Equivalent Trigonometric Functions

Four students each wrote an equation for the function shown. Who is correct?

A. $y = -\sin\theta$

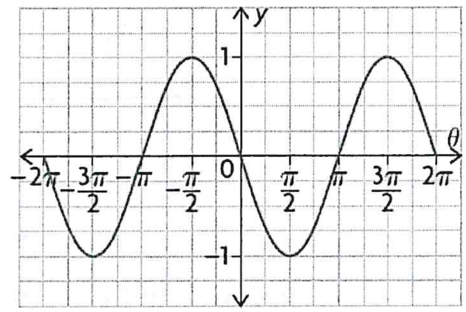
B. $y = \sin(\theta + \pi)$

C. $y = \sin(\theta - \pi)$

D. $y = \cos\left(\theta + \frac{\pi}{2}\right)$

All of them are correct !!

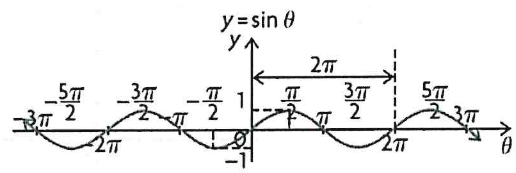
ie. pick any x-values, all 4 equations will produce the same y-value.



Equivalent Trigonometric Functions:

Two expressions may be equivalent if the <u>graph of the functions</u> created are equivalent over the entire <u>DOMAIN</u> of both functions.

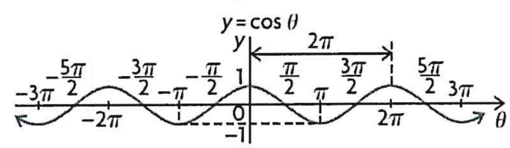
1. Using period of a function:



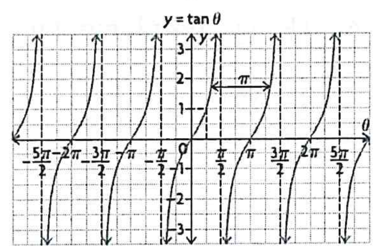
$\sin\theta = \sin(\underline{\theta + 2\pi})$

$\cos\theta = \cos(\underline{\theta + 2\pi})$

cycle begins after period.



$\tan\theta = \tan(\underline{\theta + \pi})$



Special:

$\sin\theta = \cos(\underline{\theta - \frac{\pi}{2}})$ shifted right $\frac{\pi}{2}$

$\cos\theta = \sin(\underline{\theta + \frac{\pi}{2}})$ shifted left $\frac{\pi}{2}$

2. Using Odd or Even

Sine function is <u>odd</u>. $\sin(-\theta) =$ $-\sin\theta$

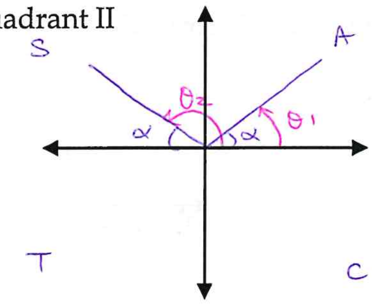
Cosine function is <u>even</u>. $\cos(-\theta) =$ $\cos\theta$

Tangent function is <u>odd</u>. $\tan(-\theta) =$ $-\tan\theta$

Recall:
Even Function $f(-x) = f(x)$
Odd Function $f(-x) = -f(x)$

3. Using CAST Rule and Related Acute Angle (α)

In Quadrant II



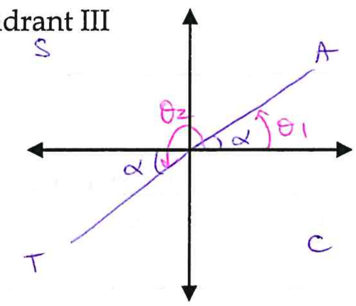
$\theta_1 = \alpha \qquad \theta_2 = \pi - \alpha$

$\sin\alpha = \sin(\pi - \alpha) \qquad \csc\alpha = \csc(\pi - \alpha)$

$\cos\alpha = -\cos(\pi - \alpha) \qquad \sec\alpha = -\sec(\pi - \alpha)$

$\tan\alpha = -\tan(\pi - \alpha) \qquad \cot\alpha = -\cot(\pi - \alpha)$

In Quadrant III



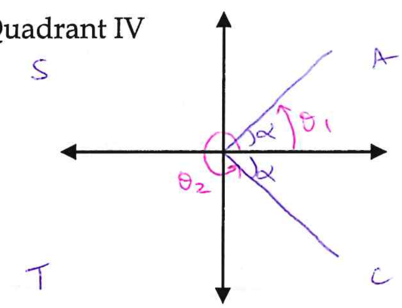
$\theta_1 = \alpha \qquad \theta_2 = \pi + \alpha$

$\sin\alpha = -\sin(\pi + \alpha) \qquad \csc\alpha = -\csc(\pi + \alpha)$

$\cos\alpha = -\cos(\pi + \alpha) \qquad \sec\alpha = -\sec(\pi + \alpha)$

$\tan\alpha = \tan(\pi + \alpha) \qquad \cot\alpha = \cot(\pi + \alpha)$

In Quadrant IV



$\theta_1 = \alpha \qquad \theta_2 = 2\pi - \alpha$

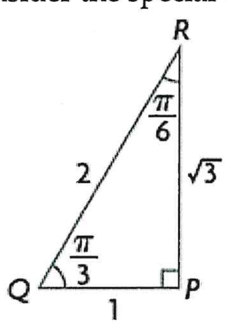
$\sin\alpha = -\sin(2\pi - \alpha) \qquad \csc\alpha = -\csc(2\pi - \alpha)$

$\cos\alpha = \cos(2\pi - \alpha) \qquad \sec\alpha = \sec(2\pi - \alpha)$

$\tan\alpha = -\tan(2\pi - \alpha) \qquad \cot\alpha = -\cot(2\pi - \alpha)$

4. Using Complementary Angles (Angles that add up to 90° or $\frac{\pi}{2}$ rad. **)**

Consider the special triangle:



$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \qquad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\tan\left(\frac{\pi}{3}\right) = \sqrt{3} \qquad \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3} \qquad \csc\left(\frac{\pi}{6}\right) = 2$

$\sec\left(\frac{\pi}{3}\right) = 2 \qquad \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$

$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3} \qquad \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$

$$\underbrace{\begin{aligned}
\sin\left(\tfrac{\pi}{3}\right) = \cos\left(\tfrac{\pi}{6}\right) &\Rightarrow \sin\theta = \cos\left(\tfrac{\pi}{2} - \theta\right) \\
\cos\left(\tfrac{\pi}{3}\right) = \sin\left(\tfrac{\pi}{6}\right) &\Rightarrow \cos\theta = \sin\left(\tfrac{\pi}{2} - \theta\right) \\
\tan\left(\tfrac{\pi}{3}\right) = \cot\left(\tfrac{\pi}{6}\right) &\Rightarrow \tan\theta = \cot\left(\tfrac{\pi}{2} - \theta\right)
\end{aligned}}_{\text{called co-functions !!}}$$

Examples:

1. Determine if the following statement is true or false. Justify your reasoning.
$$\sin(\theta)=\cos(\theta+3\pi)$$

No! by counterexample. Let $\theta=\frac{\pi}{4}$.

$LS=\sin\left(\frac{\pi}{4}\right)$
$=\frac{1}{\sqrt{2}}$

$RS=\cos\left(\frac{\pi}{4}+3\pi\right)$
$=\cos\left(\frac{13\pi}{4}\right)$
$=\frac{-1}{\sqrt{2}}$

$\therefore LS\neq RS$
$\therefore$ Not true.

2. Simplify each of the following expression in terms of one trigonometric function:

a) $\sin(x-\pi)$

$=\sin[-(\pi-x)]$ Factor negative.
$=-\sin(\pi-x)$ sine function is odd
$=-\sin x.$ using CAST rule & related acute angle (QII)

b) $\tan x+\tan(\pi-x)+\cot\left(\frac{\pi}{2}-x\right)-\tan(2\pi-x)$

$=\tan x+\underbrace{(-\tan x)}_{QII}+\underbrace{(\tan x)}_{\text{co-function}}+\underbrace{\tan x}_{QIV}.$

$=2\tan x.$

3. Write an equivalent expression for $\sin\left(\frac{3\pi}{10}\right)$,

a) using period of a function

$\sin\left(\frac{3\pi}{10}\right)=\sin\left(\frac{3\pi}{10}+2\pi\right)$
$=\sin\left(\frac{3\pi}{10}+\frac{20\pi}{10}\right)$
$=\sin\left(\frac{23\pi}{10}\right)$

b) using symmetry

$\sin\left(\frac{3\pi}{10}\right)=-\sin\left(-\frac{3\pi}{10}\right)$

c) using related acute angle

using QII

$\sin\left(\frac{3\pi}{10}\right)=\sin\left(\pi-\frac{3\pi}{10}\right)$
$=\sin\left(\frac{10\pi}{10}-\frac{3\pi}{10}\right)$
$=\sin\left(\frac{7\pi}{10}\right)$

using calculator
LS = 0.8090 RS = 0.8090
LS=RS!!

d) using cofunction identities

$\sin\left(\frac{3\pi}{10}\right)=\cos\left(\frac{\pi}{2}-\frac{3\pi}{10}\right)$
$=\cos\left(\frac{5\pi}{10}-\frac{3\pi}{10}\right)$
$=\cos\left(\frac{2\pi}{10}\right)$
$=\cos\left(\frac{\pi}{5}\right)$