was implemented in the Solver Add-In.

a) V = [1, 5, 10, 25] and A = 202.
Minimum Number of Coins Used: 10 [2, 0, 0, 8]

Excel Solver Equation:

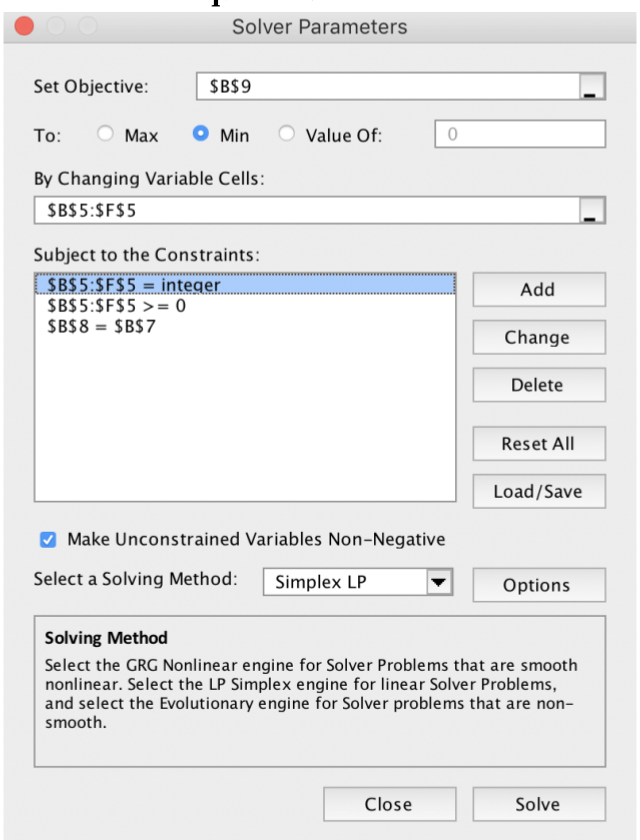


Excel Results:

	A	B	C	D	E
1	**James Hippler**				
2	**Homework 6 - Problem 4a**				
3					
4	**Coin Values**	1	5	10	25
5	**Coins Used**	2	0	0	8
6					
7	**Desired Amount:**	202			
8	**Change Total:**	202			
9	**Total Coins:**	10			

b) V = [1, 3, 7, 12, 27] and A = 293
Minimum Number of Coins Used: 14 [0, 0, 2, 3, 9]

Excel Solver Equation:



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Excel Results:

	A	B	C	D	E	F
1	**James Hippler**					
2	**Homework 6 - Problem 4b**					
3						
4	**Coin Values**	1	3	7	12	27
5	**Coins Used**	0	0	2	3	9
6						
7	**Desired Amount:**	293				
8	**Change Total:**	293				
9	**Total Coins:**	14				