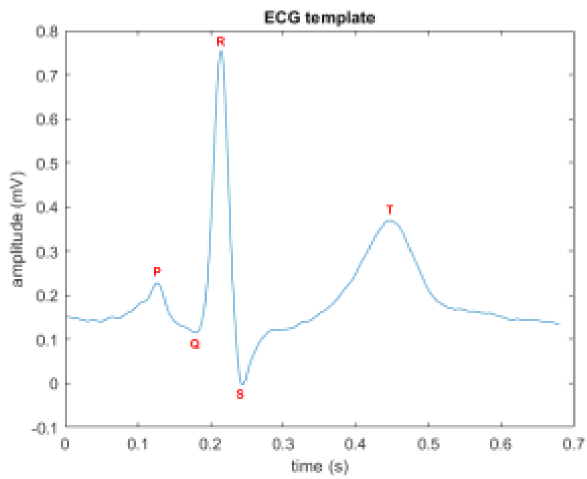
Question 01. Smoothing Filters

1.1 Moving average MA(N) filter

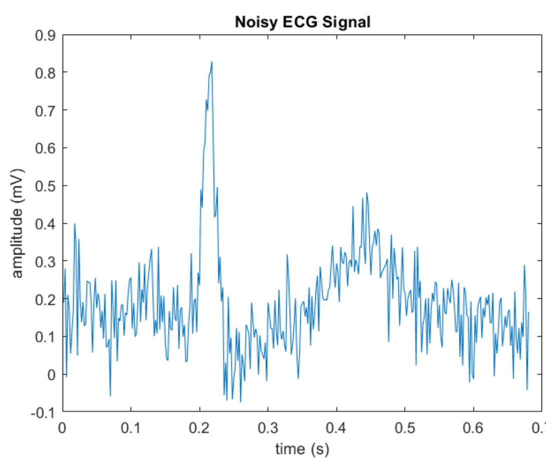
<u>Preliminaries</u>

(ii)



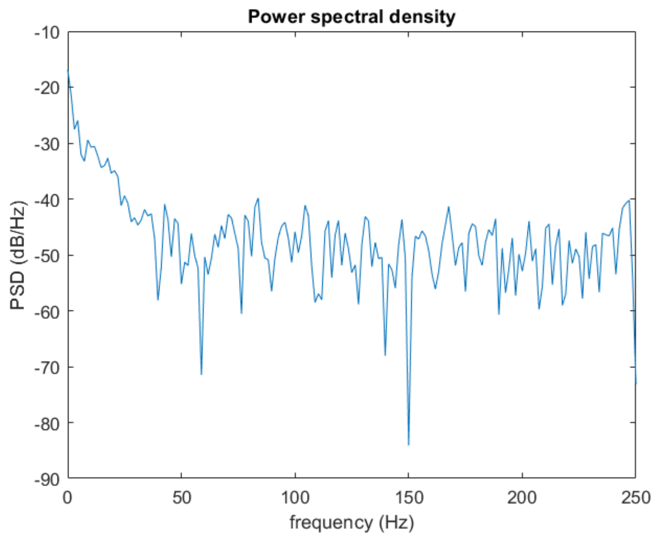
P wave: Represents atrial depolarization, which is the electrical activity leading to atrial contraction.
Q wave: The first negative deflection after the P wave, representing the initial depolarization of the interventricular septum.
R wave: A large positive deflection, reflecting ventricular depolarization as the electrical impulse spreads through the ventricles.
S wave: A downward deflection following the R wave, completing the process of ventricular depolarization.
T wave: Represents ventricular repolarization, as the heart prepares for the next cycle.

(iii)



After adding a white Gaussian noise of 5 dB the original signal has been distorted as shown in the above figure.

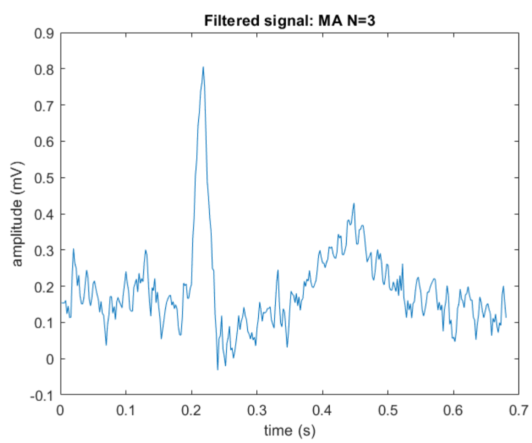
(iv)



The signal power is plotted against the frequency in the above figure.

<u>MA(3) filter implementation with a customized script</u>

(i)



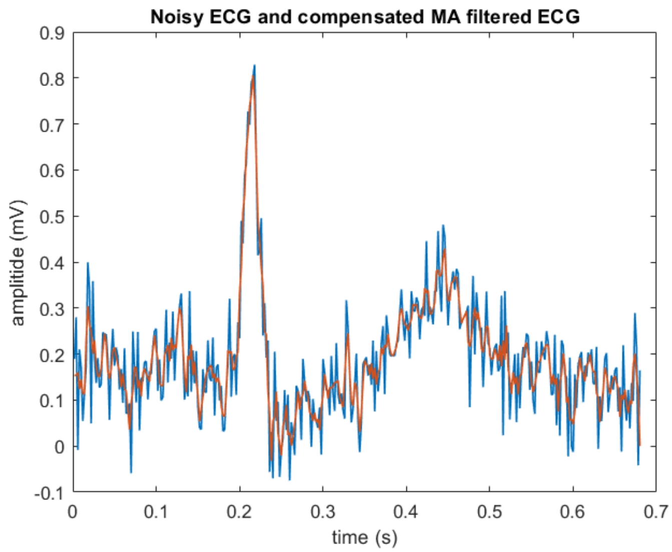
For a MA(3) filter;

$$y(n) = \frac{1}{3}\sum_{k=0}^{2} x(n - k)$$

The filtered signal is shown in the above figure.

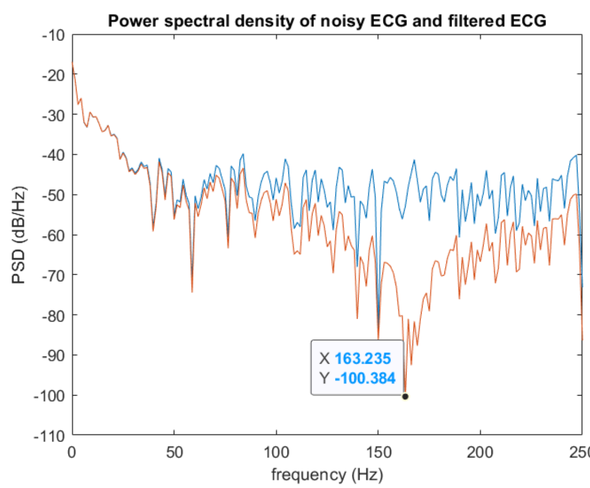
(ii) $group\ delay = \frac{N-1}{2}$

(iii)



- The blue line indicates the noisy signal. We can observe the fluctuations and sharp variations in the signal. These can distort the ECG waveform, making it harder to identify the features such as P, Q, R, S, and T waves.
- The orange line represents the filtered signal, where it is smoother and follows the overall trend of the noisy signal, but with reduced sharp fluctuations. This makes it easier to identify the ECG features.

(iv)



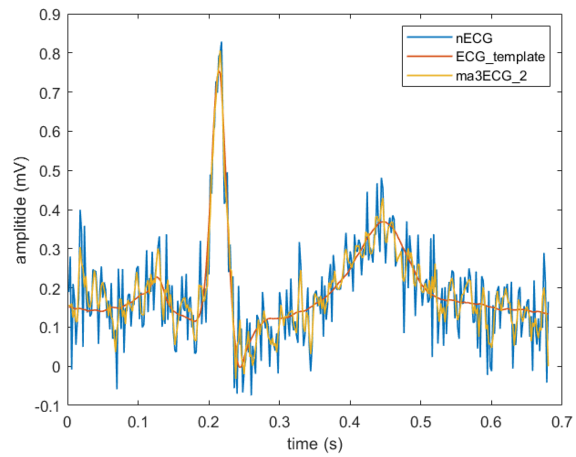
- The blue line represents the noisy signal. The presence of significant power in the high-frequency range suggests the contribution of noise, as ECG signals typically are in the low range of frequencies.
- In the orange line, which represents the filtered signal, the higher frequency ranges have less power, indicating that the filter has successfully attenuated an amount of the noise in the high-frequency range. Additionally, the power spectral density in the lower frequency range of the filtered signal, is similar to the power spectral density of the noisy signal, indicating that the filter has preserved the lower-frequency characteristics of the ECG signal. Approx. 163 Hz there

is a sudden drop in the power due to the filtering process. This is due to the notch behavior in a moving average filter according to its sampling rate.

<u>MA(3) filter implementation with the MATLAB built-in function</u>

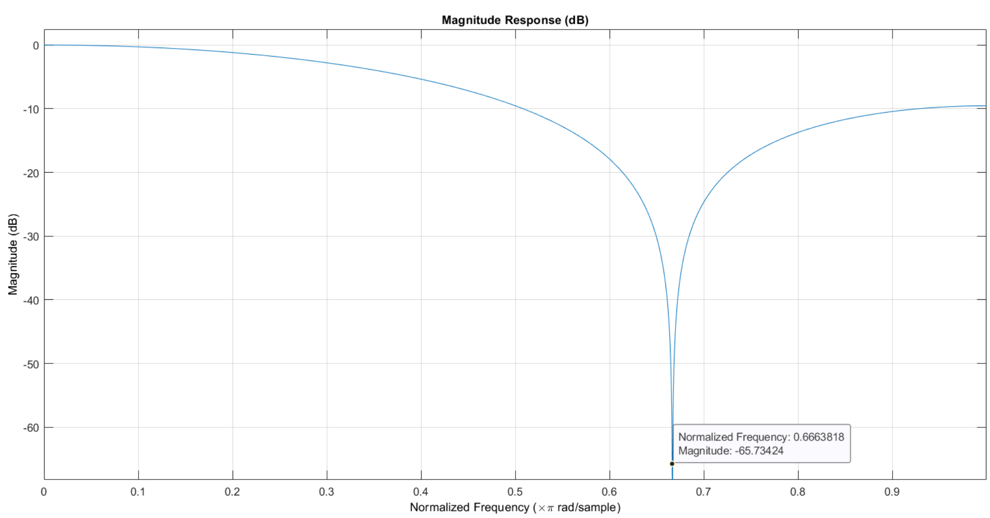
(i)

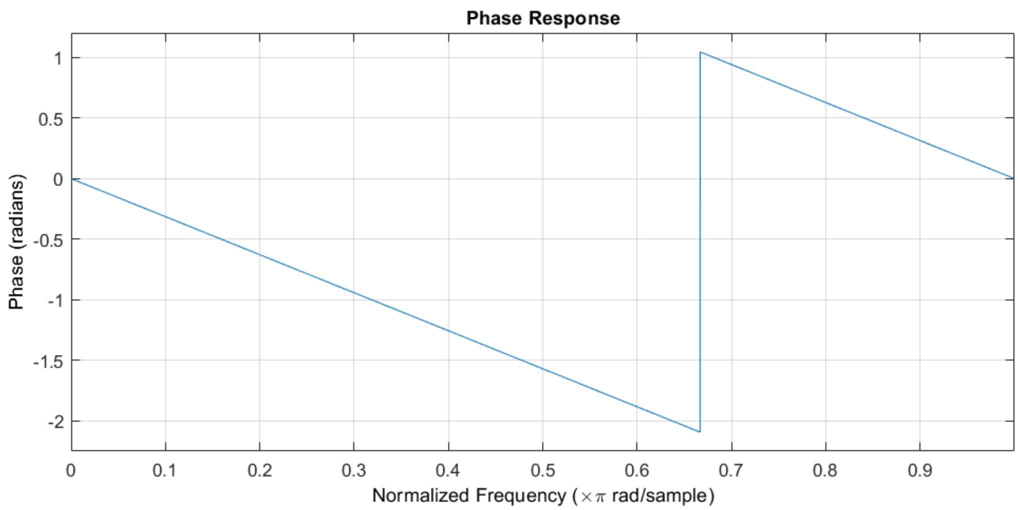
(ii)

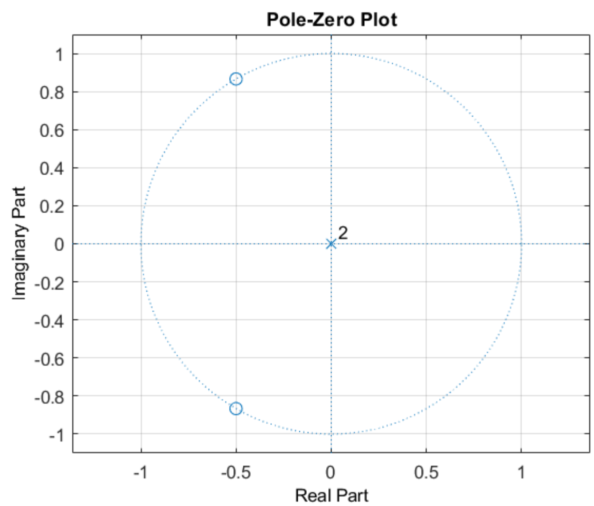


- The noisy ECG (nECG) shows a lot of sharp, erratic variations due to noise, while the template and filtered signals are much smoother and clearly highlight the ECG's important features.
- Both the template and the filtered signal (yellow line) are quite similar, suggesting that the filtering process has successfully cleaned up the noisy ECG to resemble the ideal template.

(iii)







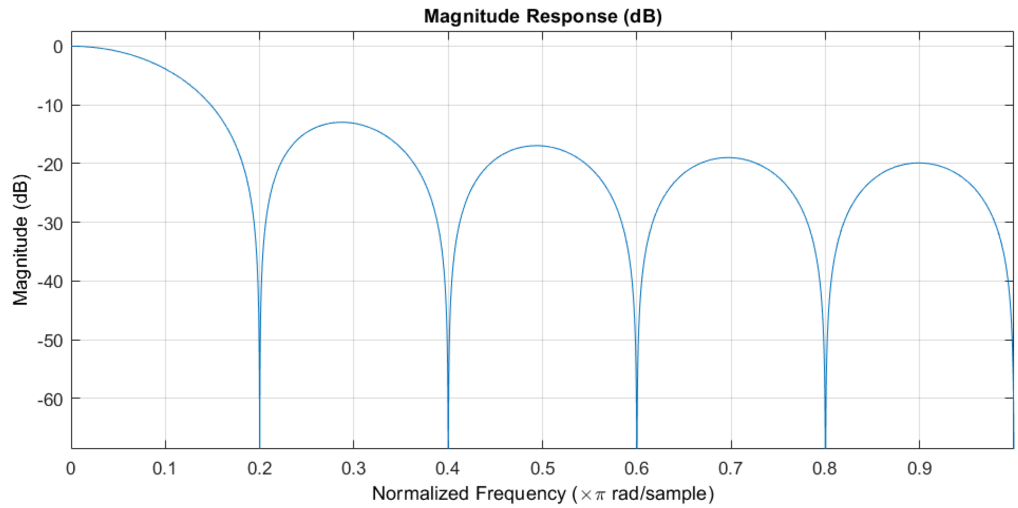
- The magnitude drop in approx. 0.6664 is due to the notch filter behavior in the MA(3) filter. The notch frequency can be calculated using,

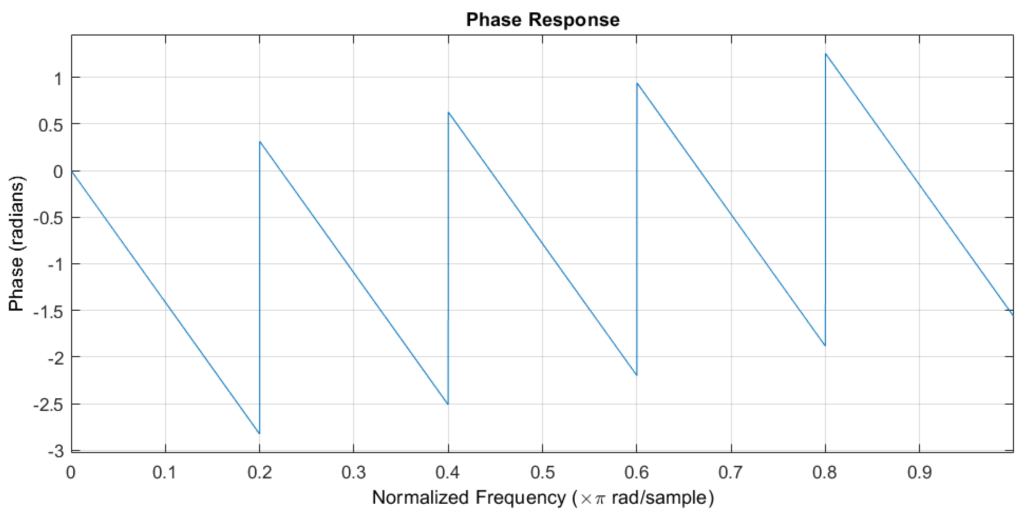
$$f = 0.5 * sampling\ frquency * 0.6664 = 166.6\,Hz$$

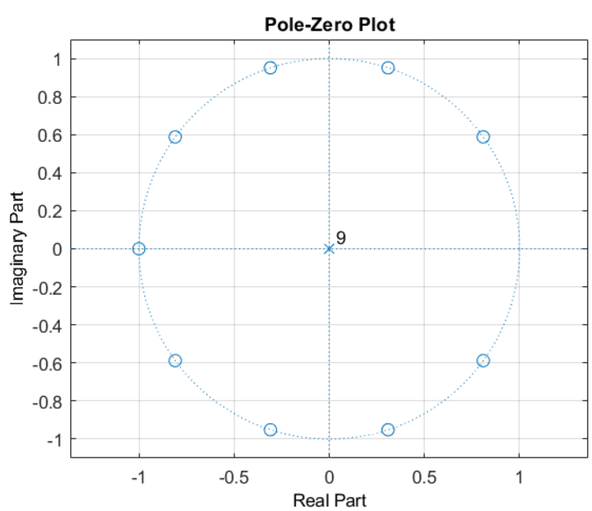
  This is nearly equivalent to the value we obtained in the PSD diagram.
- The phase shift can be seen at the same frequency and the pole-zero plot represents the zero around the same frequency in the above figures.

<u>MA(10) filter implementation with the MATLAB built-in function</u>

(i)





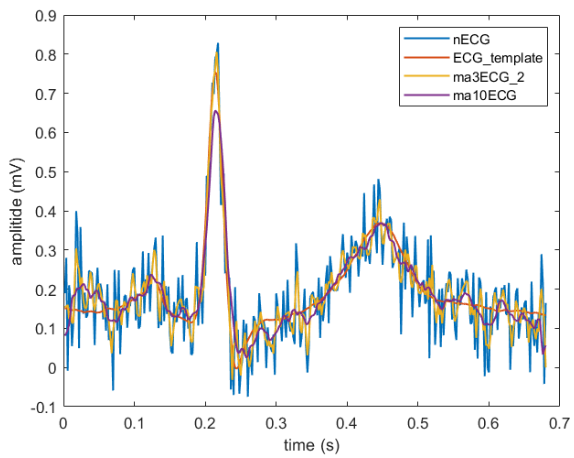


- The notch filter behavior for MA(10) can be observed in 5 frequencies in the above figure. They are 0.2, 0.4, 0.6, 0.8, 1. This is because of the filter behavior, which has the characteristic of having notch behavior on the multiples of normal frequencies,

$$f = \frac{sampling\ frequency}{N}\ Hz$$

- The magnitude response clearly represents the attenuations in each of those frequencies. This includes the removal of the power line noise frequency component at 50 Hz. Additionally, when we compare the attenuation level of the signal, we can conclude that MA(10) has a higher attenuation compared to MA(3) at higher frequencies.
- The phase response and the pole-zero plot represent the phase shifts in each frequency mentioned above.

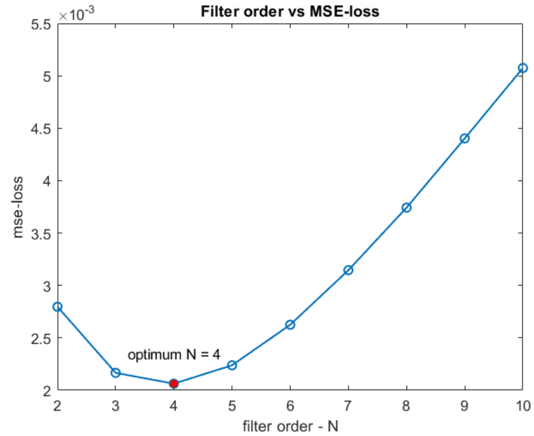
(iv)



- When we compare the MA(3) filtered signal and MA(10) filtered signal with the noisy signal, we can see a clear visible improvement due to the filtering process. Both the filtered signals have fewer fluctuations compared to the noisy signal, and the features of the original signal are easier to identify.
- When we compare the MA(3) filtered signal with the MA(10) filtered signal, the MA(10) filter has further removed the fluctuations from the noisy signal. This is due to the increased attenuation in the high-frequency range of the MA(10) filter, compared to the MA(3) filter.
- However, there are still clear deviations in the MA(10) filtered signal path when compared with the ECG_template signal. As an example, it can be seen when we compare the R wave of the MA(10) filtered signal.

<u>Optimum MA(N) filter order</u>

(ii)



(iii)

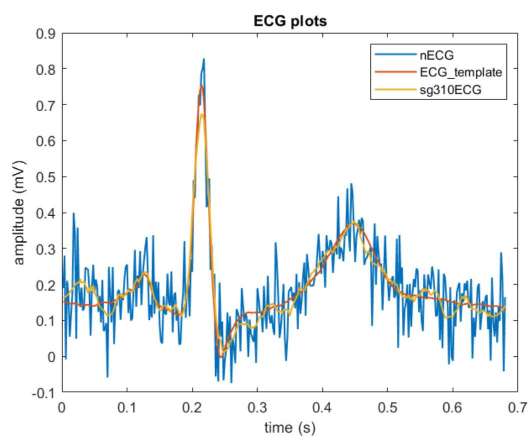
Low filter orders: With a low filter order, the MA filter might not adequately smooth out noise or fluctuations in the signal, leading to higher MSE.

High filter orders: High-order filters can introduce excessive smoothing, potentially oversimplifying the signal and ignoring significant variations, which can lead to a large MSE when comparing the estimated signal to the actual one.

1.2 Savitzky-Golay SG(N,L) filter

<u>Application of SG(N,L)</u>

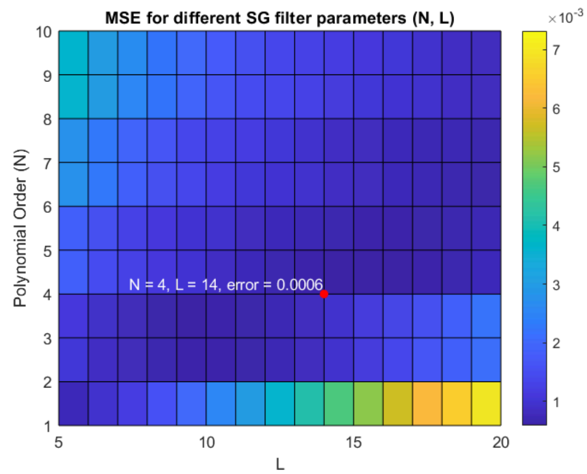
(ii)



- The filtering has removed the sharp fluctuations and smoothened the signal, compared to the noise signal. The filtered signal follows the same path as the noisy signal signal, with fewer deviations and more information about the ECG signal features.

<u>Optimum SG(N,L) filter parameters</u>

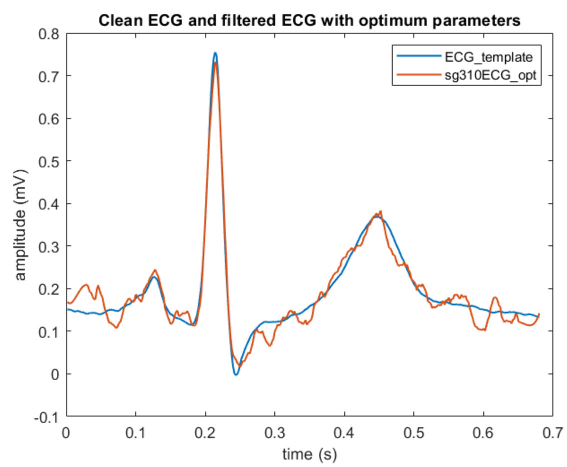
(i)



- The optimal filter parameters,
    - N = 4
    - L = 14

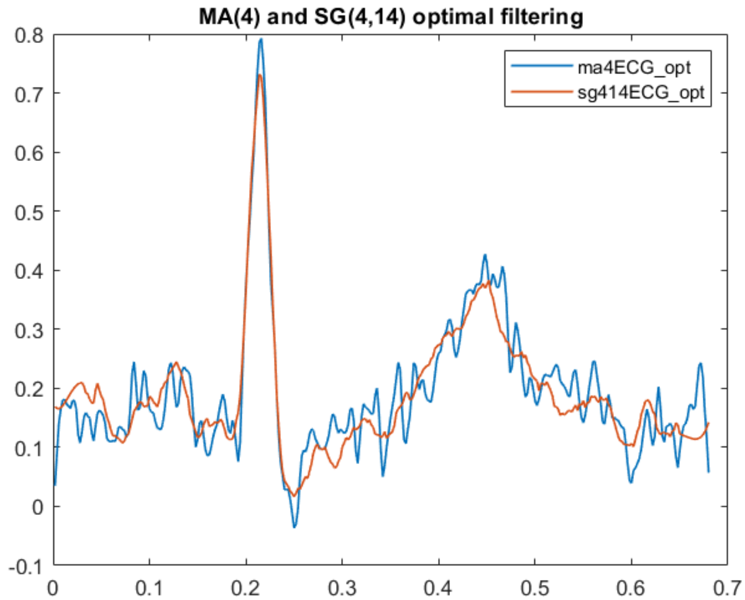
  The error is approx. 0.0006 with the above filter parameters.

(ii)



- The above figure shows that the filtered SG(4,14) signal has removed most of the noise artifacts of the noisy signal. The sharp fluctuations are removed.

(iii)



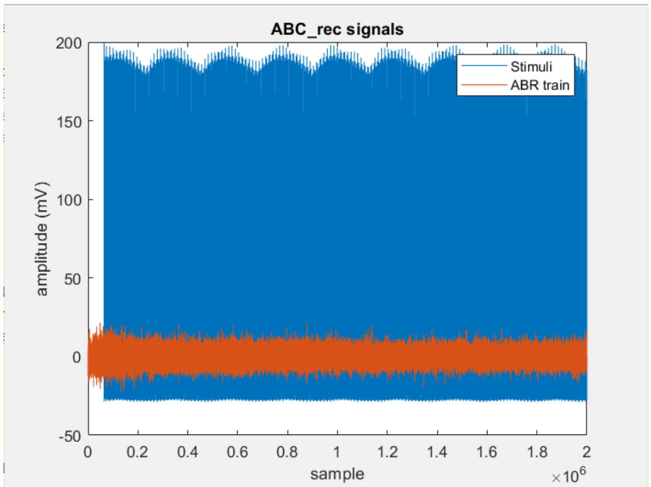
- When we compare MA(4) and SG(4,14) filtered signals, we can observe that the SG(4,14) filtered signal has less sharp fluctuations. The feature identification is relatively easier using the SG(4,14) filter.
- Computational complexity in the SG(N, L) method is higher due to the matrix operations in the calculation. It has higher complexity but a better signal perseverance.

Question 02. Ensemble Averaging

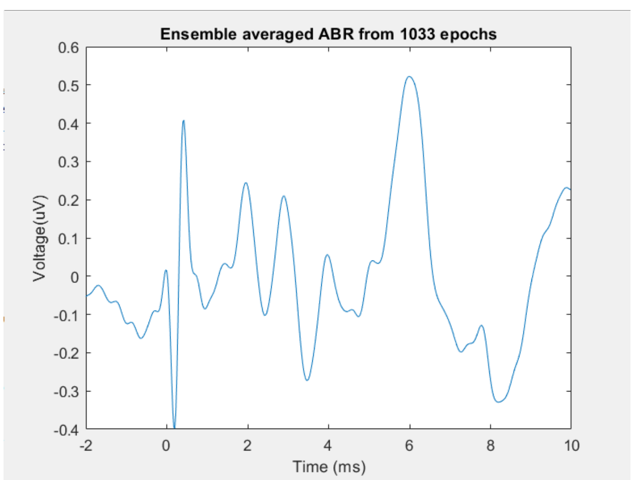
2.1. Signal with multiple measurements

a. Preliminaries

(iii)



(viii)



b. Improvement of the SNR

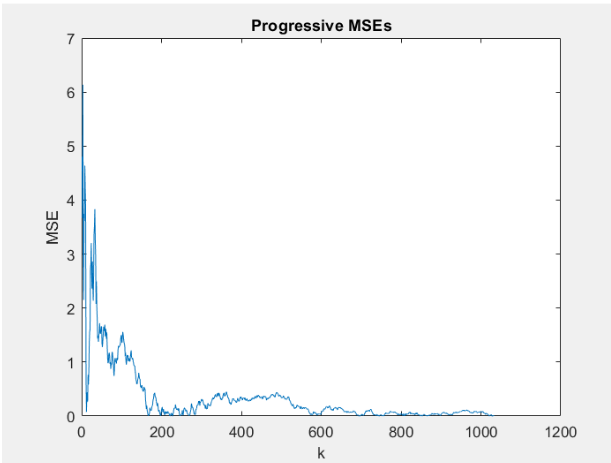
(i)

```
M = length(stim_point);
N = 399-(-80) + 1;
y_hat = ensmbl_avg;

mse_arr = zeros(1,M);

for k = 1:M
    y_curl_k = mean(epochs(:, 1:k), 2);
    mse_arr(k) = sqrt((sum(y_hat - y_curl_k).^2)/N);
end
```

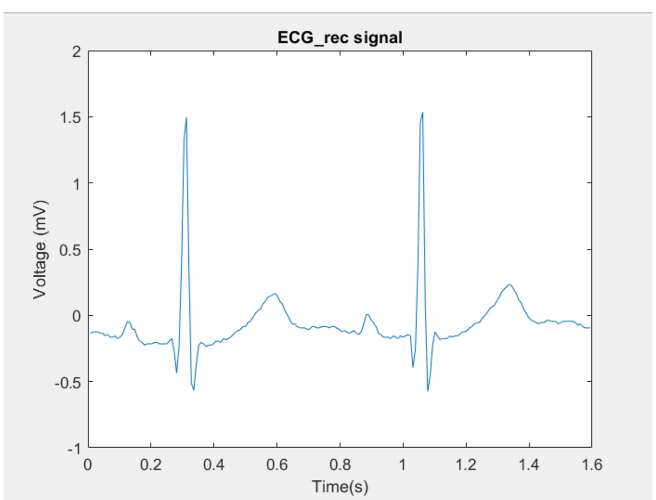
(ii) graph of MSE_k against k



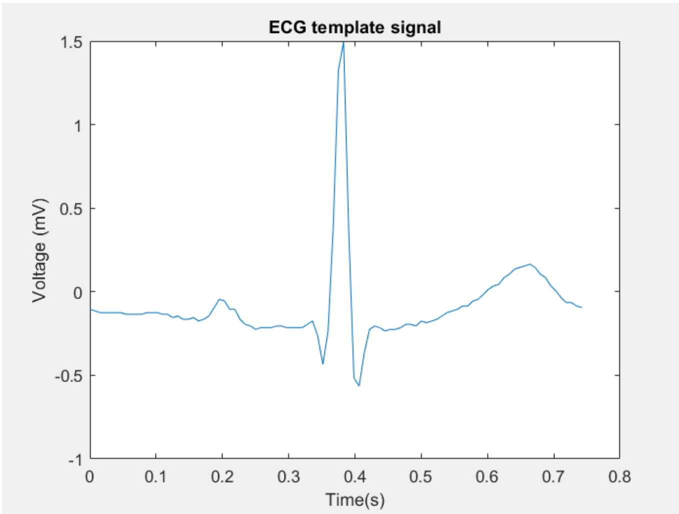
- As the number of averaged epochs k increases, the noise in the ABR data progressively cancels out, while the signal adds up constructively, leading to a reduction in MSE.
- Initially, the MSE decreases rapidly, reflecting significant noise reduction and SNR improvement. As k grows larger, the MSE flattens, showing diminishing returns in SNR improvement.
- This behavior follows the theoretical SNR improvement, where SNR increases proportionally to $\sqrt{k}$.

2.2. Signal with repetitive patterns

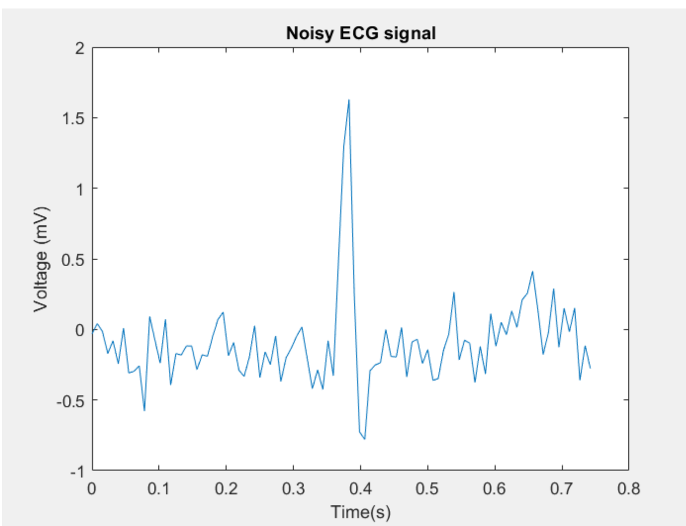
(ii)



(iii) ECG template

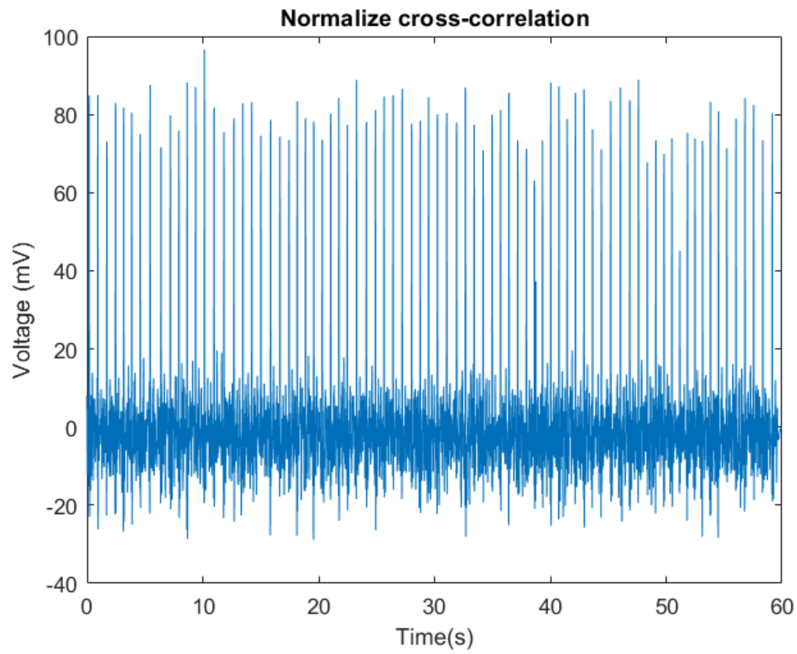


(iv) Noisy ECG signal

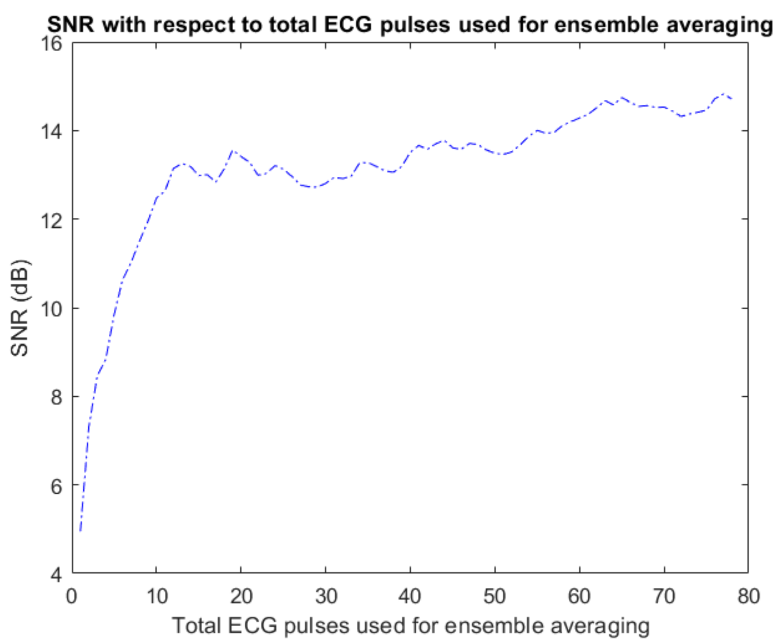


Segmenting ECG into separate epochs and ensemble averaging

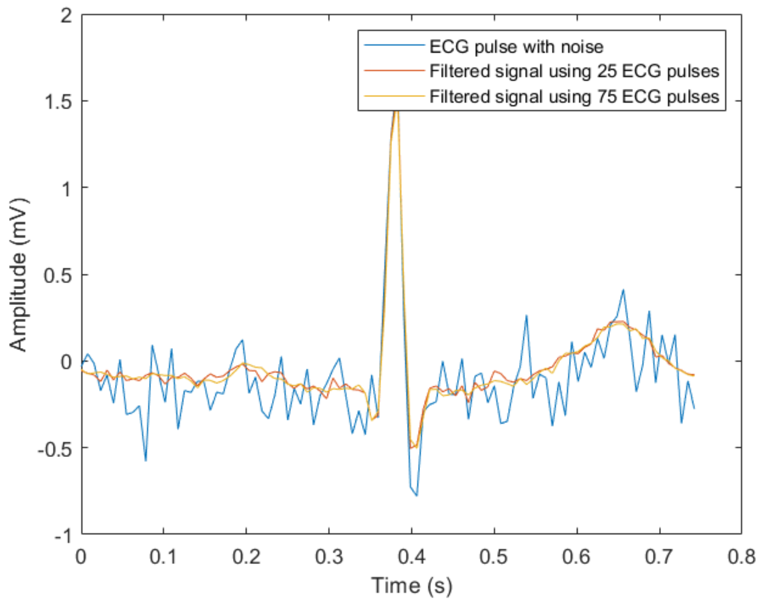
(ii)



(iv)



(v)



- The filtered signal using 75 ECG pulses has less sharp fluctuations when compared to the signal filtered using 25 ECG pulses.

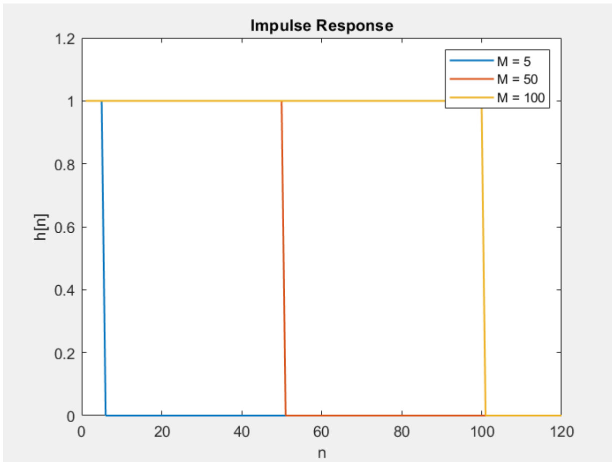
(vi)

- The correlation is less sensitive to individual noise spikes
- By using correlation to identify the ECG pulse we can avoid misinterpreting noise spikes as R waves.

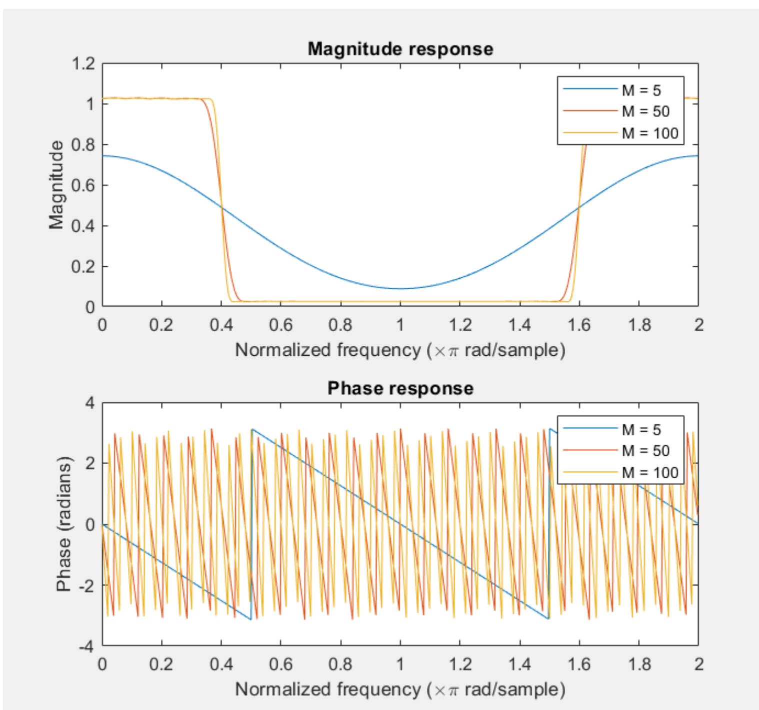
Question 03. Designing FIR filters using windows

3.1. Characteristics of window functions

(i)

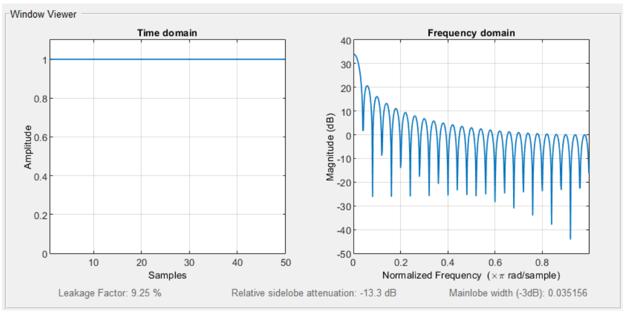


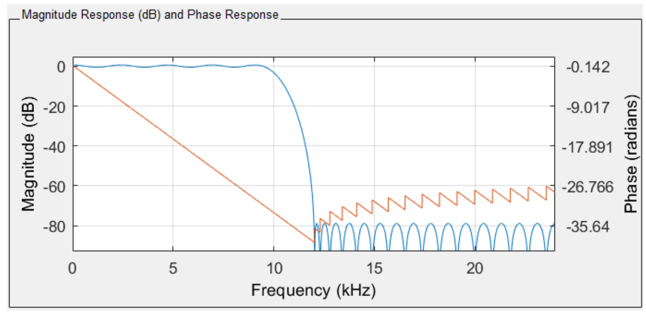
(ii)



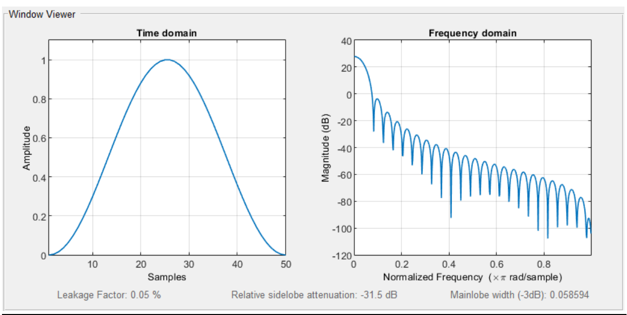
(iii) M = 50

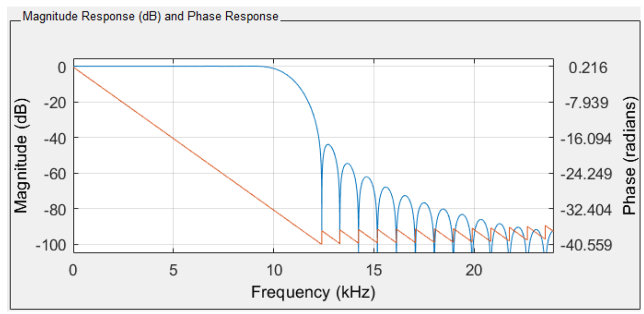
<u>Rectangular</u>



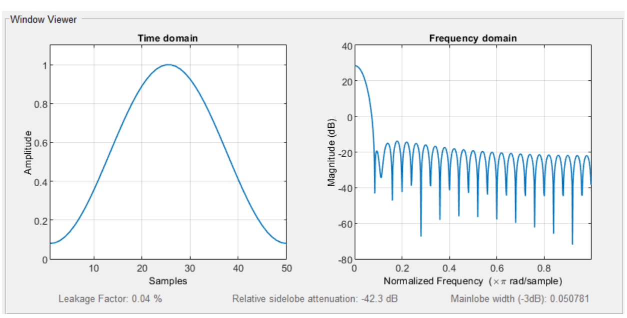


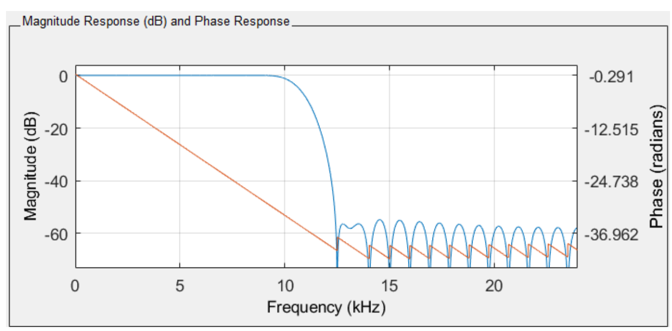
<u>Hanning</u>



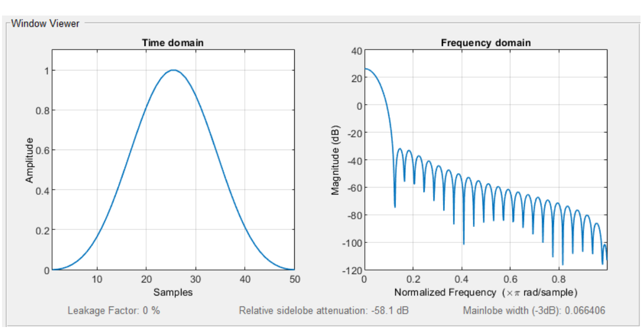


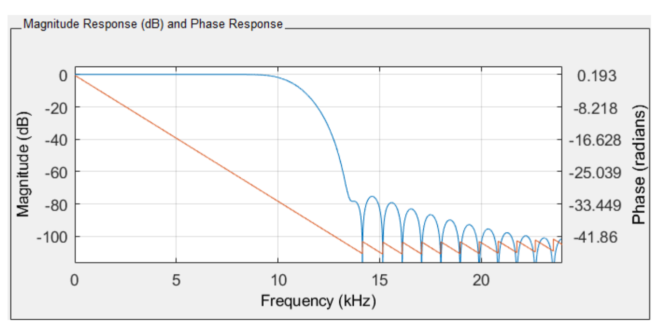
<u>Hamming</u>





<u>Blackman</u>





Morphology:

- Rectangular: Abrupt edges, no tapering.
- Hanning, Hamming, Blackman: Smooth, tapered windows with different degrees of tapering (Blackman has the most tapering).

Magnitude Response (Linear Scale):

- Rectangular: Sharp transitions but has high sidelobes.
- Hanning, Hamming, Blackman: Smoother transitions, with Blackman having the lowest sidelobes.

Magnitude Response (Logarithmic Scale):

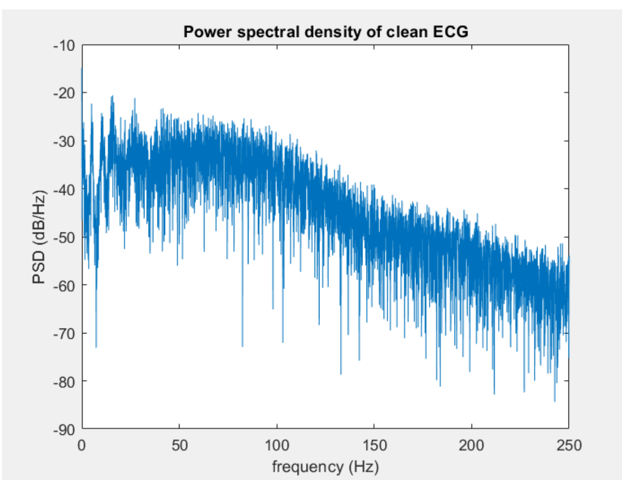
- Rectangular: High sidelobe levels, poor attenuation.
- Hanning, Hamming, Blackman: Lower sidelobes, with Blackman providing the best sidelobe attenuation.

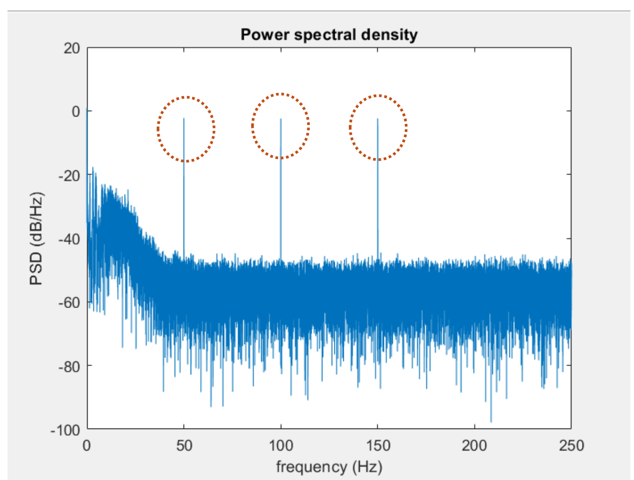
Phase Response:

- All windows produce linear phase responses as they are symmetric.

3.2. FIR Filter design and application using the Kaiser window

(i)





- The spikes in PSD of the noisy ECG signal at around 50 Hz and its harmonics are because of the added powerline noise of 50 Hz and the influence of its harmonics.
- Most ECG information is concentrated in the lower frequency range as expected in normal ECG.

(ii)

	High pass	Low pass
f_{pass}	2.0 Hz	40 Hz
f_{stop}	0.001 Hz	90 Hz
δ	0.005	0.005

	Comb filter
f_{stop}	50 Hz
f_{stop}	100 Hz
f_{stop}	150 Hz

(iii) β_{high} = 1.5099

M_{high} = 316

W_{high} = 0.004002

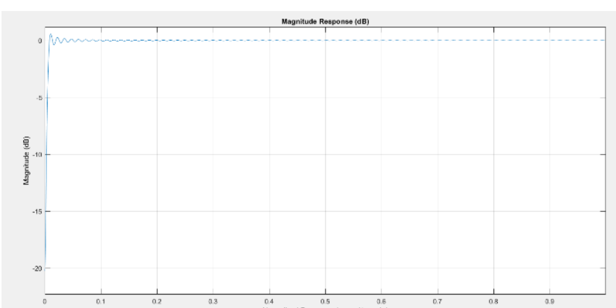
β_{low} = 1.5099

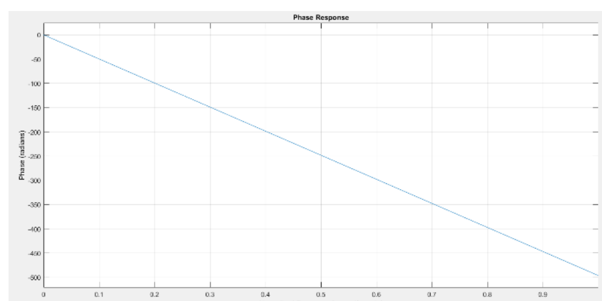
M_{low} = 13

W_{low} = 0.26

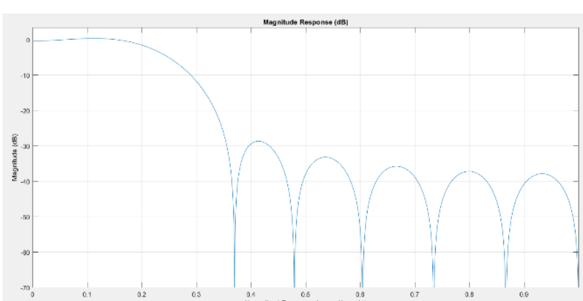
(iv)

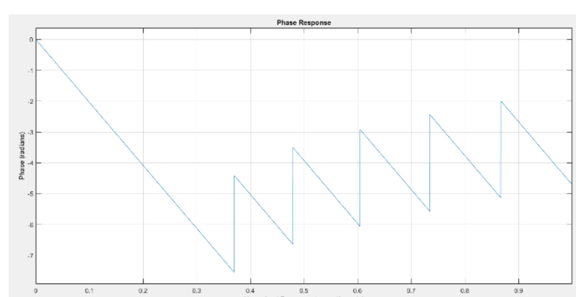
High pass filter



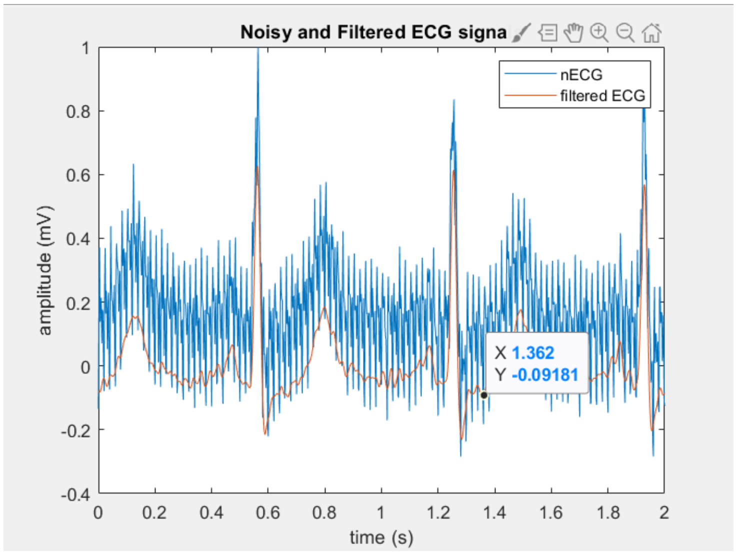


Low pass filter



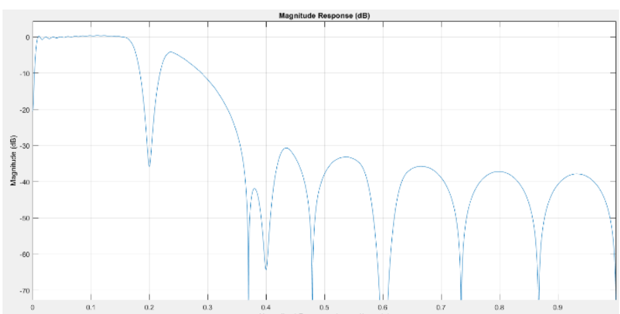


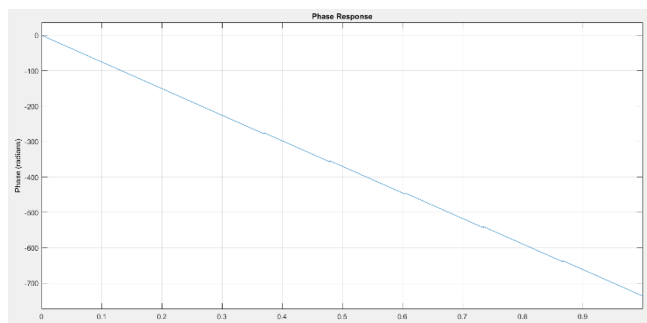
(v)



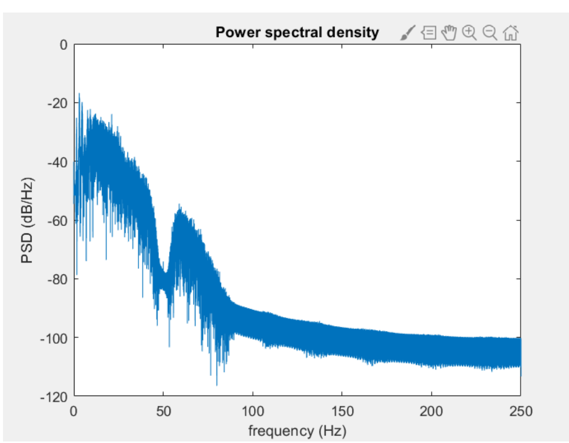
(vi)

combined filter





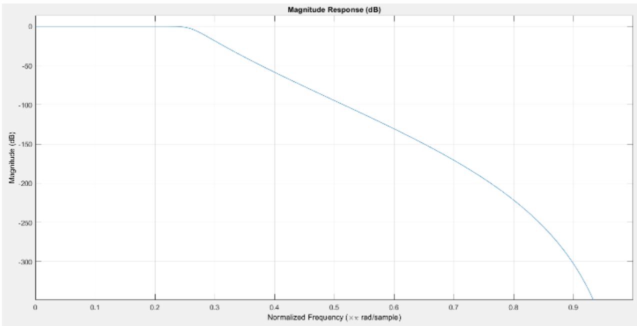
PSD of the filtered signal

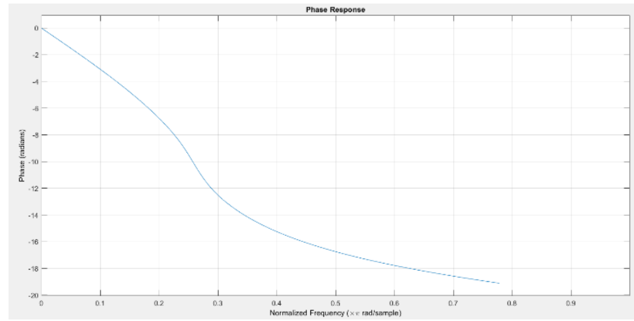


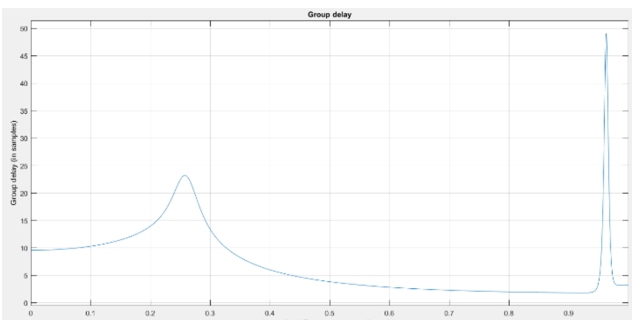
Question 04. IIR Filtering

4.1. Realizing IIR filters

(ii) low pass filter

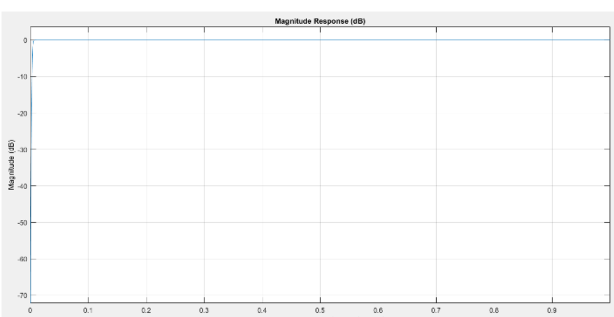


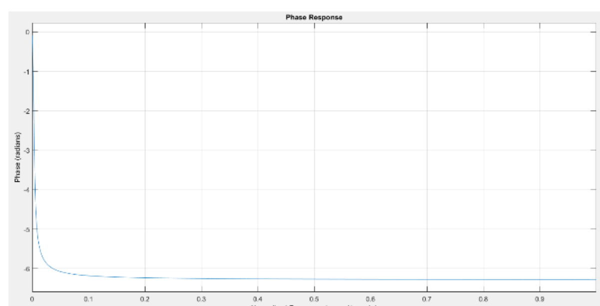


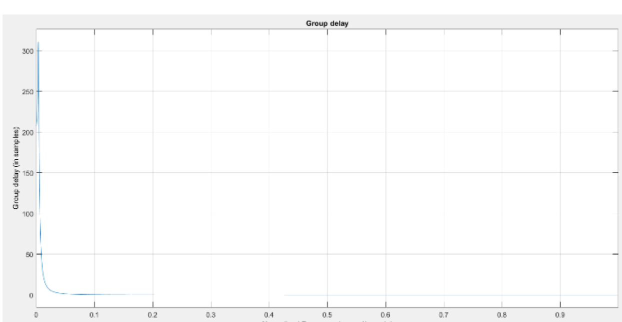


(iii)

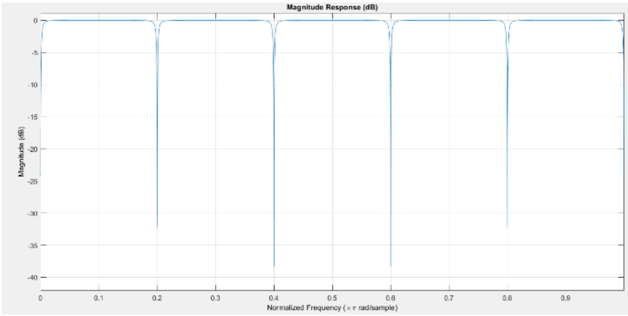
High pass filter

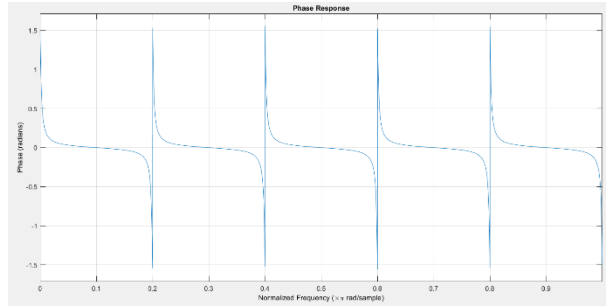


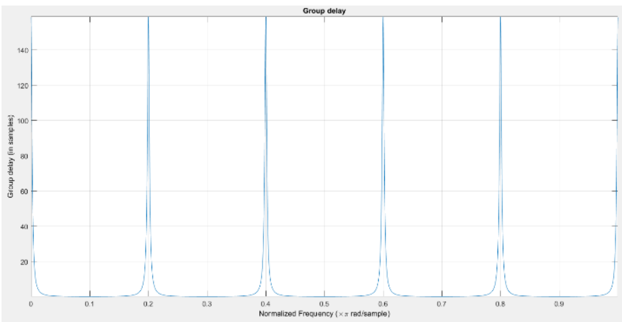




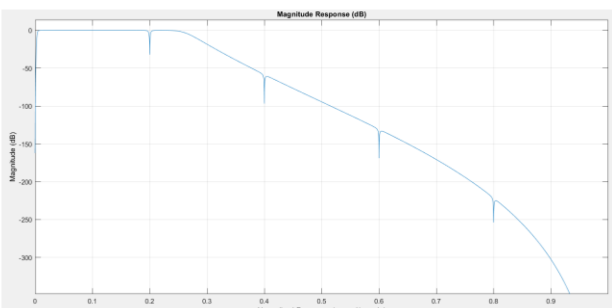
Comb filter

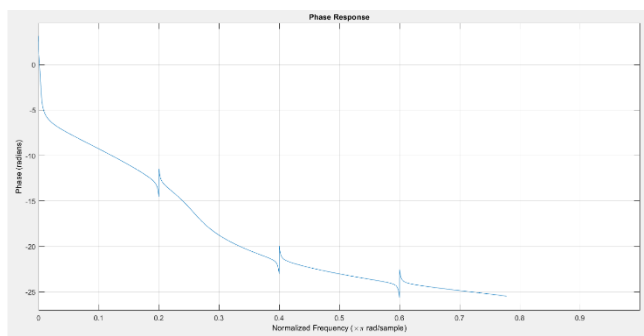


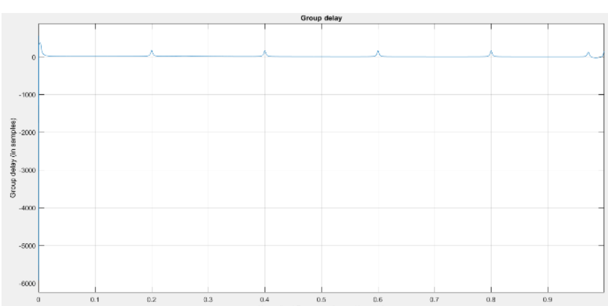




(iv) combined filter



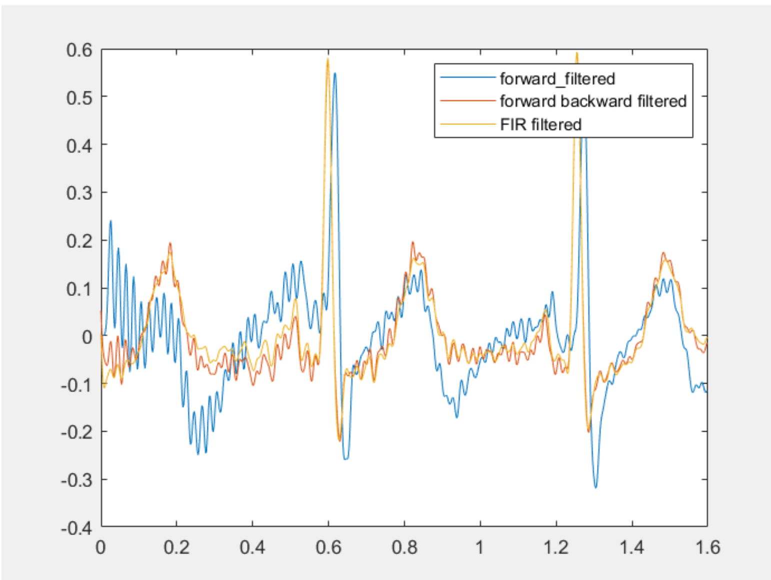




- Magnitude Response: The IIR filters exhibited sharper transitions and higher peaks, while the FIR filters provided a smoother response.
- Phase Response: The FIR filters maintained a linear phase response, ensuring minimal phase distortion, whereas the IIR filters displayed nonlinear phase characteristics.

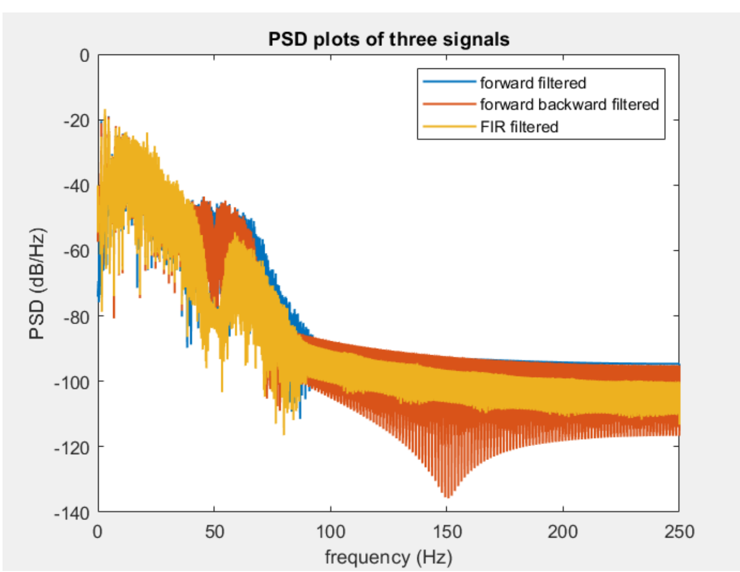
4.2. Filtering methods using IIR filters

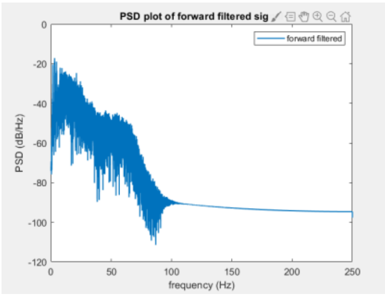
(iii)

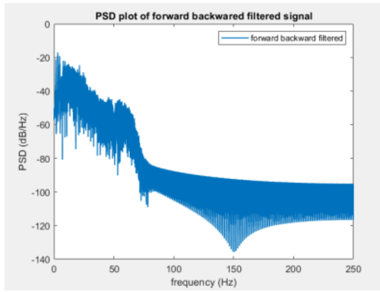


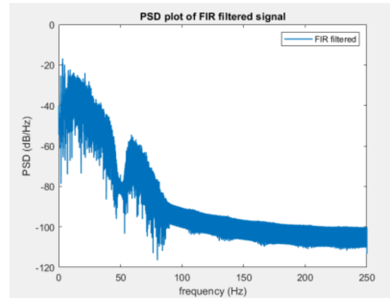
- IIR Forward Filtering: Introduces phase shift.
- IIR Forward-Backward Filtering: Zero-phase filtering, aligns better with the FIR response.
- FIR Filter: Linear phase and minimal distortion.

(iv)









- FIR Filter: Smoothest PSD due to linear phase and consistent attenuation.
- IIR Forward Filtering: Some spectral distortion due to phase shifts.
- IIR Forward-Backward Filtering: Similar to FIR but has sharper transitions.