Solving Quadratic Equations Using the Quadratic Formula

Key Concepts

- Some polynomial expressions are unfactorable with integers, such as $x^2 + 3x - 7 = 0$. [Try it, I dare you. =)]
- The Quadratic Formula can be used to solve EVERY quadratic equation, whether it has 0, 1, or 2 solutions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{where } ax^2 + bx + c = 0$$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \& \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve for x, $x \in R$.

a) $4x^2 - 9 = 0$

$$\underbrace{4}_{a}x^2 + \underbrace{0}_{b}x \underbrace{-9}_{c} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0 \pm \sqrt{(0)^2 - 4(4)(-9)}}{2(4)}$$

$$= \frac{0 \pm \sqrt{144}}{8}$$

$$= \frac{\pm 12 \div 4}{8 \div 4}$$

$$\boxed{x = \pm \frac{3}{2}}$$

$\therefore x \in \left\{\pm \frac{3}{2}\right\}$

or $x \in \left\{\frac{-3}{2}, \frac{3}{2}\right\}$

b) $1x^2 - 6x + 7 = 0$

$$\underbrace{1}_{a}x^2 \underbrace{-6}_{b}x + \underbrace{7}_{c} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 28}}{2}$$

$$= \frac{6 \pm \sqrt{8}}{2}$$

$$= \frac{6 \pm \sqrt{4 \cdot 2}}{2}$$

$$= \frac{6 \div 2 \pm 2 \div 2\sqrt{2}}{2 \div 2} \leftarrow \text{reduce}$$

$$\boxed{x = 3 \pm \sqrt{2}}$$

$x \in \{3 \pm \sqrt{2}\}$ or $x \in \{3 - \sqrt{2}, 3 + \sqrt{2}\}$

c) $1x^2 + 2x + 10 = 0$

$$\underbrace{1}_{a}x^2 + \underbrace{2}_{b}x + \underbrace{10}_{c} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 - 40}}{2}$$

$$= \frac{-2 \pm \sqrt{-36}}{2}$$

cannot $\sqrt{\text{negative}}$ real number

$\therefore$ no real solutions

$x \in \{ \quad \}$

"empty set

The Discriminant

Remember the Quadratic Formula??? $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Let's have a closer look at 3 different cases that the radicand, otherwise known as $\underline{The\ Discriminant\ D = b^2 - 4ac}$, can take on for real numbers. We can use The Discriminant to determine "Nature of the Real Roots" (type/number of real roots).

Case 1: $b^2 - 4ac < 0$ (less than 0)

How many real roots can we find using the Quadratic Formula if $b^2 - 4ac < 0$?

"there exists"

$\because$ cannot $\sqrt{\text{negative}}$ real number $\not\exists$ "no real roots"

Case 2: $b^2 - 4ac = 0$

How many real roots can we find using the Quadratic Formula if $b^2 - 4ac = 0$?

$\because b^2-4ac=0 \Rightarrow x = \frac{-b}{2a}$ $\exists$ "2 real and equal roots" or "1 real, distinct root".

Case 3: $b^2 - 4ac > 0$

How many real roots can we find using the Quadratic Formula if $b^2 - 4ac > 0$?

$\because b^2-4ac>0 \Rightarrow$ different x_1, x_2 $\exists$ "2 real and distinct roots."

Example: Describe "Nature of the Real Roots" for the following polynomial without solving the equation. (Note: Solving means to calculate the actual value of the real roots.)

$$\underset{a}{4}x^2 - \underset{b}{6}x + \underset{c}{9}$$

$D = b^2 - 4ac$

$= (-6)^2 - 4(4)(9)$

$= 36 - 144$

$\boxed{D = -108}$

$\because D < 0$,

there are no real roots.

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b) $x^2 - 6x + 7 = 0$

c) $x^2 + 2x + 10 = 0$

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Case 1: $\mathbf{b^2 - 4ac < 0}$ (less than 0)

How many real roots can we find using the Quadratic Formula if $b^2 - 4ac < 0$?

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Case 2: $\mathbf{b^2 - 4ac = 0}$

How many real roots can we find using the Quadratic Formula if $b^2 - 4ac = 0$?

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Case 3: $\mathbf{b^2 - 4ac > 0}$

How many real roots can we find using the Quadratic Formula if $b^2 - 4ac > 0$?

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Example: Describe "Nature of the Real Roots" for the following polynomial without solving the equation. (Note: Solving means to calculate the actual value of the real roots.)

$$4x^2 - 6x + 9$$