Unit 3 Review #2: Trigonometric Functions

1. Convert the following to exact radian measure.
   a) $230°$ b) $-72°$

2. Convert the following to degrees.
   a) $\frac{3\pi}{8}$ b) $2.3 radians$

3. State the principal angle, related acute angle, co-related acute angle, and two coterminal angles for each of the following. Give your answers in the same units as the given angle.
   a) $197°$ b) $-252°$ c) $1045°$
   d) $\frac{8\pi}{5}$ e) $\frac{-18\pi}{13}$ f) $\frac{53\pi}{3}$

4. Use the definition of radian measure $\theta = \frac{a}{r}$ to find the arc length, radius or angle given the other two quantities.
   a) $a = 8, r = 2$ b) $a = 3\pi, r = 4$ c) $r = 5, \theta = \frac{\pi}{6}$ d) $r = 7.3, \theta = 1.2$
   e) $\theta = \frac{3\pi}{5}, a = 4\pi$ f) $\theta = 1.3, a = 65$ g) $\theta = 3\pi, r = \frac{1}{3}$ h) $a = \frac{2\pi}{3}, \theta = 2.6$

5. Given the point $P\left(-3,\sqrt{7}\right)$ on the terminal arm of an angle, θ, in standard position, state the exact values of the six trig ratios.

6. The point $P\left(-\sqrt{33},-4\right)$ lies on the terminal arm of a rotation angle, θ, in standard position. Evaluate:
   a) $\csc\theta$ b) $3\tan\theta - 2\cot\theta$ c) $1 - \sin^2\theta$ d) $\frac{\csc\theta + \sec\theta}{\tan\theta}$

7. $\cos\theta = \frac{-40}{41}$ and θ is in QII. Provide the exact value of:
   a) $\sin\theta$ b) $\csc(-\theta)$

8. Using special triangles, provide an exact value for the each of the following expressions. Provide a labelled diagram as part of your solution.
   a) $\cos 225°$ b) $\sin\frac{7\pi}{6}$ c) $\tan(-\pi)$ d) $\sec 120°$
   e) $\csc\left(\frac{3\pi}{2}\right)$ f) $\cot(-150°)$

9. State three equivalent expressions, including one containing an acute angle, for each of the following trigonometric ratios.

a) $\sin\frac{5\pi}{3}$ b) $\csc\frac{3\pi}{4}$ c) $\cot\left(-\frac{5\pi}{6}\right)$ d) $\cos\frac{5\pi}{9}$ e) $\tan\frac{11\pi}{8}$

10. Provide the required information for the given functions:

a) $y=\frac{-3}{2}\cos\left[3\left(x-\frac{\pi}{3}\right)\right]$ range, period, co-ordinates of a maximum point

b) $y=6\sec\left(\frac{3}{4}x+\frac{3\pi}{2}\right)-2$ range, period, general expression for eq'n of VAs

c) $y=2\tan\left[\frac{\pi}{10}(x+3)\right]$ range, period, general expression for eq'n of VAs

11. Graph **two** cycles of each of the following functions. State all required information.

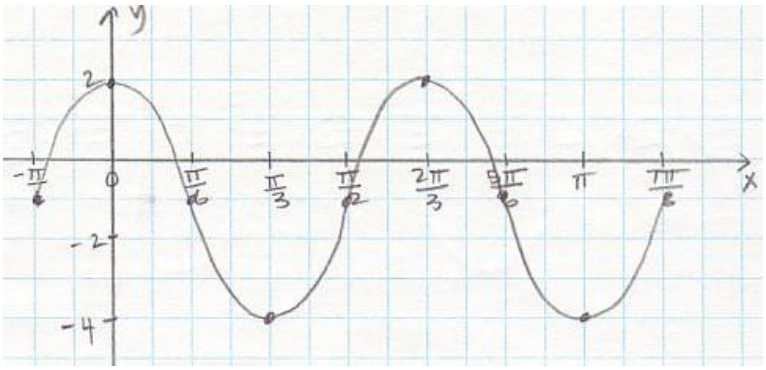
a) $y=-3\sin\left[\frac{2}{3}\left(x+\frac{3\pi}{2}\right)\right]-1$ b) $y=2\sec\left[3\left(x-\frac{\pi}{6}\right)\right]+1$

12. State the equation of a trigonometric function given:

a) cosine function, max. = 19, min. = 7, period = 5π, phase shift = $\frac{-2\pi}{3}$

b) cosecant function, equation of axis $y=3$, local max = 1, V.A.s at $\theta=\frac{\pi}{4}+\frac{\pi}{3}n,\ n\in I$

c)



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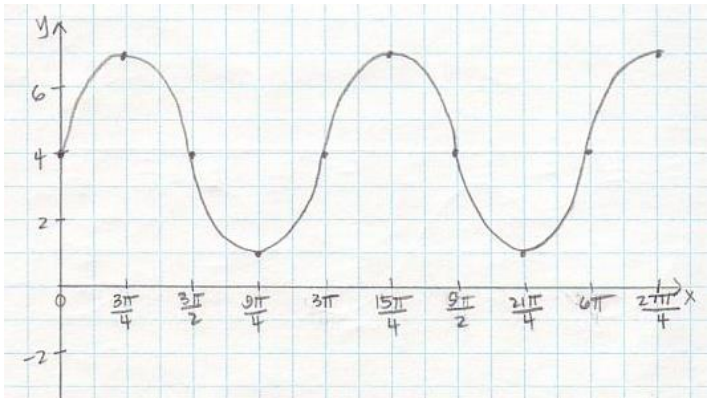

13. a) Graph $f(x) = \sec x$ for $-\pi \le x \le 2\pi$.

b) Using your graph,

i) state the equations of all vertical asymptotes

ii) state the range of $f(x)$

iii) solve $f(x) > 0$

ANSWERS:

1.a) $\frac{23\pi}{18}$ b) $\frac{-2\pi}{5}$ or $\frac{8\pi}{5}$ 2.a) $67.5°$ b) $\approx 131.8°$

3.a) $197°, 17°, 73°, 557°, 917°$ (note: many answers possible for the two coterminal angles)

b) $108°, 72°, 18°, 468°, 828°$ c) $325°, 35°, 55°, 685°, -35°$ d) $\frac{8\pi}{5}, \frac{2\pi}{5}, \frac{\pi}{10}, \frac{-2\pi}{5}, \frac{18\pi}{5}$

e) $\frac{8\pi}{13}, \frac{5\pi}{13}, \frac{3\pi}{26}, \frac{34\pi}{13}, \frac{60\pi}{13}$ f) $\frac{5\pi}{3}, \frac{\pi}{3}, \frac{\pi}{6}, \frac{11\pi}{3}, \frac{-\pi}{3}$

4.a) $\theta = 4$ b) $\theta = \frac{3\pi}{4}$ c) $a = \frac{5\pi}{6}$ d) $a = 8.76$ e) $r = \frac{20}{3}$ f) $r = 50$ g) $a = \pi$ h) $r = \frac{10\pi}{39}$

5. $\sin\theta = \frac{\sqrt{7}}{4}, \cos\theta = \frac{-3}{4}, \tan\theta = \frac{-\sqrt{7}}{3}, \csc\theta = \frac{4\sqrt{7}}{7}, \sec\theta = \frac{-4}{3}, \cot\theta = \frac{-3\sqrt{7}}{7}$

6.a) $-\frac{7}{4}$ b) $-\frac{3\sqrt{33}}{22}$ c) $\frac{33}{49}$ d) $\frac{-28-7\sqrt{33}}{16}$

7. a) $\frac{9}{41}$ b) $\frac{-41}{9}$

8. a) $\frac{-\sqrt{2}}{2}$ b) $\frac{-1}{2}$ c) 0 d) – 2 e) – 1 f) $\sqrt{3}$

9.a) $-\sin\frac{\pi}{3}, -\cos\frac{\pi}{6}, -\cos\frac{11\pi}{6}$ b) $\csc\frac{\pi}{4}, \sec\frac{\pi}{4}, -\csc\frac{5\pi}{4}$ c) $\cot\frac{\pi}{6}, \tan\frac{\pi}{3}, \tan\frac{4\pi}{3}$

d) $-\cos\frac{4\pi}{9}, -\sin\frac{\pi}{18}, -\sin\frac{17\pi}{18}$ e) $\tan\frac{3\pi}{8}, \cot\frac{\pi}{8}, \tan\frac{15\pi}{8}$

10.a) $\left\{ y \in R \mid -1.5 \le y \le 1.5 \right\}, \frac{2\pi}{3}, \left(\frac{2\pi}{3}, \frac{3}{2}\right)$ b) $\left\{ y \in R \mid y \le -8 \text{ or } y \ge 4 \right\}, \frac{8\pi}{3}, x = \frac{4\pi}{3}n, n \in I$

c) $\left\{ y \in R \right\}, 10, x = 2 + 10n, n \in I$

12.a) $y = \pm 6\cos\left[\frac{2}{5}\left(x + \frac{2\pi}{3}\right)\right] + 13$ b) $y = \pm 2\csc\left[3\left(\theta - \frac{\pi}{4}\right)\right] + 3$ c) $y = 3\sin\left[3\left(x + \frac{\pi}{6}\right)\right] - 1$

d) $y = 3\sin\left(\frac{2}{3}x\right) + 4$

13.b) i) $x = -\frac{\pi}{2}, x = \frac{\pi}{2}, x = \frac{3\pi}{2}$ ii) $\left\{ y \in R \middle| y \le -1 \text{ or } y \ge 1 \right\}$

iii) $\left\{ x \in R \middle| -\frac{\pi}{2} < x < \frac{\pi}{2} \text{ or } \frac{3\pi}{2} < x \le 2\pi \right\}$