Proving Trigonometric Identities – Day 2

Identities Involving Reciprocal, Quotient, And Pythagorean Identities

1. Prove:

a) $\sin x \tan x = \sec x - \cos x$

b) $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

c) $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$

d) $\sec^2 x - \sec^2 y = \tan^2 x - \tan^2 y$

e) $\dfrac{\tan a + \tan b}{\cot a + \cot b} = \tan a \tan b$

f) $(\sec\theta - \cos\theta)(\csc\theta - \sin\theta) = \dfrac{\tan\theta}{1 + \tan^2\theta}$

g) $\cos^6\theta + \sin^6\theta = 1 - 3\sin^2\theta + 3\sin^4\theta$

h) $\sec^6\theta - \tan^6\theta = 1 + 3\tan^2\theta\sec^2\theta$

i) $\cos^2 x\cos^2 y + \sin^2 x\sin^2 y + \sin^2 x\cos^2 y + \sin^2 y\cos^2 x = 1$

Identities Involving Compound Angle Formulas

2. Prove:

a) $1 + \cot x \tan y = \dfrac{\sin(x+y)}{\sin x \cos y}$

b) $\cos(x+y)\cos y + \sin(x+y)\sin y = \cos x$

c) $\sin x - \tan y \cos x = \dfrac{\sin(x-y)}{\cos y}$

d) $\cos\left(\dfrac{3\pi}{4} + x\right) + \sin\left(\dfrac{3\pi}{4} - x\right) = 0$

e) $\dfrac{\tan\left(\dfrac{\pi}{4} + x\right) - \tan\left(\dfrac{\pi}{4} - x\right)}{\tan\left(\dfrac{\pi}{4} + x\right) + \tan\left(\dfrac{\pi}{4} - x\right)} = 2\sin x \cos x$

f) $\sin(a+b)\sin(a-b) = \cos^2 b - \cos^2 a$

g) $\tan(a+b)\tan(a-b) = \dfrac{\sin^2 a - \sin^2 b}{\cos^2 a - \sin^2 b}$

h) $\dfrac{\tan(a-b) + \tan b}{1 - \tan(a-b)\tan b} = \tan a$

i) $\sin(5\theta) = \sin\theta\left(\cos^2(2\theta) - \sin^2(2\theta)\right) + 2\cos\theta\cos(2\theta)\sin(2\theta)$

Identities Involving Double Angle Formulas

3. Prove:

a) $2\csc(2\theta) = \sec\theta\csc\theta$

b) $\dfrac{1+\cos a}{\sin a} = \cot\dfrac{a}{2}$

c) $\dfrac{\cos(2x)}{1+\sin(2x)} = \tan\left(\dfrac{\pi}{4} - x\right)$

d) $2\cot(2b) = \cot b - \tan b$

e) $\sec x - \tan x = \tan\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$

f) $\dfrac{1-\cos 2x + \sin 2x}{1+\cos 2x + \sin 2x} = \tan x$

g) $4(\cos^6\theta + \sin^6\theta) = 1 + 3\cos^2(2\theta)$

h) $\left(\dfrac{\sin 2x}{1+\cos 2x}\right)\left(\dfrac{\cos x}{1+\cos x}\right) = \tan\dfrac{x}{2}$

i) $\cos^6\theta - \sin^6\theta = \cos(2\theta)\left(1 - \dfrac{1}{4}\sin^2(2\theta)\right)$

All Types Of Identities

4. Prove:

a) $\sin^2 x + \cos^4 x = \cos^2 x + \sin^4 x$

b) $\tan x - \cot x = (\tan x - 1)(\cot x + 1)$

c) $\cos x = \sin x \tan^2 x \cot^3 x$

d) $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

e) $\sin^4 x + \cos^4 x = \sin^2 x(\csc^2 x - 2\cos^2 x)$

f) $\sin x = 1 - 2\sin^2\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$

g) $\cos^3 a + \sin^3 a = (\cos a + \sin a)(1 - \sin a\cos a)$

h) $\sin(8x) = 8\sin x\cos x\cos(2x)\cos(4x)$

i) $\sin(a+b) + \sin(a-b) = 2\sin a\cos b$

j) $\dfrac{1}{1-\sec b} + \dfrac{1}{1+\sec b} = -2\cot^2 b$

k) $\cos^4 A - \sin^4 A = 1 - 2\sin^2 A$