

1.

a) Yes. This is definitely possible.

Example:

$$f(x) = 3x^2$$

$$g(x) = x^4$$

$$h(x) = \frac{f(x)}{g(x)}$$

$$= \frac{3x^2}{x^4}$$

$$= \frac{3}{x^2}$$

It is possible to turn 2 functions that have no restrictions into a function with a restriction by combining them with the quotient method. In this case the restriction is $x \neq 0$.

b) This is possible as well.

Example:

$$f(x) = \frac{x^3+1}{x} \leftarrow \text{restriction } x \neq 0$$

$$g(x) = x^2 \leftarrow \text{no restriction}$$

$$f(x) \cdot g(x) = \left(\frac{x^3+1}{x} \right) (x^2)^x$$

$$= (x^3+1)(x)$$

$$= x^4 + x$$

As you can see when the functions are combined by multiplication, the restriction goes away.

2.

a) $h(x) = 4x^2 - 9$

i) sum

$$f(x) = 2x^2 - 9 \rightarrow h(x) = 4x^2 - 9$$
$$g(x) = 2x^2$$

ii) prod

$$f(x) = (2x - 3)$$
$$g(x) = (2x + 3) \rightarrow h(x) = 4x^2 - 9$$

iii) quotient

$$f(x) = (2x - 3)^2$$
$$g(x) = (2x - 3) \quad (2x + 3)^2$$

$$h(x) = 4x^2 - 9$$

iv) comp

$$f(x) = 4x$$

$$g(x) = x^2 - \frac{9}{4}$$

$$f[g(x)] = 4\left(x^2 - \frac{9}{4}\right)$$

$$h(x) = 4x^2 - 9$$

$$\begin{aligned}
 b) \text{ AROC} &= \frac{F(\infty_2) - F(\infty_1)}{\infty_2 - \infty_1} \quad (-2, 1) \\
 &= \frac{F(1) - F(-2)}{1 - (-2)} \\
 &= \frac{(4(1)^2 - 9) - (4(-2)^2 - 9)}{3} \\
 &= \frac{(-5)(7)}{3} \\
 &= -\frac{35}{3}
 \end{aligned}$$

est. IROC:

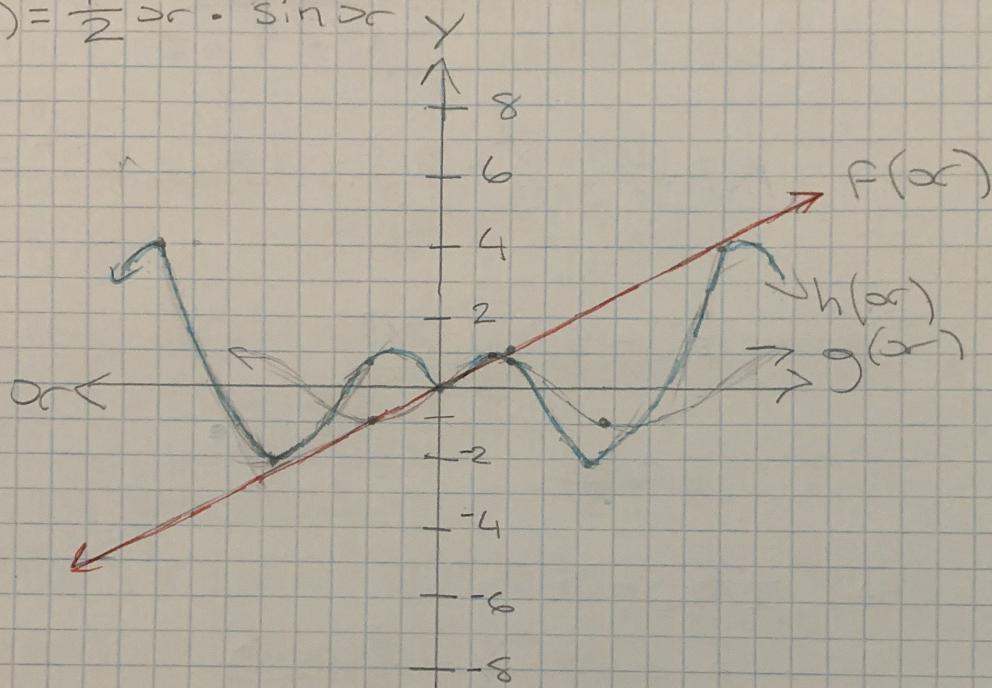
$$\begin{aligned}
 &= \frac{F(\infty_2) - F(\infty_1)}{\infty_2 - \infty_1} \quad (0.9, 1.1) \\
 &= \frac{F(1.1) - F(0.9)}{1.1 - 0.9} \\
 &= \frac{(4(1.1)^2 - 9) - (4(0.9)^2 - 9)}{0.2} \\
 &= \frac{(-4.16) - (-5.76)}{0.2} \\
 &= \frac{1.6}{0.2} \\
 &= 8
 \end{aligned}$$

3.

$$f(x) = \frac{1}{2} \cos x$$

$$g(x) = \sin x$$

$$h(x) = \frac{1}{2} \cos x + \sin x$$



Similarities to original function:

- amplitude increases with max value of $y = \frac{1}{2} \cos x$ (sinusoidal)
- Function oscillates like sin (behavior)
- Period is the same as sin (except at differences).

- goes through the origin similar to how cos does
- reflected on the y axis