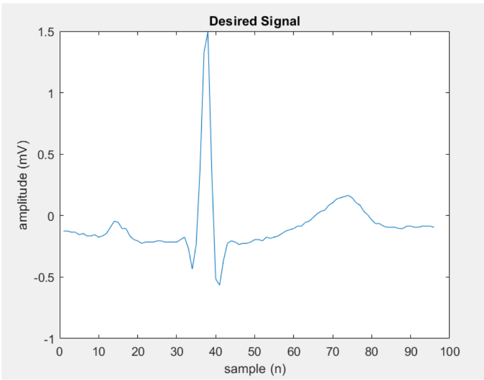
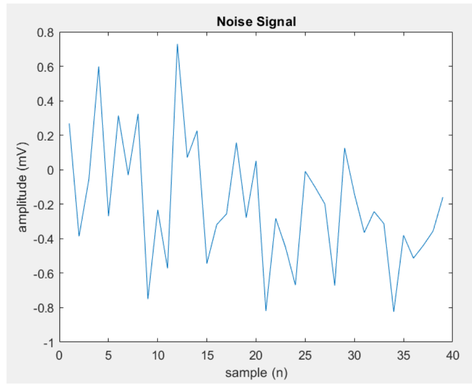
1. Wiener filtering

1.1 Discrete time-domain implementation of the Wiener filter

Part 1



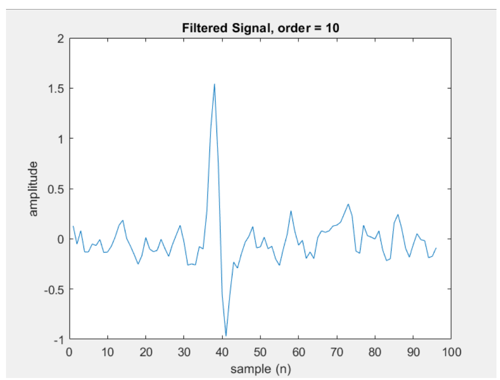
Clean ECG single beat



Isoelectric segment from noisy ECG

a. filter order, M=10

```
weights vector:
    0.2991
    0.1827
   -0.0306
   -0.0467
   -0.0506
    0.0092
   -0.0322
   -0.0492
   -0.0151
    0.0084
```

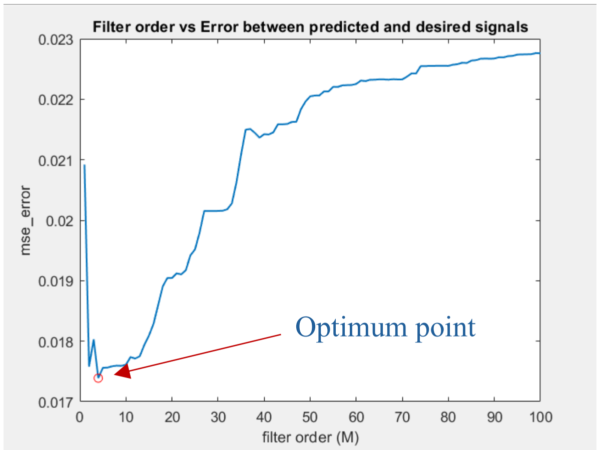


Filtered ECG single beat

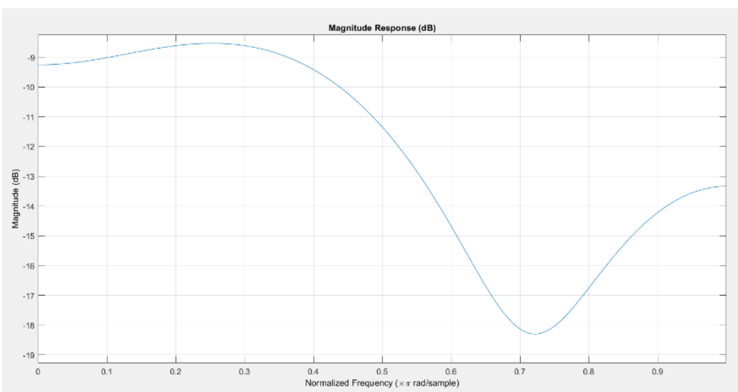
b.

```
ans =

    'optimum order: 4'

optimum filter coefficients:
    0.2453
    0.1174
    0.0347
   -0.0528
```

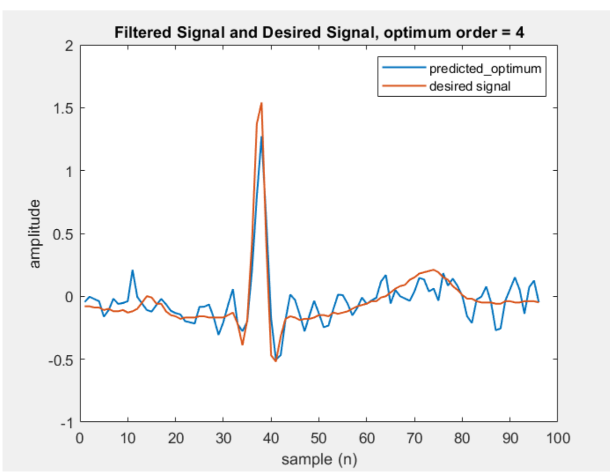
- To find the optimum filter order, the graph of filter order vs the mean squared error between the desired signal and the predicted signal was used. The obtained error variation plot is given below.



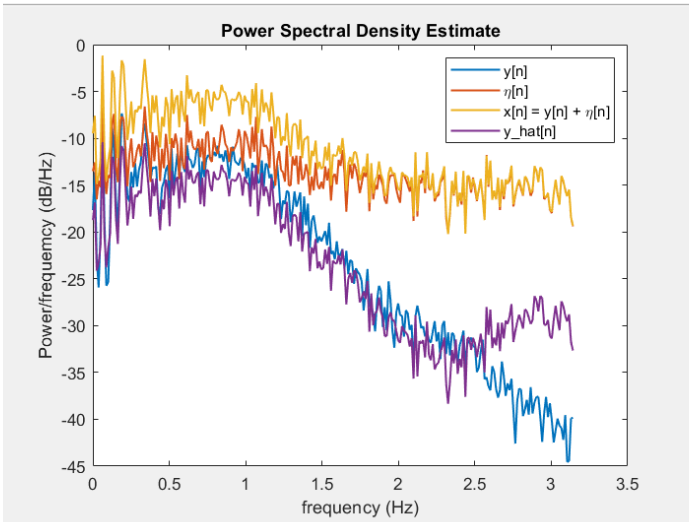
- According to the above graph, the optimum filter order is 4. Magnitude response of the Wiener filter of order 4 is given by,



c.

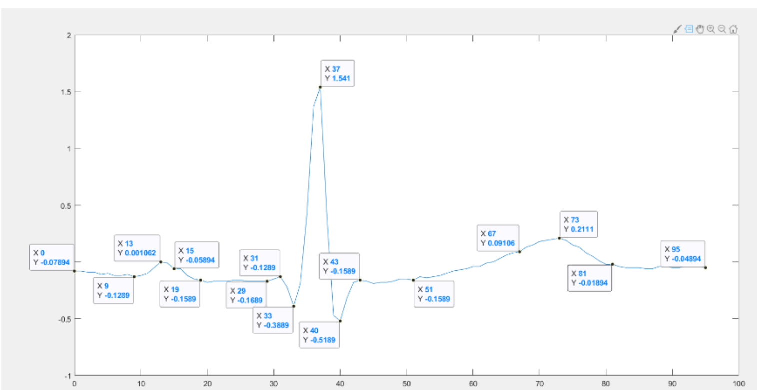


d. Power spectral density estimate plot of the signals is given below.

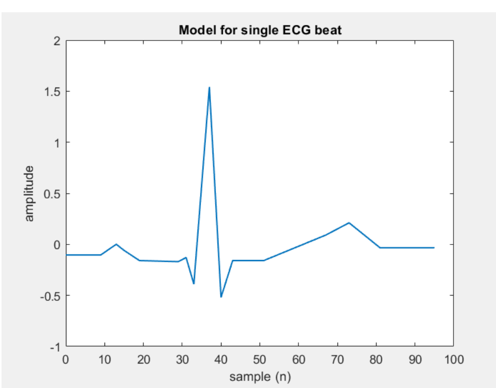


Part 2

- To generate the linear model, first, a set of key points are selected from the ECG template and then a linear interpolation is performed.

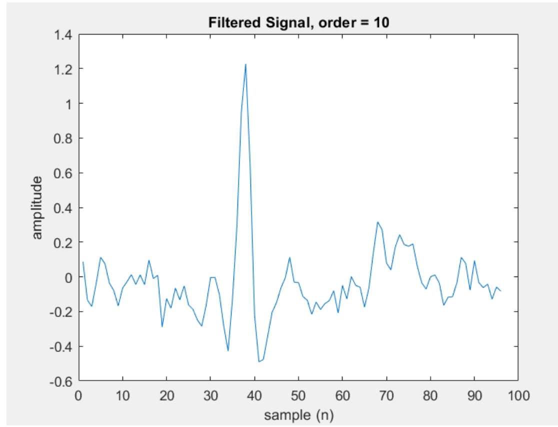


- So, the obtained linear model is,



a.

```
weights vector:
    0.2882
    0.1366
    0.0533
   -0.0428
   -0.0392
   -0.0301
   -0.0138
    0.0354
    0.0115
   -0.0396
```



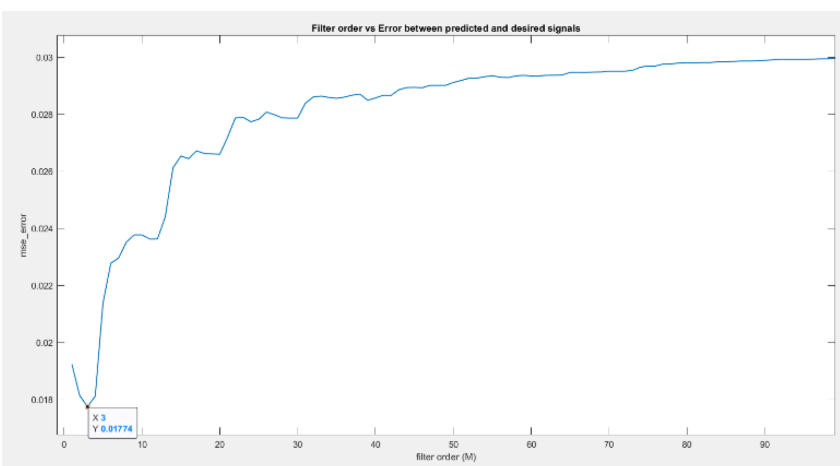
Filtered ECG single beat

b.

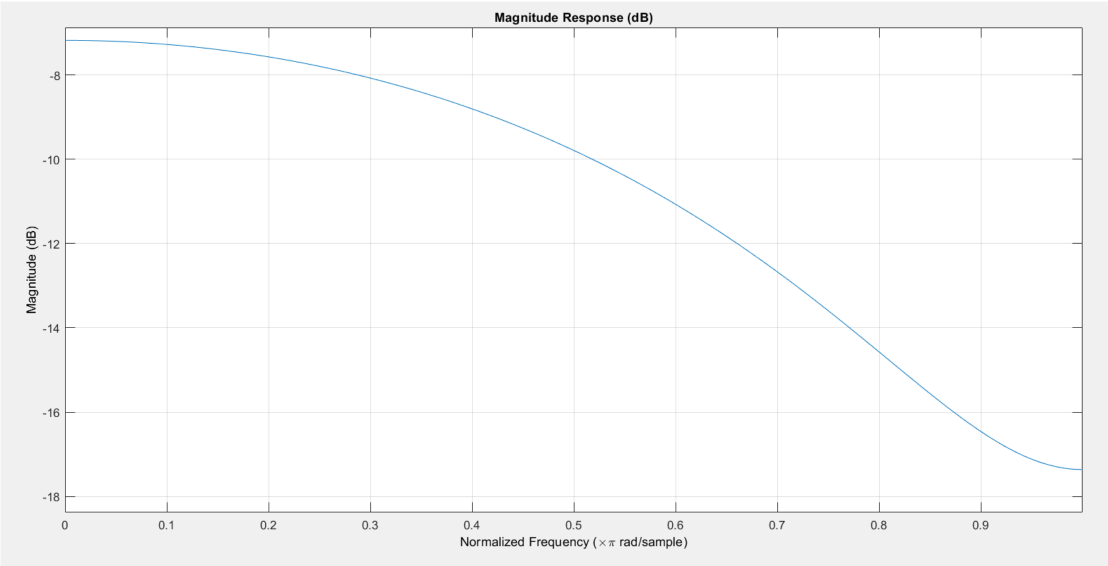
```
optimum order:
     3
```

```
optimum filter coefficients:
    0.2684
    0.1799
   -0.0100
```

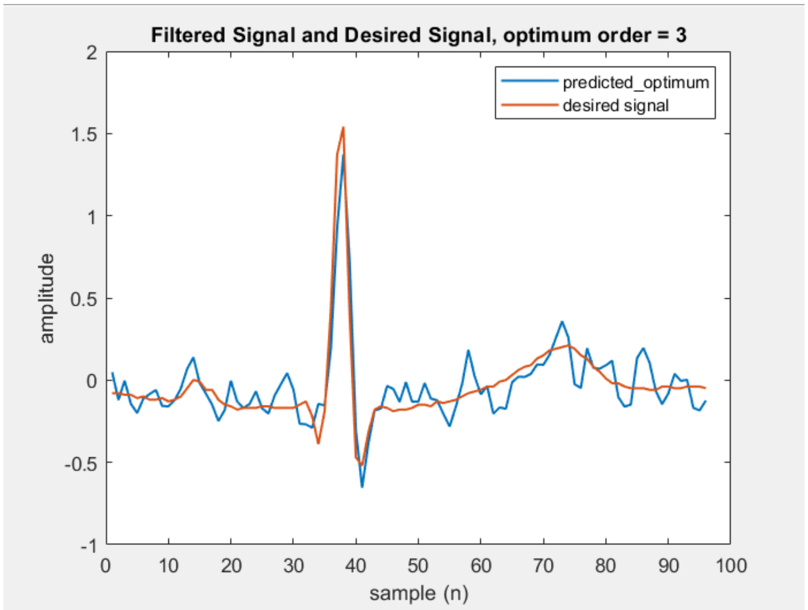
- Filter order vs MSE error between predicted and desired signals is given below.



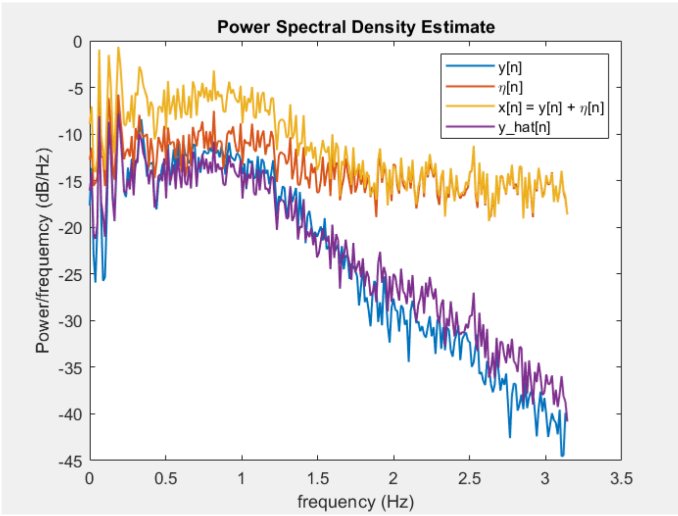
- The magnitude response of the Wiener filter of the optimum filter order 3 is given below.



c. The filtered signal for optimum Winer filter order is,

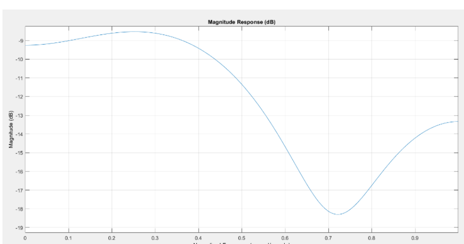


d. The power spectral density estimate plot for the signals is given by,

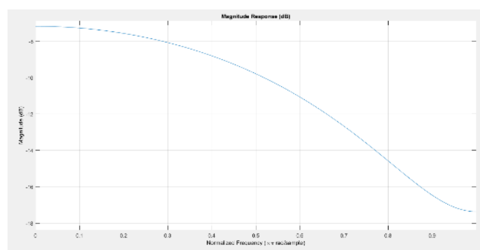


e.

- When observing spectra, the PSD of predicted signal using ECG signal linear model is closer to the PSD plot of the actual signal than the predicted signal using an ECG beat from y[n].



Filter 1: Part (1)



Filter 2: Part (2)

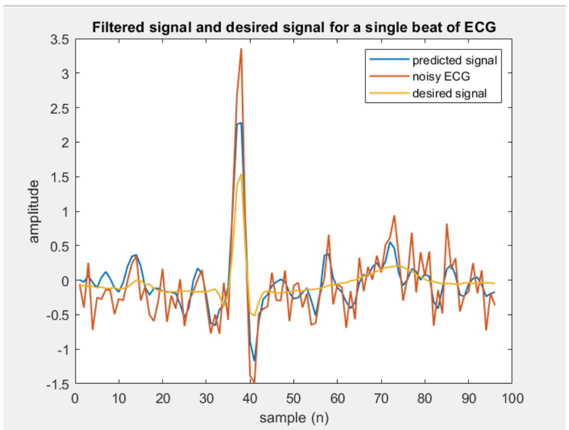
- The magnitude response of **Filter 1** shows a band-reject characteristic at normalized frequencies around 0.7π rad/sample, effectively removing narrowband noise in that region. There is a relatively long transition band. The filter causes minimal distortion to frequencies below approximately 0.4π rad/sample, with moderate distortion in the higher frequencies above 0.9π rad/sample.

- The magnitude response of **Filter 2** resembles a low pass filter with a relatively long transition band. This filter causes minimal distortions to frequencies approximately below 0.4π rad/sample and distorts higher frequency components.

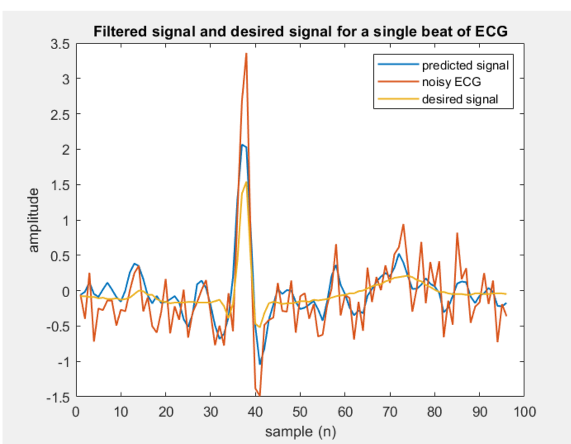
1.2 Frequency domain implementation of the Wiener filter

a. Visual comparison of noisy ECG, clean (desired) ECG and the predicted signal for a single ECG beat is given below.

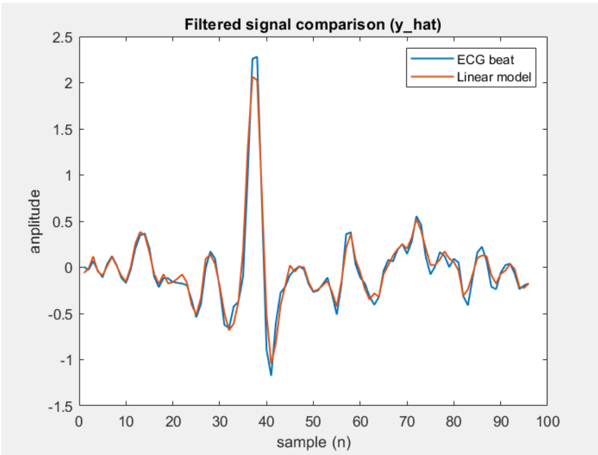
<u>Part 1</u>



<u>Part 2</u>



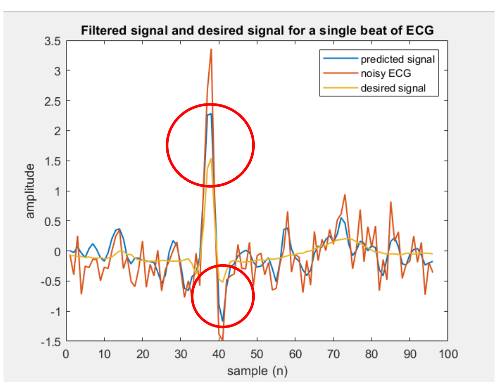
b. A visual comparison of the predicted signals in both methods (Part 1 and Part 2) is given below.



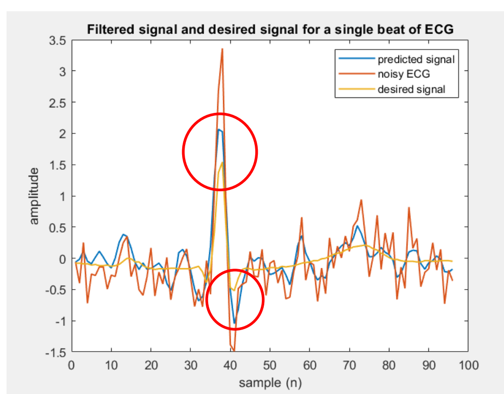

```
part 1 mse error = 0.053607
part 2 mse error = 0.050113
```

- Error in Part 1 > Error in Part 2

- The predicted signal in Part 1 has a higher amplitude than the predicted signal in Part 2 as well as the desired signal.



Signal comparison in Part 1



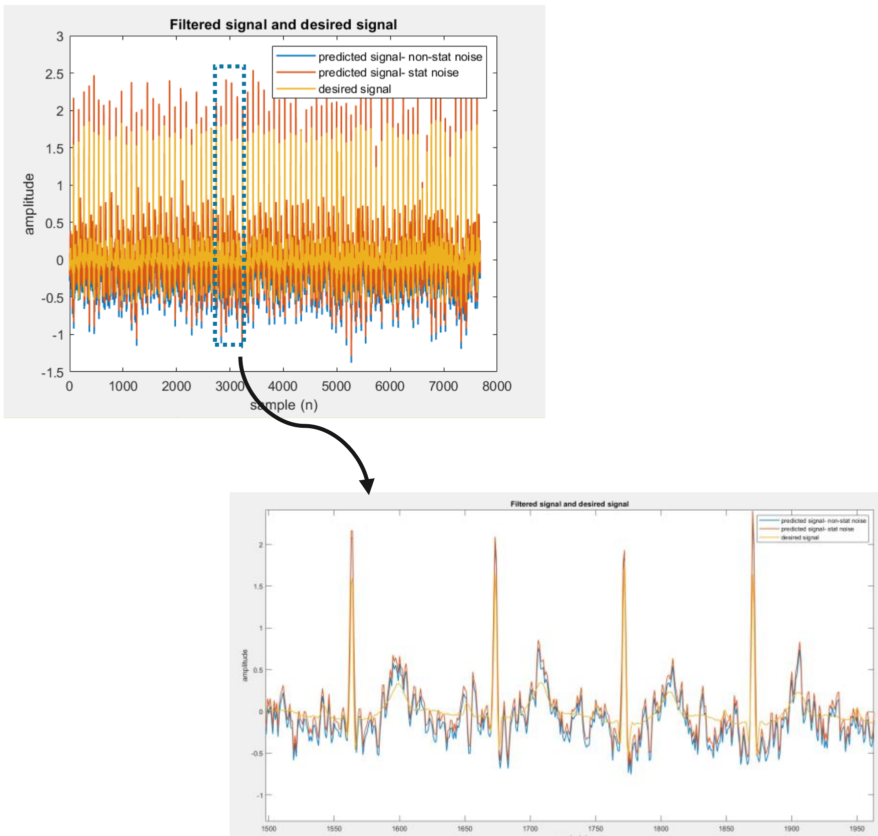
Signal comparison in Part 2

- By looking at the plots, we can see that the filter in Part 1 performs poor compared to the filter in Part 2 in noise removal.

1.3 Effect of non-stationary noise on Wiener filtering

a. and b.

- The method used here is the frequency domain filtering with ECG template linear model

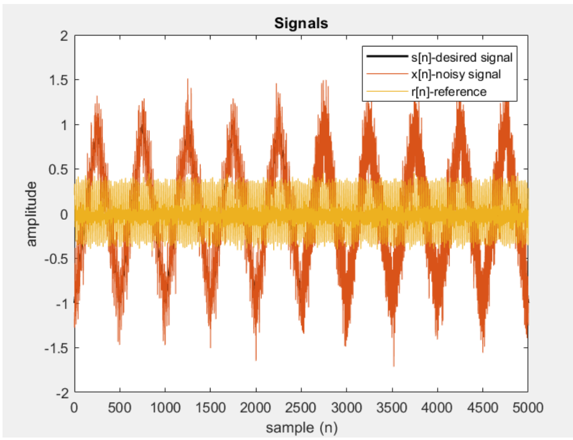


A magnified view of the
ECG signal

- High frequency noise is still present in the predicted signal.

2. Adaptive filtering

2.1 LMS Method



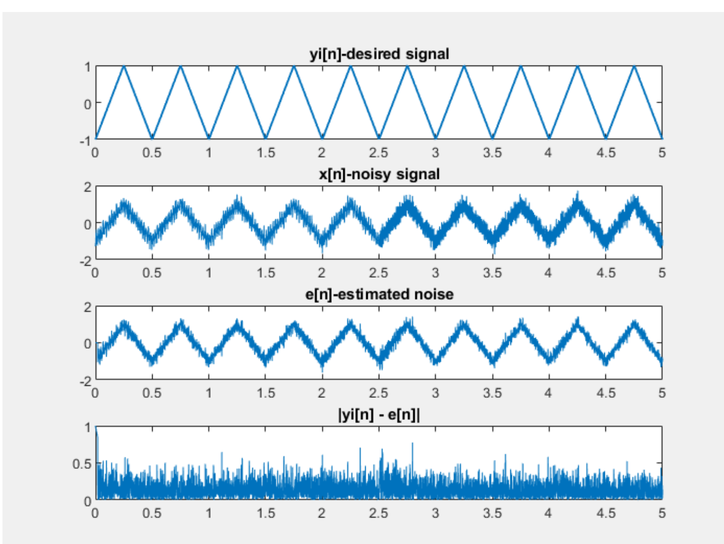
a. A separate function named *LMS_filter* was implemented for this

b. The filter order is set to 20. Then the valid range for μ is calculated using the following formulae,

$$0 < \mu < 2/\lambda_{max} \quad \text{-------------} \ (1)$$

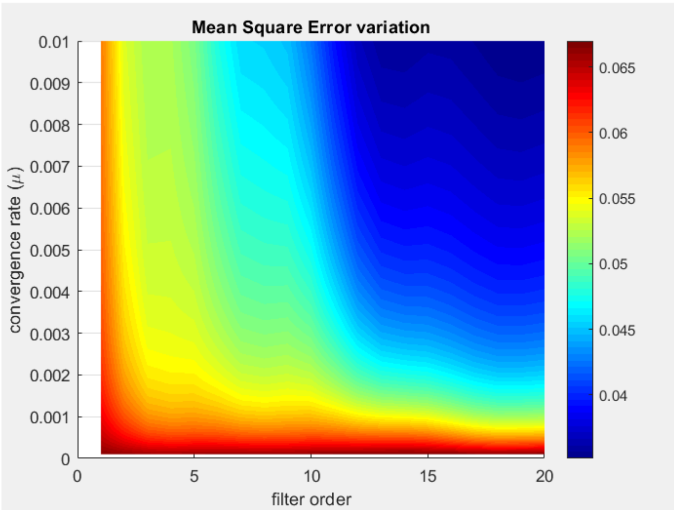
$$\lambda_{max} = 20 \times filter\_order \times P_x \quad \text{-------------} \ (2)$$

where P_x is the power of the input signal



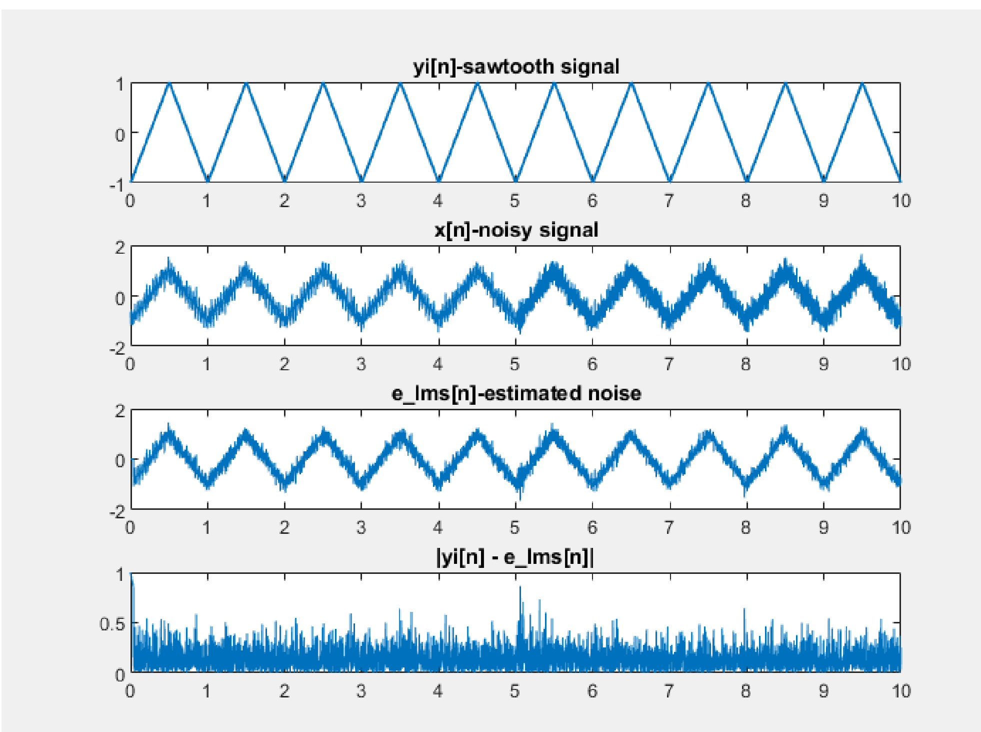

```
upper bound = 0.012574
mu = 0.009022
```

c. The top view of the surface plot of MSE error against different combinations of filter orders (M) and converging rates (μ) is given below.




```
best combination is, filter order 19.000000 and mu 0.010000
```

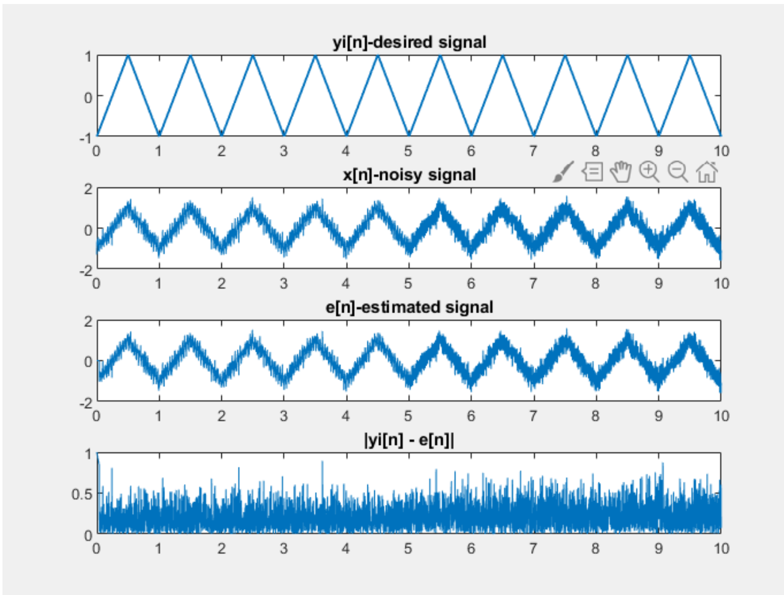
- The signal results for the optimum LMS filter parameters are given below



2.2 RLS method

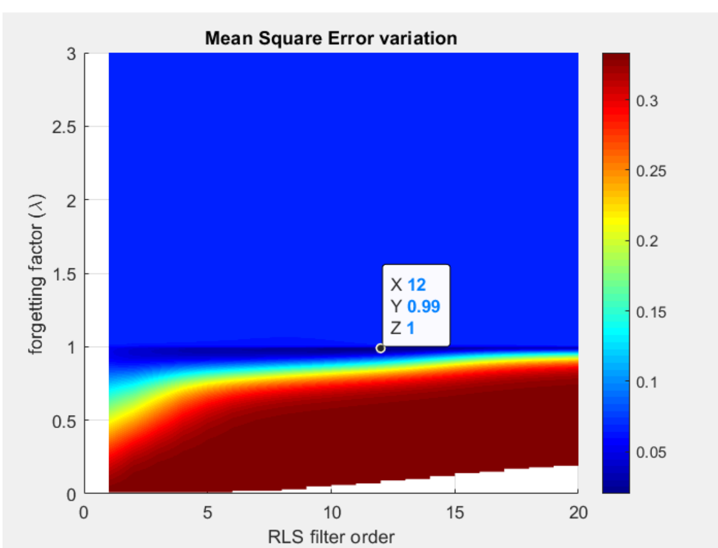
a A separate function named *RLS_filter* was implemented for this.

b The results for filter order = 20 and lambda = 3 are given below.



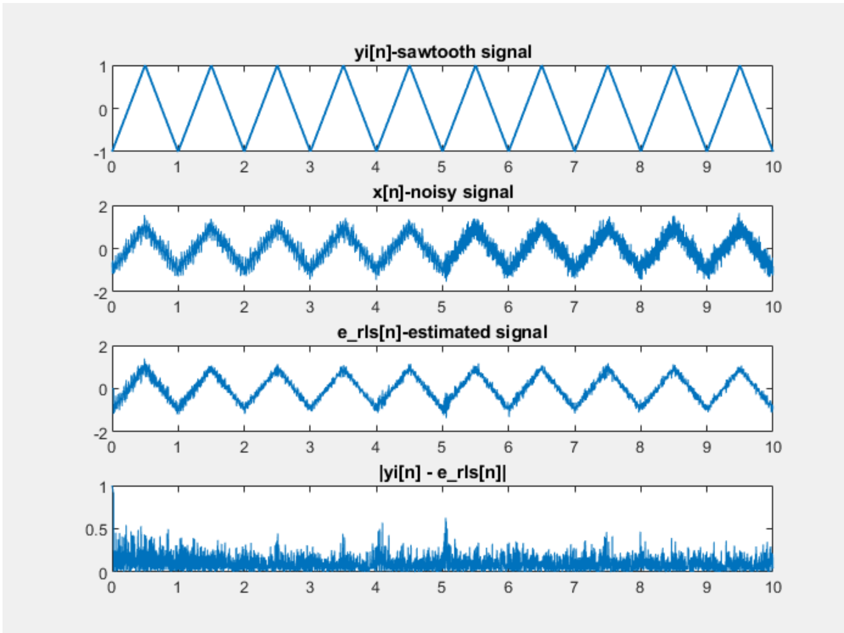
- In LMS filter lower μ values (<1) tend to give better signal estimations while in RLS filter, λ tends to give better estimations.

c The top view of the surface plot of MSE error against different combinations of filter orders (M) and forgetting factors (λ) is given below

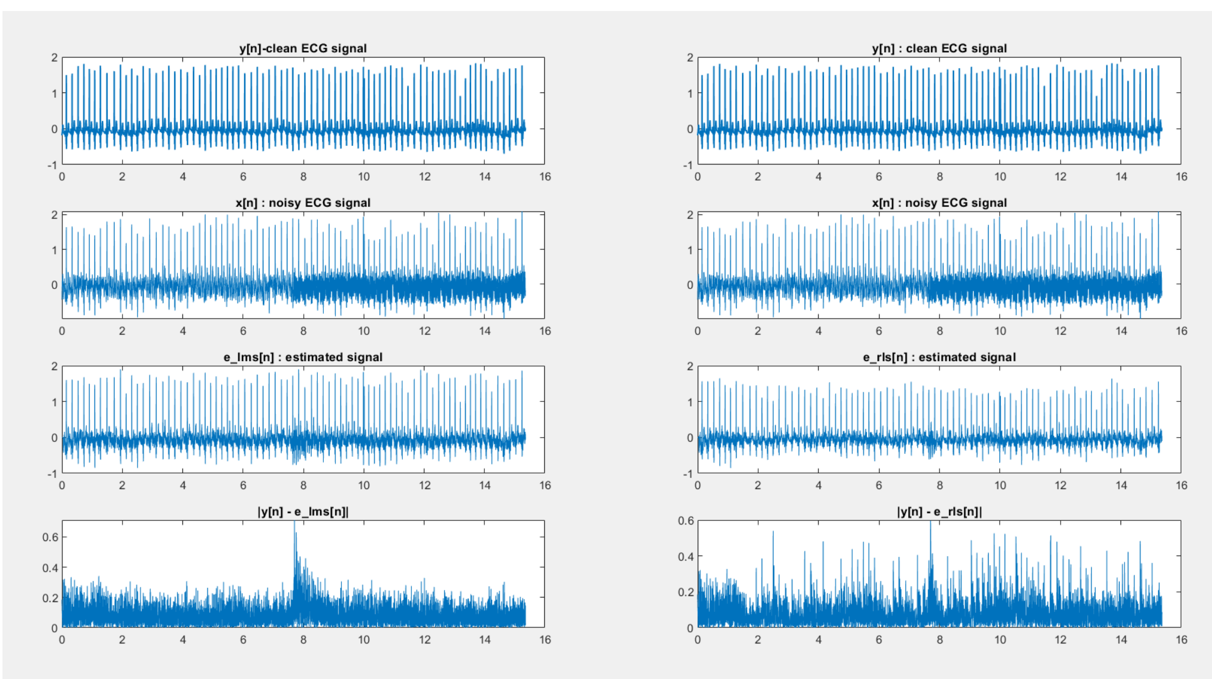


```
best combination is, filter order 12.000000 and lambda 0.990000
```

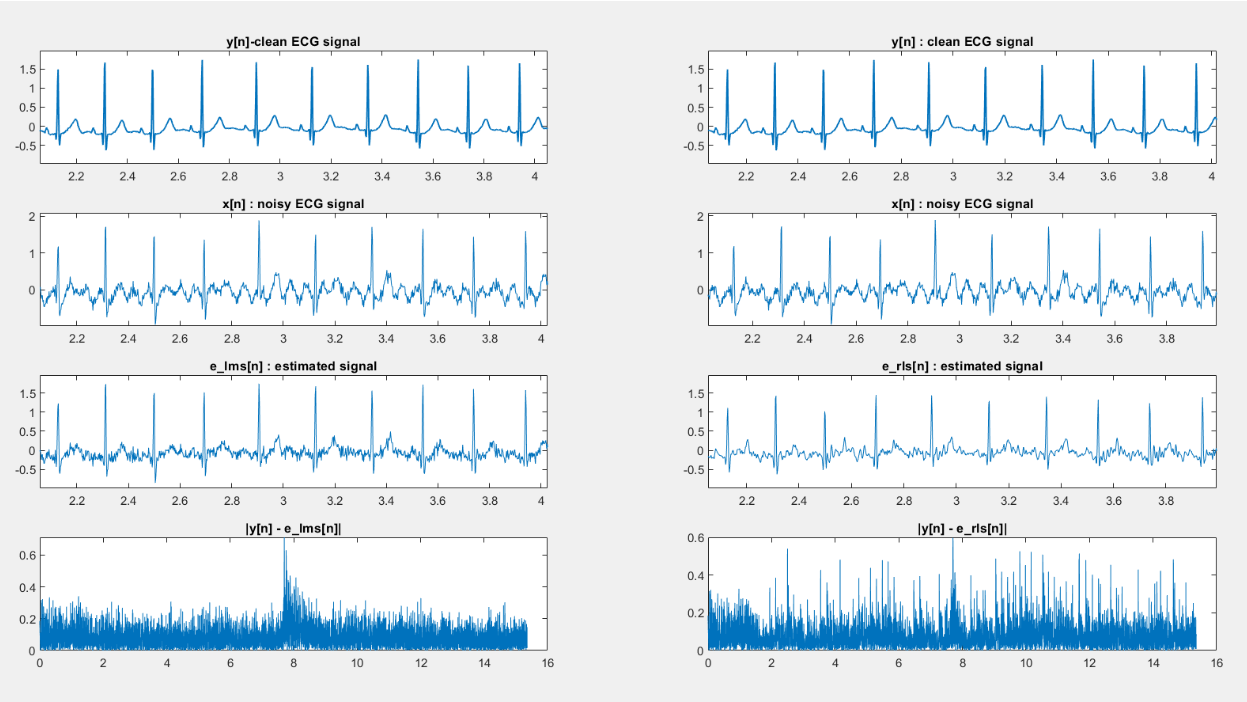
- The signal results for the optimum RLS filter parameters are given below.



d.



- A zoomed version of the above plot is given below.



- By observing the results, we can see that the high frequency noise in the noisy ECG signal is better cancelled by RLS filter than the LMS filter.

- However, the RLS filter results in a higher amplitude distortion of the signal than the LMS filter.