Curve Sketching – Polynomial and Rational Functions

For each of the following functions, perform a complete curve analysis. **Note: Some of the solutions are provided but not all.**

1. For each of the following functions state the factored form of the equation, the degree, calculate the y-intercept, state the x-intercepts and their multiplicities, the end behaviour, domain and range (if possible). Then, sketch the function neatly.

a) $f(x) = (x+1)(x-2)$

b) $y = (x+2)(x-1)(x+3)$

c) $y = (x-2)(x+3)(x+1)(x-4)$

d) $f(x) = (1-x)(x+2)^2$

e) $g(x) = -(x^2-6x+9)(x+1)^3$

f) $h(x) = -2(x-3)^3(2x-1)$

g) $y = x^3 - x^2 + 2x - 2$

h) $f(x) = x^3 + 3x^2 - 4$

i) $g(x) = -4x^4 + 38x^3 - 126x^2 + 162x - 54$

2. For each of the following functions write the fully factored and simplified equations, state the **coordinates** of all Removable Discontinuities (holes), Intercepts (x and y), and domain & range (if possible). Then, sketch a detailed graph.

a) $f(x) = \dfrac{2x+1}{x-1}$

b) $f(x) = \dfrac{25-x^2}{x^2-6x+5}$

c) $f(x) = \dfrac{5}{2x^2-5x-3}$

d) $f(x) = \dfrac{3x^2-12x+12}{x^2-4}$

e) $y = \dfrac{x^2-4}{x^3-1}$

f) $f(x) = \dfrac{2x^2-7x-4}{x^2-16}$

g) $g(x) = \dfrac{x^2-5x+6}{x-5}$

ANSWERS:

1.a)degree = 2; y-int = – 2; x-ints = – 1, 2; as $x \to \infty,\ f(x) \to \infty$ and as $x \to -\infty,\ f(x) \to \infty$
b) degree = 3; y-int = – 6; x-ints = – 2, 1, – 3 ; as $x \to \infty,\ y \to \infty$ and as $x \to -\infty,\ y \to -\infty$
c) degree = 4; y-int = 24; x-ints = 2, – 3, – 1, 4; as $x \to \infty,\ y \to \infty$ and as $x \to -\infty,\ y \to \infty$
d) degree = 3; y-int = – 4; x-ints = 1, – 2 (order 2); as $x \to \infty,\ f(x) \to -\infty$ and as $x \to -\infty,\ f(x) \to \infty$
(**NOTE**: must factor -ive out in first set of brackets)
e) degree = 5; y-int = – 9; x-ints = 3 (order 2), – 1(order 3); as $x \to \infty,\ g(x) \to -\infty$ and as $x \to -\infty,\ g(x) \to \infty$
f) degree = 4; y-int = – 54; x-ints = 3 (order 3), ½ ; as $x \to \infty,\ h(x) \to -\infty$ and as $x \to -\infty,\ h(x) \to -\infty$

2. a) eq'n is already factored/simplified
RD: none x-int: $(-0.5, 0)$ y-int: $(0, -1)$
VA: $x = 1$ (as $x \to 1^-$, $f(x) \to -\infty$, as $x \to 1^+$, $f(x) \to \infty$)
HA: $y = 2$ (as $x \to -\infty$, $f(x) \to 2$ from below, as $x \to \infty$, $f(x) \to 2$ from above)
domain: $\{x \in R \mid x \neq 1\}$ range: $\{y \in R \mid y \neq 2\}$

b) $f(x) = \dfrac{(5+x)(5-x)}{(x-5)(x-1)} = \dfrac{-(x+5)}{x-1}, x \neq 5$
RD: $(5, -2.5)$ x-int: $(-5, 0)$ y-int: $(0, 5)$
VA: $x = 1$ (as $x \to 1^-$, $f(x) \to \infty$, as $x \to 1^+$, $f(x) \to -\infty$)
HA: $y = -1$ (as $x \to -\infty$, $f(x) \to -1$ from above, as $x \to \infty$, $f(x) \to -1$ from below)
domain: $\{x \in R \mid x \neq 1, 5\}$ range: $\{y \in R \mid y \neq -1, -2.5\}$

c) $f(x) = \dfrac{5}{(2x+1)(x-3)}$
RD: none x-int: none y-int: $(0, -1.667)$
VA: $x = -0.5$ (as $x \to -0.5^-$, $f(x) \to -\infty$, as $x \to -0.5^+$, $f(x) \to \infty$)
$x = 3$ (as $x \to 3^-$, $f(x) \to -\infty$, as $x \to 3^+$, $f(x) \to \infty$)
HA: $y = 0$ (as $x \to -\infty$, $f(x) \to 0$ from above, as $x \to \infty$, $f(x) \to 0$ from above)
domain: $\{x \in R \mid x \neq -0.5, 3\}$ range: $\{y \in R \mid y \leq -0.816, 0 < y\}$

d) $f(x) = \dfrac{3(x-2)(x-2)}{(x-2)(x+2)} = \dfrac{3(x-2)}{x+2}, x \neq 2$
R.D. $(2,0)$; y-int $(0,-3)$;
V.A. $x = -2$ (as $x \to -2^-$, $f(x) \to \infty$, as $x \to -2^+$, $f(x) \to -\infty$,);
H.A. $y = 3$ (as $x \to -\infty$, $f(x) \to 3$ from above, as $x \to \infty$, $f(x) \to 3$ from below)
domain: $\{x \in R \mid x \neq \pm 2\}$ range: $\{y \in R \mid y \neq 0,3\}$