Weighted Density for The Win: Accurate Subspace Density Clustering

APPENDIX

A. Details of the Equations

TABLE I
FREQUENTLY USED NOTATIONS.

Symbol	Meaning
k	The number of neighbors considered
m	The number of clusters
W	The weights matrix
$D = \{d_{ij}\}$	The distances of the pairs of data points in X
$SNN(i) = (x_1, \dots)$	The set of shared nearest neighbors of point i
$S_w(ij)$	The weighted SNN similarity between points i and j
$P = (P_1, \dots, P_n)$	The weighted local density
$\Delta = (\Delta_1, \dots, \Delta_n)$	The weighted distance from a larger density point
$\Gamma = (\Gamma_1, \dots, \Gamma_n)$	The weighted decision value the element-wise product of ρ and Δ
$X = (x_1, \dots, x_n)$	The dataset with x_i as its i-th data point
$N(i) = (x_1, \dots, x_k)$	The set of k-nearest neighbors of point i
$Z(i) = (x_1, \dots, x_k)$	The set of k points with the highest similarity to point i

Table I presents the major symbols and notations of the equations used in the paper.

B. Details of the Datasets

TABLE II
THE FEATURES OF COMMON-USED SYNTHETIC AND UCI DATASETS, N DENOTES THE NUMBER OF SAMPLES, D DENOTES THE NUMBER OF DIMENSIONS, AND NC DENOTES THE NUMBER OF NATURAL CLUSTERS BASED ON GROUND-TRUTHS.

ID	Type	Datasets	<N, D, NC>	ID	Type	Datasets	<N, D, NC>
1	Real	Sonar	<208, 60, 2>	7	Synthetic	Pathbased	<300, 2, 3>
2	Real	Wine	<178, 13, 3>	8	Synthetic	Aggregation	<788, 2, 7>
3	Real	Iris	<150, 4, 3>	9	Synthetic	Jain	<373, 2, 2>
4	Real	Heart	<303, 13, 2>	10	Synthetic	Spiral	<312, 2, 3>
5	Real	Pima	<768, 8, 2>	11	Synthetic	Moon	<210, 2, 2>
6	Real	Leaf	<340, 15, 30>	12	Synthetic	CMC	<1002, 2, 3>

The statistics of the 12 experimental datasets are shown in Table II. [1]–[4] "Sonar", "Wine", "Iris", "Heart", "Pima" and Leaf datasets are collected from the UCI machine learning repository. Datasets "Pathbased", "Aggregation", "Jain" and "Spiral" are from University of Eastern Finland, "Moon" and "CMC" datasets are collected from GitHub.

C. Parameters Settings of Algorithms

Some parameters need to be pre-specified for these compared clustering algorithms. For WKM approach, the parameter β for updating the attribute weights is set at 2 [5]. The number of clusters k is set according to the labels of the datasets. DBSCAN has two important parameters, the maximum radius Eps and the minimum points MinPts. d_c is the cutoff distance of DPC and its variants, which is set to be a distance at which 2%–3% of points are included on average [12]. K for SNN-DPC and DenMune represents the number of nearest neighbors. Similarly, this applies to the WDSC algorithm. T_r of DPC-CE controls the sensitivity of the connectivity estimate, while P_r adjusts the distance penalty to balance connectivity with spatial distance. δ_t of VDPC is used to select representatives. The parameter k in ICKDP controls the selection of local centers to optimize clustering performance, and K refers to the number of K-nearest neighbors in the dataset. The parameters configurations of regarding compared algorithms and proposed methods are listed in Table III.

TABLE III
PARAMETER CONFIGURATIONS OF ALGORITHMS.

Type	Algorithms	Parameters setting	Reference
Conventional	WKM	input k, β=2	[5]
	DBSCAN	Eps = 0.033 - 1.9, Minpts = 4 - 61	[6]
Representative	SNNDPC	$d_c = 2\% - 3\%$, $5 \le K \le 30$	[7]
	DenMune	$1 \le K \le 50$	[8]
Sota	DPC-CE	$d_c = 2\%$, $T_r = 0.25, P_r = 0.3$	[9]
	VDPC	$d_c = 0.2\% - 50\%$, $\delta_t = 0.04 - 40$	[10]
	ICKDP	$d_c = 2\%$, $K/4 \le k < K/2$	[11]
Ours	WDSC	$d_c = 2\% - 3\%$, $5 \le K \le 30$	-

TABLE IV
ARI AND FMI PERFORMANCE OF WDSC AND SEVEN COUNTERPARTS.

ARI								
Data	**W-KM**	**DBSCAN**	**SNNDPC**	**DenMune**	**DPC-CE**	**VDPC**	**ICKDP**	**WDSC**
Pathbased	0.3113	0.4735	0.9294	0.9239	0.4623	**1**	0.4311	0.9699
Aggregation	0.5577	0.7997	0.9731	0.9941	0.9978	**1**	0.9978	0.9920
Jain	0.4445	0.4215	0.5612	**1**	**1**	**1**	0.6965	**1**
Spiral	-0.0048	0.4079	**1**	0.6887	**1**	**1**	0.0414	**1**
Moon	0.7748	0.0403	**1**	**1**	0.1426	0.3807	0.3574	**1**
CMC	0.0749	0.8434	0.8198	**1**	0.2140	0.1476	0.4490	**1**
Sonar	-0.0047	0.0012	0.0533	0.0001	0.0253	0.0052	0.0443	**0.0581**
Wine	0.8620	0.2524	0.8992	0.8819	0.3715	0.2860	0.3715	**0.9637**
Iris	0.6943	0.6789	0.9038	0.7455	0.6634	0.7074	0.7445	**0.9222**
Heart	0.1851	0.0626	0.1508	0.1410	0.1723	0.1723	0.1507	**0.4382**
Pima	0.0206	0.0199	0.0131	0.0107	0.0119	**0.1707**	0.0787	0.0996
Leaf	0.2576	0.0238	0.2510	0.0592	0.0114	0.0100	0.0430	**0.2620**
Total Rank	**69**	**77**	**41**	**48**	**57**	**45**	**58**	**18**
Avg Rank	**5.75**	**6.42**	**3.42**	**4.00**	**4.75**	**3.75**	**4.83**	**1.50**
Rank	**7**	**8**	**2**	**4**	**5**	**3**	**6**	**1**
FMI								
Data	**W-KM**	**DBSCAN**	**SNNDPC**	**DenMune**	**DPC-CE**	**VDPC**	**ICKDP**	**WDSC**
Pathbased	0.5457	0.5980	0.9529	0.9489	0.6652	**1**	0.6508	0.9799
Aggregation	0.6508	0.8842	0.9789	0.9953	0.9983	**1**	0.9983	0.9937
Jain	0.8200	0.6210	0.8123	**1**	**1**	**1**	0.8739	**1**
Spiral	0.3275	0.5650	**1**	0.8297	**1**	**1**	0.4184	**1**
Moon	0.9084	0.4114	**1**	**1**	0.6470	0.7116	0.7030	**1**
CMC	0.5048	0.8143	0.8896	**1**	0.6235	0.4759	0.6862	**1**
Sonar	0.5144	0.0012	0.5702	**0.7070**	0.5132	0.6653	0.5217	0.6087
Wine	0.9233	0.2524	0.9330	0.9215	0.5834	0.7009	0.5834	**0.9759**
Iris	0.9233	0.6789	0.9355	0.8321	0.7824	0.8084	0.8306	**0.9479**
Heart	0.5543	0.0626	0.6177	0.4556	0.6299	0.6299	0.6251	**0.7231**
Pima	0.5275	0.0199	**0.7106**	0.7080	0.7093	0.5550	0.7050	0.6274
Leaf	0.3167	0.0238	0.2930	0.0858	0.0811	0.1133	0.0946	**0.3452**
Total Rank	**68**	**88**	**38**	**41**	**53**	**42**	**60**	**23**
Avg Rank	**5.67**	**7.33**	**3.17**	**3.42**	**4.42**	**3.50**	**5.00**	**1.92**
Rank	**7**	**8**	**2**	**3**	**5**	**4**	**6**	**1**

D. Clustering Performence of WDSC

All the results are averaged by 10 runs of the experiments. The bold numbers indicate the best performance for a given dataset and metric. WDSC consistently outperforms other algorithms on most datasets, as can be seen in Table IV where

WDSC's column has the most bold entries, highlighting its validity and reliability. **Observations: (1)** Although WDSC cannot reach 1 in the pathbased and aggregation datasets, it is the first and second closest algorithm to 1. **(2)** Real datasets like "Wine", "Heart", "Leaf" present a different challenge as they often contain noise and may have clusters without clear boundaries. The WDSC algorithm demonstrates strong performance on these datasets as well, suggesting that it is capable of dealing with the complexities of real-world data. **Conclusion:** WDSC's superior performance across diverse datasets can be attributed to its innovative hybrid approach that combines subspace and density-based clustering with adaptive weighting and iterative optimization.

E. Significance Experiment Results

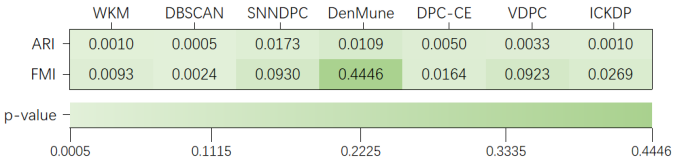

Fig. 1. p-values of Wilcoxon signed rank test in comparing out method against the other methods on ARI and FMI metrics.

In the significance experiments, we compared the scores of the WSDC algorithm with other clustering algorithms on different datasets and evaluation metrics (ARI and FMI) using the Wilcoxon Signed Rank Test [13]. This was done to verify whether there is a statistically significant difference or advantage of our proposed algorithm over other clustering algorithms. The experimental results are shown in Fig. 1. In this experiment, we set the significance level at 0.05 to balance error control and result sensitivity. The larger the p-values, the darker the color.

F. Effectiveness of the Core Components of WDSC

TABLE V
ABLATION STUDY OF WDSC ON EIGHT DATASETS. THE BEST RESULTS IN TERM OF ARI AND FMI VALIDITY METRIC IS MARKED IN **BOLDFACE**.

Data	Baseline		WDC		WDSC	
	ARI	FMI	ARI	FMI	ARI	FMI
Pathbased	0.9294	0.9529	**0.9699**	**0.9799**	**0.9699**	**0.9799**
Aggregation	0.9731	0.9789	**0.9920**	**0.9937**	**0.9920**	**0.9937**
Jain	0.5612	0.8123	0.6017	0.8310	**1.0000**	**1.0000**
Sonar	0.0533	0.5702	**0.1026**	0.5518	0.0581	**0.6087**
Wine	0.8992	0.9330	0.9325	0.9552	**0.9637**	**0.9759**
Heart	0.1508	0.6177	0.4037	0.7028	**0.4382**	**0.7231**
Pima	0.0131	**0.7106**	0.0826	0.6047	**0.0996**	0.6274
Leaf	0.2510	0.2930	0.2575	0.3245	**0.2620**	**0.3452**

We assessed the effectiveness of the core components of WDSC, namely the weight computation method and the learning mechanism. The effectiveness of the weighted approach was verified by comparing the representative density peak clustering algorithm SNNDPC (Baseline) with its weighted version, SNNDPC+W (Baseline+W), hereinafter referred to as WDC. The learning mechanism's effectiveness was validated by comparing WDC with the complete version of WDSC. Results shown in Table V demonstrate that WDC outperforms SNNDPC on most datasets, proving the efficacy of the weight computation method, while WDSC significantly surpasses WDC, confirming the success of the learning mechanism.

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