Test Review

1. A bacterial colony with an initial population of 300 doubles every day. Which equation models this exponential growth?

A $P = 2 \times 300^n$ **B** $P = 300 \times \left(\frac{1}{2}\right)^n$

C $P = 200 \times 3^n$ **D** $P = 300 \times 2^n$

4. A radioactive substance with an initial mass of 250 mg has a half-life of 1 year.
   - **a)** Write an equation to relate the mass of radioactive material remaining to time.
   - **b)** What mass will remain after 10 years?
   - **c)** How long will it take for the sample to decay to 20% of its initial mass? Explain how you arrived at your answer.

5. Refer to question 4.
   - **a)** Show how you can write the equation from part a) in another way.
   - **b)** Explain why the two equations are equivalent.

6. Evaluate. Express as a fraction in lowest terms.

a) 10^{-1} **b)** 4^{-2} **c)** $3^{-2} + 9^{-1}$

d) $5^{-3} + 5^0$ **e)** $\left(\frac{1}{5}\right)^{-1}$ **f)** $\left(\frac{3}{4}\right)^{-3}$

7. Simplify. Express your answers using only positive exponents.

a) $(x^{-2})(x^{-1})(x^0)$ **b)** $(3km^2)(2k^{-2}m^{-2})$

c) $w^{-3} \div w^{-2}$ **d)** $\frac{u^{-2}v^3}{u^{-3}v^{-2}}$

e) $(z^{-3})^{-2}$ **f)** $(2ab^{-1})^{-2}$

8. Evaluate.

a) $\sqrt[3]{64}$ **b)** $\sqrt[4]{625}$ **c)** $\sqrt[5]{-3125}$

d) $\left(\frac{1}{64}\right)^{\frac{1}{6}}$ **e)** $27^{\frac{2}{3}}$ **f)** $(-1000)^{\frac{4}{3}}$

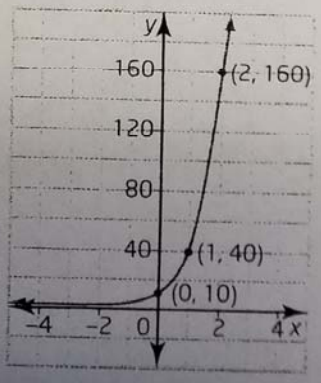
g) -4^{-3} **h)** $\left(\frac{3}{4}\right)^{-2}$ **i)** $\left(-\frac{27}{125}\right)^{-\frac{2}{3}}$

9. The length, x, in centimetres, by which a spring with spring constant k is stretched or compressed from its rest position is related to its stored potential energy, U, in joules (J), according to the equation $x = (2Uk^{-1})^{\frac{1}{2}}$.
   - **a)** Use the power of a power rule to write this equation in a different form.
   - **b)** Write the equation in radical form, using a single radical.
   - **c)** A spring with spring constant 10 has 320 J of stored energy. By how much is this spring stretched?

3.4 Properties of Exponential Functions, pages 178 to 187

10. **a)** Graph the function $y = 27\left(\frac{1}{3}\right)^x$.
    - **b)** Identify the
      - **i)** domain **ii)** range
      - **iii)** x- and y-intercepts, if they exist
      - **iv)** intervals of increase/decrease
      - **v)** equation of the asymptote

11. Determine the equation for the exponential graph shown.

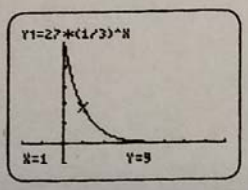


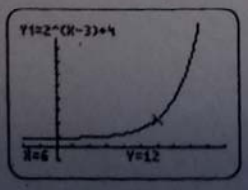
12. **a)** Sketch the function $y = 2^{x-3} + 4$.
    - **b)** Identify the
      - **i)** domain
      - **ii)** range
      - **iii)** equation of the asymptote

13. Describe the transformation or transformations that map the base function $y = 5^x$ onto each given function.

a) $y = 2(5^x)$ **b)** $y = 5^{2x}$

Answers:

1. D
2. Answers may vary.
3. a) 1 b) Answers may vary.
4. a) $A = 250\left(\frac{1}{2}\right)^n$, where n is the number of years and A is the amount radioactive material remaining, in milligrams.
   b) 0.244 mg c) approximately 2.3 years
5. Answers may vary. Sample answers:
   a) $A = 250(2^{-n})$
   b) Since $b^{-x} = \frac{1}{b^x}$, which can be written as $\left(\frac{1}{b}\right)^x$, then $\left(\frac{1}{2}\right)^n = 2^{-n}$, so the equations are equivalent.
6. a) $\frac{1}{10}$ b) $\frac{1}{16}$ c) $\frac{2}{9}$
   d) $\frac{126}{125}$ e) 5 f) $\frac{64}{27}$
7. a) $\frac{1}{x^3}$ b) $\frac{6}{k}$ c) $\frac{1}{w}$
   d) uv^5 e) z^6 f) $\frac{b^2}{4a^2}$
8. a) 4 b) 5 c) -5
   d) $\frac{1}{2}$ e) 9 f) 10 000
   g) $-\frac{1}{64}$ h) $\frac{16}{9}$ i) $\frac{25}{9}$
9. a) $x = 2^{\frac{1}{2}}U^{\frac{1}{2}}k^{-\frac{1}{2}}$
   b) $x = \sqrt{\frac{2U}{k}}$ c) 8 cm
10. a) 

    b) i) $\{x \in \mathbb{R}\}$ ii) $\{y \in \mathbb{R}, y > 0\}$
    iii) no x-intercepts; y-intercept 27
    iv) The function is decreasing over its domain.
    v) $y = 0$
11. $y = 10(2)^{2x}$ or $y = 10(4^x)$
12. a) 

    b) i) $\{x \in \mathbb{R}\}$
    ii) $\{y \in \mathbb{R}, y > 4\}$
    iii) $y = 4$
13. a) vertical stretch by a factor of 2
    b) horizontal compression by a factor of $\frac{1}{2}$
    c) reflection in the x-axis and the y-axis
    d) reflection in the y-axis, horizontal compression by a factor of $\frac{1}{2}$, translation of 2 units left