1.Prove that D42={S42, D} is a complemented lattice by finding the complements of all the elements where Sn is the set of all divisors of the positive integer n and D is the relation of division.

To prove that $D_{42} = \{S_{42}, D\}$ is a complemented lattice, we need to show that for each element in the lattice, there exists a unique complement. In this context, the complement is defined in relation to the lattice’s join and meet operations.

Let’s denote the join operation as $\vee$ (least upper bound) and the meet operation as $\wedge$ (greatest lower bound).

1. Meet and Join:

- The meet operation $\wedge$ corresponds to the intersection of sets.
- The join operation $\vee$ corresponds to the union of sets.

2. Elements of D_{42}:

- S_{42} is the set of all divisors of 42.
- D is the relation of division.

3. Finding Complements:

- The complement of an element x is denoted as x' and satisfies $x \wedge x' = \bot$ (the bottom element) and $x \vee x' = \top$ (the top element).

4. Complements in D_{42}:

- $S_{42}' = D$ because $S_{42} \cap D = D$ and $S_{42} \cup D = S_{42}$.
- $D' = S_{42}$ because $D \cap S_{42} = D$ and $D \cup S_{42} = S_{42}$.

5. Prove Complemented Lattice Properties:

- $S_{42} \wedge D = D$ and $S_{42} \vee D = S_{42}$ (meet and join operations are well-defined).
- $S_{42}' \wedge S_{42} = \bot$ and $S_{42}' \vee S_{42} = \top$ (complements exist for each element).

Therefore, $D_{42} = \{S_{42}, D\}$ is a complemented lattice, where $S_{42}' = D$ and $D' = S_{42}$.