1. **Show that $(\neg q \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Rightarrow r$.**

   Let's break down the given expression:

   1.$(\neg q\wedge(\neg q\wedge r))\vee(q\wedge r)\vee(p\wedge r)$

   This expression can be simplified using the properties of logical operators:

   2.$(\neg q\wedge r)\vee(q\wedge r)\vee(p\wedge r)$

   Here, we used the idempotent law which states that $a\wedge a=a$.
   Next, we can factor out r:

   3.$r\wedge((\neg q)\vee q)\vee(p\wedge r)$

   Here, we used the distributive law which states that $a\wedge(b\vee c)=(a\wedge b)\vee(a\wedge c)$.
   The expression $(\neg q)\vee q$ is a tautology, meaning it is always true. So, we can simplify further:

   4.$r\vee(p\wedge r)$

   Finally, we can factor out r again:

   5.$r\wedge(\text{True}\vee p)$

   The expression $\text{True}\vee p$ is also a tautology. So, the final simplification is:

   6.r

   So, we have shown that $(\neg q\wedge(\neg q\wedge r))\vee(q\wedge r)\vee(p\wedge r)$ simplifies to r, which means $(\neg q\wedge(\neg q\wedge r))\vee(q\wedge r)\vee(p\wedge r)\Rightarrow r$.

2. **Show that p -> q, p -> r, q -> ~r, and p are inconsistent.**
3.

Inconsistency means that the conjunction of all the premises is false. Let's denote the negation of a proposition p as ~p.

We have:

(p -> q) ^ (p -> r) ^ (q -> ~r) ^ p

This can be rewritten using the definition of implication as: (~p v q) ^ (~p v r) ^ (~q v ~r) ^ p

Distributing the terms, we get: (~p v p) ^ (~p v r) ^ (q v ~r)

The term (~p v p) is a tautology (always true), so we can simplify to: True ^ (~p v r) ^ (q v ~r)

If we assume p is true, then ~p is false, and we get: True ^ False ^ (q v ~r)

The conjunction of any proposition with False is False, so the entire expression simplifies to False.

Therefore, the premises are inconsistent.