**Methods used in analyse the spectrum of the signal**

1. **Fast Fourier Transform (FFT)**

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| **Advantages** | Efficient and fast computation  Widely used |
| **Disadvantages** | Shares limitations with FT  Lack of time resolution for non-stationary signals |
| **Applications** | Real-time signal processing  Audio and image processing |

In FFT common approach is to assume that the signal is stationary over short time intervals.

* The signal is divided into small segments or windows. Each segment is short enough that the signal can be approximated as stationary within that interval.
* The FFT is applied to each segment to analyse the frequency content. This gives a snapshot of the frequency components present in the signal during that short interval.
* This process is repeated by sliding the window over the entire signal. Each windowed segment is analysed using FFT, creating a series of frequency spectra over time.

**Why FFT is Suitable for Spectrum Analysers**

* FFT is computationally efficient, making it possible to perform real-time or near-real-time analysis of signals.
* FFT provides detailed frequency resolution, allowing for the identification of closely spaced frequency components within each window.

1. **Fourier Transform (FT)**

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| **Advantages** | Provides exact frequency representation  Suitable for stationary signals |
| **Disadvantages** | Not effective for non-stationary signals  Cannot provide time information |
| **Applications** | Analysing signals that are constant over time |

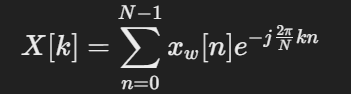
The Fourier Transform (FT) is a mathematical technique used to transform a time-domain signal into its frequency-domain representation.

**Steps for Using Fourier Transform in Signal Processing**

* 1. Obtain the analogue signal from the source (e.g., a microphone, sensor, or other measurement device).
  2. Apply an analogue filter to the signal to remove unwanted noise or to focus on a particular frequency band.
  3. Convert the continuous analogue signal into a discrete digital signal using an Analog-to-Digital Converter (ADC).
  4. Ensure the sampling rate (fs) is at least twice the highest frequency component (fmax​) in the signal to avoid aliasing.
  5. The ADC produces a series of discrete-time samples x[n] from the analogue signal.
  6. Apply a window function (e.g., Hamming, Hanning, Blackman) to the discrete samples to reduce spectral leakage.
  7. Mathematically, apply window function w[n] to the signal,

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* 1. Since practical implementation involves discrete data, compute the DFT of the windowed signal. The DFT is defined as,

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where N is the number of samples, xw[n] is the windowed signal, X[k] is the DFT of the signal, and k is the frequency bin index.

* 1. The result X[k] is a complex number representing the amplitude and phase of the frequency component k. Compute the magnitude spectrum separately and phase spectrum separately.

**Why FT is Not Used Over FFT**

* The Fast Fourier Transform (FFT) is a more efficient algorithm that reduces the computational complexity from (N^2) to (NlogN), making it suitable for real-time applications.
* The FFT is designed for discrete signals, making it suitable for digital data processing.
* FFT algorithms can be efficiently implemented in hardware, making them ideal for real-time signal processing in devices like spectrum analysers**.**

1. **Short-Time Fourier Transform (STFT)**

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| **Advantages** | Provides time-frequency representation  suitable for analysing quasi-stationary signal |
| **Disadvantages** | Fixed window size limits resolution trade-off between time and frequency |
| **Applications** | Speech analysis, Time-varying signal analysis |

**Similarities between FFT and STFT**

* Both methods involve segmenting the signal into short intervals.
* Both apply the FFT to these short segments to analyse the frequency content.
* Both assume the signal is approximately stationary within each short segment.

**Short-Time Fourier Transform (STFT) Process**

The STFT provides a way to analyse the time-varying frequency content of a signal by breaking it down into short segments and applying the FFT to each segment.

* consider a discrete-time signal x[n].
* Select a window function w[n] of length N (e.g., Hamming, Hanning, or Gaussian window).
* Divide x[n] into overlapping segments. The amount of overlap is determined by the hop size R (The hop size R is the number of samples by which the analysis window is shifted along the signal for each subsequent FFT computation. If the window length is N and the hop size is R, the overlap between consecutive windows is N−R samples.)
* For each segment k, multiply the signal segment by the window function.
* Here, k is the segment index and n ranges from 0 to N−1.
* Apply the FFT to each windowed segment to transform it into the frequency domain.
* This gives the frequency spectrum for the segment centred around time kR.
* The resulting spectra from each windowed segment Xk[f] are combined to form a time-frequency representation of the signal.

**Why FFT is Preferred in Spectrum Analysers over STFT**

* + The STFT inherently involves the concept of windowing the signal and applying the FFT to each window. This adds a layer of complexity.
  + FFT in Spectrum Analysers ( Traditional spectrum analysers) often use a simpler approach where the signal is fed through a filter bank or a similar system to isolate frequency components, followed by an FFT.
  + FFT algorithms are highly optimized for hardware implementation. Digital signal processing (DSP) chips and specialized FFT processors can perform FFTs very efficiently, making them ideal for real-time spectrum analysis.
  + While STFT can be implemented in hardware, it requires additional steps to manage the windowing process, which can increase complexity and resource usage.
  + STFT is particularly useful in applications where the time-frequency representation is critical, such as in audio signal processing, speech analysis, and other non-stationary signal analyses.
  + Traditional spectrum analysers focus primarily on frequency resolution and the identification of frequency components. The primary goal is to measure and display the frequency spectrum.
  + For a given signal x[n], the entire signal or a long segment of the signal is transformed using the FFT. This approach assumes that the signal is stationary over the duration of the segment being analysed.
  + The signal may be passed through a series of band-pass filters to isolate specific frequency ranges. Each filter outputs a narrowband signal corresponding to a specific portion of the spectrum.
  + The FFT is then applied to the output of each filter to analyse the frequency content within that band.
  + A window function may still be applied to reduce spectral leakage, but the focus is on analysing the entire signal or large segments rather than small, overlapping windows as in STFT.

1. **Wavelet Transform**

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| **Advantages** | Multi-resolution analysis  Suitable for both stationary and non-stationary signals |
| **Disadvantages** | More complex to implement and understand |
| **Applications** | Image compression  Biomedical signal processing |

The wavelet transform addresses the limitations of the STFT by using a multi-resolution approach.

* Variable Window Size used. It uses small windows for analysing high-frequency components (fast-changing) and large windows for low-frequency components (slow-changing).
* This allows for detailed time information for high frequencies and detailed frequency information for low frequencies.

**Advantages of Wavelet Transform**

* Wavelet transform provides better localization in both time and frequency domains for signals with varying characteristics.
* Suitable for non-stationary signals where the frequency content changes over time.

**Why FFT is Preferred in Spectrum Analysers over wavelet transform**

1. Traditional spectrum analysers primarily focus on providing a detailed frequency spectrum of the signal. The primary goal is to identify and measure the frequency components present in the signal.
2. The assumption is that the signal can be approximated as stationary over short segments, making the FFT suitable for this purpose.
3. FFT is computationally efficient and well-suited for real-time processing. This makes it ideal for applications requiring quick and repeated analysis.
4. FFT is simpler to implement in both hardware and software compared to wavelet transform.
5. Spectrum analysers often utilize hardware that is specifically optimized for FFT computations, ensuring fast and reliable performance.
6. Implementing wavelet transform requires more computational resources and memory management, which can complicate hardware design and increase costs.
7. Many practical applications of spectrum analysers involve signals that are either stationary or can be approximated as stationary over short intervals. This aligns well with the capabilities of FFT.
8. FFT provides high-frequency resolution, which is critical for identifying closely spaced frequency components in the spectrum.
9. **Hilbert-Huang Transform (HHT)**

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| **Advantages** | Adaptive and data-driven  Effective for non-linear and non-stationary signals |
| **Disadvantages** | Computationally intensive  Relatively new and less standardized |
| **Applications** | Geophysical data analysis  Biomedical signal analysis |

The Hilbert-Huang Transform (HHT) is a method for analysing non-stationary and nonlinear time series data.

It consists of two main components.

1. Empirical Mode Decomposition (EMD)
2. Hilbert Spectral Analysis

**How HHT Works**

**1. Empirical Mode Decomposition (EMD)**

EMD is a process used to decompose a signal into a set of intrinsic mode functions (IMFs) and a residual.

* + The signal is iteratively decomposed into IMFs. Each IMF represents a simple oscillatory mode with well-defined frequency and amplitude.
  + For each IMF, the sifting process involves extracting oscillatory components by removing the trend from the signal.

This process continues until a stopping criterion is met.

IMFs are functions with well-defined instantaneous frequency and amplitude. They are obtained by removing the high-frequency noise and extracting oscillatory components.

* + After extracting all IMFs, the residual is the remaining part of the signal that cannot be further decomposed.

**2. Hilbert Spectral Analysis**

Once the signal is decomposed into IMFs, Hilbert spectral analysis is applied to each IMF.

* The Hilbert transform is used to obtain the instantaneous frequency and amplitude of each IMF. This provides a time-frequency representation of the signal.
* Combining the instantaneous frequencies and amplitudes from each IMF gives the Hilbert spectrum, which shows how the frequency content of the signal evolves over time.

**Drawbacks of HHT**

* The EMD process can be computationally intensive, especially for long signals or signals with complex characteristics. This can make it challenging to apply in real-time or on large datasets.
* EMD can suffer from mode mixing, where different frequency components are combined into a single IMF or a single frequency component appears in multiple IMFs. This can complicate the interpretation of the results.
* The performance of EMD can be sensitive to the choice of parameters and stopping criteria. This can lead to variability in the decomposition results.
* Compared to methods like FFT, which have well-established theoretical foundations, HHT's theoretical basis is less rigorous and still an area of ongoing research.
* FFT is highly efficient and well-optimized for real-time applications. It is simpler to implement in both hardware and software compared to HHT.
* FFT provides a clear frequency spectrum with high resolution, making it ideal for applications where the primary goal is to identify and measure frequency components.
* FFT algorithms are well-supported by specialized hardware and DSP chips, providing reliable and fast performance for traditional spectrum analysis.
* FFT has a strong theoretical foundation and well-established properties, making it a reliable choice for many signal processing tasks.

1. **Periodogram**

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| **Advantages** | Simple to implement  Provides a straightforward estimate of the power spectral density |
| **Disadvantages** | High variance of the spectral estimate  Limited frequency resolution for short data segments |
| **Applications** | Preliminary spectral analysis  Power spectral density estimation |

The periodogram is a technique used to estimate the power spectral density (PSD) of a signal. The power spectral density shows how the power of a signal is distributed across different frequency components.

**How the Periodogram Works**

1. Split the signal into overlapping or non-overlapping segments. Each segment is treated as a separate time series.
2. Apply a window function (e.g., Hamming, Hanning) to each segment to reduce edge effects and leakage. This step is optional but helps mitigate spectral leakage.
3. For each windowed segment, compute the Fast Fourier Transform (FFT) to convert the time-domain segment into the frequency domain. This gives the frequency components of the segment.
4. Compute the squared magnitude of the FFT output to get the power spectrum for each segment. This represents the power of each frequency component in the segment. P(f)=∣X(f)∣ 2 Where X(f) is the FFT of the segment, and P(f) is the power spectral density estimate.
5. If multiple segments are used, average the power spectra from each segment to obtain a smoother and more reliable estimate of the power spectral density.
6. This averaging reduces variance and provides a clearer representation of the signal's frequency content.

**Drawbacks of the Periodogram**

* The periodogram can produce noisy estimates of the power spectral density, especially if the segments are short. This high variance can obscure the true frequency content of the signal.
* Spectral leakage occurs when the signal's energy spills over into adjacent frequency bins due to the finite length of the windowed segments. This can distort the frequency spectrum and affect the accuracy of the analysis.
* The accuracy of the periodogram is sensitive to the choice of window size and overlap. Selecting an inappropriate window size can either lead to poor frequency resolution or high variance.
* The periodogram does not provide time-frequency information.
* For complex or non-stationary signals, methods that offer better time-frequency localization, such as wavelet transform or Hilbert-Huang Transform (HHT), are often preferred.
  + 1. **Welch’s Method**

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| **Advantages** | Reduces variance of the spectral estimate  Averaging improves frequency resolution |
| **Disadvantages** | Slightly more complex than a basic periodogram  Smoothing can reduce peak sharpness |
| **Applications** | Power spectral density estimation in noisy environments |

Welch’s method is an improvement over the traditional periodogram for estimating the power spectral density (PSD) of a signal. It addresses some of the limitations of the periodogram, such as high variance and spectral leakage.

**How Welch’s Method Works**

* Split the signal into overlapping segments. Each segment is treated as a separate time series. The overlapping helps to reduce the variance of the spectral estimate.
* Apply a window function (e.g., Hamming, Hanning) to each segment to minimize edge effects and leakage. The window function smooths the edges of the segments and reduces the discontinuities at the segment boundaries.
* For each windowed segment, compute the Fast Fourier Transform (FFT) to convert the time-domain segment into the frequency domain.
* Compute the squared magnitude of the FFT output to obtain the power spectrum for each segment.
* Average the power spectra from all segments to obtain a more stable estimate of the power spectral density. This averaging reduces the variance of the estimate and provides a clearer frequency representation. Advantages of Welch’s Method
* By averaging the periodograms of multiple overlapping segments, Welch’s method reduces the variance of the power spectral density estimate. This provides a smoother and more reliable estimate.
* The use of window functions helps to reduce spectral leakage, where energy from one frequency component spills into adjacent frequencies.
* Welch’s method allows for a trade-off between frequency resolution and variance by adjusting the segment length and overlap. This can lead to a more accurate frequency analysis compared to a single-periodogram approach.

**Why Welch’s Method Might Not Always Be Preferred Over FFT**

* Welch’s method focuses on providing a more reliable power spectral density estimate, which is useful when the primary goal is accurate frequency analysis. For applications requiring time-frequency information, other methods like STFT or wavelet transform might be preferred.
* Traditional FFT is computationally efficient and provides a clear frequency spectrum, especially in real-time applications. Welch’s method, while reducing variance, involves additional computation due to multiple segment FFTs.
* In real-time applications where, quick frequency analysis is required, traditional FFT might be preferred due to its simplicity and efficiency.
* For signals with complex or rapidly changing characteristics, methods like wavelet transform or HHT might be more suitable as they offer better time-frequency localization.
  + 1. **Autoregressive (AR) Methods**

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| **Advantages** | High-resolution spectral estimates  Suitable for short data records |
| **Disadvantages** | Model order selection can be challenging  Computational complexity increases with model order |
| **Applications** | Speech and audio signal analysis |

**Some significant spectrum analysers**

1. **Siglent SSA3021X-TG**:
   * **Frequency Range**: 9 kHz to 3.2 GHz
   * **Displayed Average Noise Level (DANL)**: -161 dBm/Hz (normalized to 1 Hz)
   * **Phase Noise**: < -98 dBc/Hz @ 1 GHz, 10 kHz offset
   * **Resolution Bandwidth (RBW)**: 1 Hz to 1 MHz (in 1-3-10 sequence)
   * [SSA3021X-TG](https://siglentna.com/wp-content/uploads/dlm_uploads/2017/10/SSA3000X_DataSheet_DS0703X_E05A.pdf).
2. **Rigol DSA815-TG**:
   * **Frequency Range**: 9 kHz to 1.5 GHz
   * **DANL**: Ranges from -135 dBm to -161 dBm (normalized to 1 Hz)
   * **Phase Noise**: -80 to -98 dBc/Hz @ 10 kHz offset
   * **RBW**: 10 Hz minimum
   * [DSA815-TG](https://www.rigolna.com/products/spectrum-analyzers/dsa800/dsa815-tg/featured/).
3. **RF Explorer 6G Combo Plus**:
   * **Frequency Range**: 50 kHz to 6.1 GHz (expandable up to 7.5 GHz with Wideband license)
   * **Internal LNA**: 25 dB gain
   * **Attenuator**: Selectable 30 dB (up to 60 dB on the right connector)
   * [Learn more](http://j3.rf-explorer.com/6gcomboplus).
4. **Tektronix RSA507A**:
   * **Frequency Range**: 9 kHz to 18 GHz
   * **Capture Bandwidth**: 40 MHz
   * **Dynamic Range**: -160 dBm to +20 dBm
   * [RSA507A](https://www.tek.com/en/products/spectrum-analyzers/rsa500).

**Comparison of common frequency spectrum analysers**

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| Spectrum Analyzer | Frequency Range | Resolution Bandwidth (RBW) |
| Rohde & Schwarz FSW Series | 2 Hz to 85 GHz | 1 Hz to 10 MHz |
| Keysight N9030B PXA | 2 Hz to 50 GHz | 1 Hz to 8 MHz |
| Tektronix RSA5000 Series | 1 Hz to 26.5 GHz | 1 Hz to 10 MHz |
| Rigol RSA5000 Series | 9 kHz to 6.5 GHz | 1 Hz to 10 MHz |
| Siglent SSA3000X Plus Series | 9 kHz to 7.5 GHz | 1 Hz to 10 MHz |
| Keysight N9344C | 9 kHz to 20 GHz | 1 Hz to 3 MHz |
| Rohde & Schwarz FPH Series (Handheld) | 5 kHz to 26.5 GHz | 1 Hz to 10 MHz |
| Rigol DSA815-TG | 9 kHz to 1.5 GHz | 100 Hz to 1 MHz |
| Siglent SSA3021X | 9 kHz to 2.1 GHz | 1 Hz to 1 MHz |

1. **Rohde & Schwarz FSW Series**:
   * **Frequency Range**: Up to 50 GHz
   * **Displayed Average Noise Level (DANL)**: As low as -165 dBm/Hz
   * **Phase Noise**: < -98 dBc/Hz @ 10 kHz offset
   * **Third-Order Intercept**: +10 dBm
   * [Datasheet](https://www.rohde-schwarz.com/us/brochure-datasheet/fsw/).
2. **Keysight N9030B PXA**:
   * **Frequency Range**: 2 Hz to 50 GHz
   * **DANL**: As low as -165 dBm/Hz
   * **Phase Noise**: < -98 dBc/Hz @ 10 kHz offset
   * **Third-Order Intercept**: +10 dBm
   * [N9030B PXA](https://www.keysight.com/us/en/product/N9030B/pxa-signal-analyzer-multi-touch-2-hz-50-ghz.html).
3. **Tektronix RSA5000 Series**:
   * **Frequency Range**: Up to 7.5 GHz
   * **DANL**: As low as -165 dBm/Hz
   * **Phase Noise**: < -98 dBc/Hz @ 10 kHz offset
   * **Third-Order Intercept**: +14 dBm
   * [RSA5000](https://www.tek.com/en/datasheet/spectrum-analyzers-datasheet).
4. **Rigol RSA5000 Series**:
   * **Frequency Range**: Up to 7.5 GHz
   * **DANL**: As low as -165 dBm/Hz
   * **Phase Noise**: < -98 dBc/Hz @ 10 kHz offset
   * **Third-Order Intercept**: +10 dBm
   * [Rigol RSA5000](https://www.rigolna.com/products/spectrum-analyzers/rsa5000/).
5. **Siglent SSA3000X Plus Series**:
   * **Frequency Range**: Up to 7.5 GHz
   * **DANL**: As low as -165 dBm/Hz
   * **Phase Noise**: < -98 dBc/Hz @ 10 kHz offset
   * **Resolution Bandwidth (RBW)**: Minimum 1 Hz
   * [SSA3000X Plus](https://int.siglent.com/u_file/document/SSA3000X%20Plus_DataSheet_DS0703P_E03A.pdf).
6. **Keysight N9344C**:
   * **Frequency Range**: Up to 20 GHz
   * **DANL**: As low as -161 dBm/Hz
   * **Phase Noise**: < -98 dBc/Hz @ 10 kHz offset
   * **Third-Order Intercept**: +10 dBm
   * [N9344C](https://www.keysight.com/us/en/product/N9030B/pxa-signal-analyzer-multi-touch-2-hz-50-ghz.html).
7. **Rohde & Schwarz FPH Series (Handheld)**:
   * **Frequency Range**: Up to 6 GHz
   * **DANL**: As low as -161 dBm/Hz
   * **Phase Noise**: < -98 dBc/Hz @ 10 kHz offset
   * **Third-Order Intercept**: Not specified
   * [FPH Series](https://www.rohde-schwarz.com/us/brochure-datasheet/fsw/).

**Comparison of some high frequency spectrum analysers**

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| Spectrum Analyzer | Frequency Range | Resolution Bandwidth (RBW) |
| Keysight N9041B UXA | 3 Hz to 110 GHz | 1 Hz to 8 MHz |
| Rohde & Schwarz FSW85 | 2 Hz to 85 GHz (up to 500 GHz with external mixers) | 1 Hz to 10 MHz |
| Keysight N9030B PXA with external mixers | 2 Hz to 50 GHz (up to 1.1 THz with external mixers) | 1 Hz to 8 MHz |
| Anritsu MS2760A | 9 kHz to 110 GHz | 10 Hz to 10 MHz |
| Tektronix RSA7100A | 16 kHz to 110 GHz (with external mixers) | 1 Hz to 40 MHz |

1. **Keysight N9041B UXA**:
   * **Frequency Range**: 2 Hz to 110 GHz
   * **DANL @1 GHz**: -174 dBm
   * **Maximum Analysis Bandwidth**: 1 GHz
   * [N9041B UXA](https://www.keysight.com/us/en/product/N9041B/uxa-signal-analyzer-multi-touch-2-hz-110-ghz.html).
2. **Rohde & Schwarz FSW85**:
   * **Frequency Range**: Up to 85 GHz
   * **DANL**: As low as -165 dBm/Hz
   * **Phase Noise**: < -98 dBc/Hz @ 10 kHz offset
   * [FSW85](https://www.rohde-schwarz.com/us/brochure-datasheet/fsw/).
3. **Keysight N9030B PXA with external mixers**:
   * **Frequency Range**: 2 Hz to 50 GHz
   * **DANL**: As low as -134 dBc/Hz @ 1 GHz, 10 kHz offset
   * **Real-Time Bandwidth**: 320 MHz
   * [N9030B PXA](https://www.keysight.com/us/en/product/N9030B/pxa-signal-analyzer-multi-touch-2-hz-50-ghz.html).
4. **Anritsu MS2760A**:
   * **Frequency Range**: 9 kHz to 170 GHz
   * **Dynamic Range**: High performance for advanced design verification
   * **Real-Time Bandwidth**: 320 MHz (MS2760A) or 800 MHz (MS2762A)
   * [MS2760A](https://www.anritsu.com/test-measurement/products/ms2760a).
5. **Tektronix RSA7100A**:
   * **Frequency Range**: 16 kHz to 14/26.5 GHz
   * **Real-Time Bandwidth**: 320 MHz (standard) or 800 MHz (optional)
   * **Real-Time Performance**: 419 ns for 100% Probability of Intercept
   * [RSA7100A](https://www.tek.com/en/datasheet/rsa7100a).