

**Unit 4 Review: Trigonometry Equations and Identities**

1. Using compound angle formulas, state the exact value of:

a)  $\sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$       b)  $\cos\left(\frac{7\pi}{6} - \frac{7\pi}{4}\right)$       c)  $\tan\left(\frac{\pi}{6} - \frac{5\pi}{4}\right)$       d)  $\cot\left(\frac{19\pi}{12}\right)$

e)  $\csc\left(-\frac{17\pi}{12}\right)$       f)  $\sec\left(\frac{7\pi}{12}\right)$       g)  $\sin 80^\circ \cos 50^\circ - \cos 80^\circ \sin 50^\circ$

h)  $\frac{\tan 40^\circ + \tan 110^\circ}{1 - \tan 40^\circ \tan 110^\circ}$

2. Using double angle formulas, state the exact value of:

a)  $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$       b)  $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}$       c)  $1 - 2\sin^2 22.5^\circ$

d)  $2\cos^2\frac{7\pi}{24} - 1$       e)  $2\sin\frac{\pi}{4}\cos\frac{\pi}{4}$       f)  $\sin 67.5^\circ$

3. Given  $\sin \alpha = \frac{5}{13}$ ,  $\cos \beta = -\frac{3}{5}$ ,  $\alpha$  is in QII,  $\beta$  is in QIII, evaluate:

a)  $\cos(\alpha + \beta)$       b)  $\tan(\alpha + \beta)$

4. Given  $\cos \beta = \frac{40}{41}$ ,  $\tan b = \frac{12}{5}$ ,  $\beta$  is in QI,  $b$  is in QIII, evaluate:

a)  $\sin(\beta - b)$       b)  $\cos(\beta + b)$

5. Solve the following equations, for  $0 \leq x \leq 2\pi$  providing exact values or rounding to 2 decimal places where applicable.

a)  $4\cos x - 2\sqrt{3} = 0$       b)  $5 - 3\cot x = 4$       c)  $\frac{2\csc x}{3} + 5 = 9$

d)  $4\sin^3 x = 3\sin x$       e)  $\cos^2 x - \sin^2 x = 2 - 5\cos x$       f)  $\sin(2x) + \frac{\sqrt{2}}{2} = 0$

g)  $\cos(3x) + \frac{1}{2} = 0$       h)  $\sin^2 x + 3\cos^2 x = 0$       i)  $\sin(4x) - \sin(2x) = 0$

6. If  $\sin y = \cos 3y$ , determine an exact value for the angle  $y$ .

7. Solve the following equations on the specified interval, providing exact values or rounding to 2 decimal places where applicable.

a) $\cos x = 0.135, [0, 2\pi]$	b) $\tan(2x) = 3.8, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
c) $\cos(2x) = \frac{-1}{2}, [-\pi, \pi]$	d) $2\cos^2 x + \cos x = 0, [0, 2\pi]$
e) $\cos(2x) - \sin x = 0, [-\pi, \pi]$	f) $\sin^2 x - 3\cos^2 x = 0, \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$
g) $6\sin^2 x + \sin x - 1 = 0, [0, 2\pi]$	h) $(\sec x - 2)(\sec x + 1) = 0, [-\pi, \pi]$
i) $\sin(4x) + \sin(2x) = 0, [-2\pi, 0]$	j) $2\cos x + 5\sin x = 0, [0, 2\pi]$
k) $(3\csc x + 1)^2 = 16, [-\pi, \pi]$	

8. The tangent of the acute angle  $\alpha$  is 0.75, and the tangent of the acute angle  $\beta$  is 2.4. Without using a calculator, determine the value of  $\sin(\alpha - \beta)$  and  $\cos(\alpha + \beta)$ .

9. The angle  $x$  lies in the interval  $\frac{\pi}{2} \leq x \leq \pi$ , and  $\sin^2 x = \frac{4}{9}$ . Determine the value of each of the following. Round your answers to four decimal places.

a) $\sin 2x$	b) $\cos 2x$	c) $\cos \frac{x}{2}$	d) $\sin 3x$
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10. Solve the equation  $\frac{1}{1 + \tan^2 x} = -\cos x$  for  $x$  in the interval  $0 \leq x \leq 2\pi$ .

11. Prove the following identities.

a) $\sin^2 x + \cos^4 x = \cos^2 x + \sin^4 x$	b) $\tan x - \cot x = (\tan x - 1)(\cot x + 1)$
c) $\cos x = \sin x \tan^2 x \cot^3 x$	d) $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$
e) $\sin^4 x + \cos^4 x = \sin^2 x(\csc^2 x - 2\cos^2 x)$	f) $\sin^3 x + \cos^3 x = (1 - \sin x \cos x)(\sin x + \cos x)$
g) $\cos\left(\frac{\pi}{12} - x\right) \sec \frac{\pi}{12} - \sin\left(\frac{\pi}{12} - x\right) \csc \frac{\pi}{12} = 4 \sin x$	
h) $\tan(x - y) + \tan(y - z) = \frac{\sec^2 y (\tan x - \tan z)}{(1 + \tan x \tan y)(1 + \tan y \tan z)}$	
i) $\sin(8x) = 8 \sin x \cos x \cos(2x) \cos(4x)$	j) $\sin x = 1 - 2 \sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$
k) $\sin(x + y) + \sin(x - y) = 2 \sin x \cos y$	l) $\tan x + \tan(\pi - x) + \cot\left(\frac{\pi}{2} + x\right) = \tan(2\pi - x)$

**ANSWERS:**

1.a)  $\frac{\sqrt{2}-\sqrt{6}}{4}$  b)  $\frac{-\sqrt{6}+\sqrt{2}}{4}$  c)  $\sqrt{3}-2$  d)  $\sqrt{3}-2$  e)  $\sqrt{6}-\sqrt{2}$  f)  $-\sqrt{2}-\sqrt{6}$  g)  $\frac{1}{2}$  h)  $\frac{-\sqrt{3}}{3}$

2.a)  $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$  b)  $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$  c)  $\cos 45^\circ = \frac{\sqrt{2}}{2}$  d)  $\frac{\sqrt{2}-\sqrt{6}}{4}$  e)  $\sin \frac{\pi}{2} = 1$  f)  $\frac{\sqrt{2+\sqrt{2}}}{2}$

3.a)  $\frac{56}{65}$  b)  $\frac{33}{56}$

4.a)  $\frac{435}{533}$  b)  $\frac{-92}{533}$

5.a)  $\frac{\pi}{6}, \frac{11\pi}{6}$  b) 1.25, 4.39 c) 0.17, 2.97 d)  $0, \pi, 2\pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$  e)  $\frac{\pi}{3}, \frac{5\pi}{3}$

f)  $\frac{5\pi}{8}, \frac{13\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}$  g)  $\frac{2\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}, \frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}$  h) no sol'ns

i)  $0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi, 2\pi, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

6.  $\frac{\pi}{8}$

7.a) 1.44, 4.85 b) -0.91, 0.66 c)  $\frac{-2\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$  d)  $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

e)  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{-\pi}{2}$  f)  $\frac{-\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$  g)  $\frac{7\pi}{6}, \frac{11\pi}{6}, 0.34, 2.80$  h)  $-\pi, \frac{-\pi}{3}, \frac{\pi}{3}, \pi$

i)  $-2\pi, \frac{-3\pi}{2}, -\pi, \frac{-\pi}{2}, 0, \frac{-\pi}{3}, \frac{-2\pi}{3}, \frac{-4\pi}{3}, \frac{-5\pi}{3}$  j) 2.76, 5.90 k) -2.50, -0.64

8.  $-\frac{33}{65}, -\frac{16}{65}$

9. a)  $-\frac{4\sqrt{5}}{9}$  b)  $\frac{1}{9}$  c)  $\sqrt{\frac{3-\sqrt{5}}{6}}$  d)  $\frac{22}{27}$

10.  $x = \frac{\pi}{2}, \pi, \text{ or } \frac{3\pi}{2}$