

Problem Set HW 4

Siting Chang

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1 VC Dimesion

1.1 part a

VC dimension of H is 3.

It is trivial to show that there exists one or two points on the plane that can be shattered by H . We now show that H is able to shatter 3 points which implies that the VC dimension of H is at least 3.

Select three points with coordinates $A(1,1)$, $B(-1,1)$ and $C(-1, -1)$. There are 8 possible combination of labels which H are able to classify all. The orgin and radius choices are listed as follows for each combination. The order is: label of point A, label of point B, label of point C, origin, radius.

- +, +, +, (0,0), 2
- -, -, -, (0,0), 0.5
- +, -, -, (1,0), 1
- +, -, +, (1,-1), 2
- -, +, -, (-1,1), 1
- -, +, +, (-1,0), 1
- -, -, +, (-1,-1), 0.5
- +, +, -, (0,1), 1

Next, we show that H is not able to shatter any four points on the plane which means that the VC dimension of H is less than 4.

There are two possible situations when randomly choose four points:

- four points form a convex hull. This situation cannot be classified by any hypotese in H when the opposing points with the largest distance both have positive lables and the other two have negative labels.
- three points form a convex hull and one point is inside the hull. This situation cannot be classified when the first three points (on the convex hull) have positive label and the fourth point has negative label.

1.2 part b

VC dimension of H is $2k$.