Dispersion (Measure of Variation)

Dispersion is a ~~method~~ measure of Variation

Eg:

A	B
200	250
200	150
200	240
200	260
200	100

$\Sigma A = 1000$ $\qquad$ $\Sigma B = 1000$

	A	B
Total	1000	1000

$Avg = \frac{1000}{5} = 200$ $\qquad$ $Avg = \frac{1000}{5} = 200$

methods:-

1) Range(R)

2) Quartile deviation (QD)

3) mean deviation (MD)

4) standard deviation (σ)

(i) Variance

(ii) Coefficient of variation (CV)

I. Range:-

(i) Range $= R = L - S$

(ii) Coeffient of Range $= \dfrac{L-S}{L+S}$

L = Largest value

S = Smallest value

II Quartile Deviation :-

(i) Intera Quartile Range $= Q_3 - Q_1$
(IQR)

(ii) Semi Inter Quartile deviation (or) (QD) $= \dfrac{Q_3 - Q_1}{2}$

(iii) Coefficient of Quartile deviation (QD) $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

Where,

Q_1 = lower Quartile = 25%

Q_3 = Upper Quartile = 75%

Standard deviation :-

* Measure of studying variation

The square root of the means of square deviation from the arithemetic mean is called standard deviation and it is denoted by σ.

(i) I. Individual :-

$$\sigma = S.D = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

N = No. of Values

(ii) Variance $= (S.D)^2 = \sigma^2$

(iii) Coefficient of Variation (CV) $= \frac{\sigma}{\bar{x}} \times 100$

1) Calculate standard deviation, variance and coefficient of variation from the following.

x	x^2
14	196
22	484
9	81
15	225
20	400
$\Sigma x = 80$	$\Sigma x^2 = 1386$

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Sol:-

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$N = 5$

$$= \sqrt{\frac{1386}{5} - \left(\frac{80}{5}\right)^2} \Rightarrow \sqrt{277.2 - 256}$$

$$= \sqrt{21.2} \Rightarrow 4.6043$$

(ii) Variance $= (S.D)^2 = \sigma^2$

$$= (4.6)^2 \Rightarrow 21.16$$

(iii) $C.V = \frac{\sigma}{x} \times 100$

$$= \frac{4.6}{16} \times 100 \Rightarrow 28.75\%$$

II Discrete :-

$$\sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

f = frequency, Σf = total frequency

x = Var

2) Calculate standard deviation and coeffient of Variation

Marks (x)	No. of Stud (f)	f^2
10	8	64
20	12	144
30	20	400
40	10	100
50	7	49
60	3	9
	$\Sigma f = 60$	$\Sigma f^2 = 766$

x^2	fx	fx^2
100	80	800
400	240	4800
900	600	18000
1600	400	16000
2500	350	17500
3600	180	10800

$\Sigma fx = 1850$ $\Sigma fx^2 = 67900$

$$= \sqrt{\frac{67900}{60} - \left(\frac{1850}{60}\right)^2}$$

$$= \sqrt{1131.6 - 950.48} \Rightarrow \sqrt{181.12}$$

$= 13.458$ //

(ii) $CV = \frac{\sigma}{\bar{x}} \times 100$

$\bar{x} = \frac{1850}{60}$

$= \frac{13.45}{30.83} \times 100 \Rightarrow$

$\bar{x} = 30.83$

$= 43.626 \Rightarrow 43.61$ //

Continious :-

(i) $\sigma = \sqrt{\frac{\Sigma fm^2}{\Sigma f} - \left(\frac{\Sigma fm}{\Sigma f}\right)^2}$

$\Sigma f = N =$ Total of frequency

$m =$ mid point

$= \frac{\text{upperlimit} + \text{lowerlimit}}{2}$

$\Rightarrow \bar{x}$ - Mean $= \frac{\Sigma fm}{\Sigma f}$

1) Calculate σ from the following.

C.I	f	m	m^2	fm	fm^2
0-5	2	2.5	6.25	12.5	156.25
5-10	4	7.5	56.25	2215	50625
10-15	6	12.5	156.25	937.5	878906.25
15-20	8	17.5	306.25	2450	6002500
	$\Sigma f = 20$			$\Sigma fm = 3625$	$\Sigma fm^2 = 6932187.5$

$\bar{x} = \frac{\Sigma fm}{\Sigma f} = \frac{3625}{20} \Rightarrow \bar{x} = 181.25$

$\sigma = \sqrt{\frac{6932187.5}{20} - \left(\frac{3625}{20}\right)^2}$

$= \sqrt{346609.37 - 32851.56}$

$= \sqrt{313757.81} \Rightarrow \sqrt{313757.81}$

$\sigma = 560.140$

1) A sample of 35 values has mean standard Deviation and SD = 4
A second sample 65 values has mean 70 and SD = 5

sol:-

sample I	sample II
$N_1 = 35$	$N_2 = 65$
$\bar{x}_1 = 80$	$\bar{x}_2 = 70$
$\sigma_1 = 4$	$\sigma_2 = 5$

Sol:-

Combined SD $= \sigma_{12} = \sqrt{\dfrac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$

$d_1^2 = (\bar{x}_1 - \bar{x}_{12})^2$

$d_2^2 = (\bar{x}_2 - \bar{x}_{12})^2$

$\bar{x}_{12} = \dfrac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2} \Rightarrow \dfrac{35(80) + 65(70)}{35 + 65}$

$= 73.5$

$d_1 = (\bar{x}_1 - \bar{x}_{12}) \Rightarrow (80 - 73.5) = 6.5$

$d_1^2 = (6.5)^2 = 42.25$

$d_2 = (\bar{x}_1 - \bar{x}_{12}) \Rightarrow d_2 = (70 - 73.5)$

$= -3.5$

$d_2^2 = (-3.5)^2$

$d_2^2 = 12.25$

Combined SD $= \sigma_{12} = \sqrt{\dfrac{35(16 + 42.25) + 65(25 + 12.25)}{35 + 65}}$

$= 6.671$

2) Calculate mean, median, mode and SD.

CI	f	m	fm	fm²	cf
2-4	3	3	9	81	3
4-6	4	5	20	400	7
6-8	2	7	14	196	9
8-10	1	9	9	181	10
	$\Sigma f = 10$		$\Sigma fm = 52$	$\Sigma fm^2 = 858$	$\Sigma cf = 29$

mean $= \bar{x} = \dfrac{\Sigma fm}{\Sigma f} \Rightarrow \bar{x} = \dfrac{52}{10} \Rightarrow 5.2$

median $= L + \left[\dfrac{\frac{N}{2} + cf}{f} \times i\right]$; Mode $= Z = L + \left[\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right]$

1) Mean Deviation and Coefficient of Mean Deviation

X	$\lvert D \rvert = \lvert x - 37 \rvert$
10	$\lvert 10-37 \rvert = 27$
15	$\lvert 15-37 \rvert = 22$
20	$\lvert 20-37 \rvert = 17$
25	$\lvert 25-37 \rvert = 12$
36	$\lvert 36-37 \rvert = 1$
49	$\lvert 49-37 \rvert = 12$
50	$\lvert 50-37 \rvert = 13$
60	$\lvert 60-37 \rvert = 23$
68	$\lvert 68-37 \rvert = 31$
$\Sigma x = 333$	$\Sigma \lvert D \rvert = 158$

Mean Deviation $= \boxed{\dfrac{\Sigma \lvert D \rvert}{N}}$

$N = 9$

$\lvert D \rvert = \lvert x - \bar{x} \rvert$

$\boxed{\bar{x} = \dfrac{\Sigma x}{N}} = \dfrac{333}{9} = 37$

(i) Mean Deviation $= \dfrac{\Sigma \lvert D \rvert}{N} = \dfrac{158}{9}$

$= 17.5$

(ii) Coefficient of M.D $= \dfrac{M.D}{\bar{x}} = \dfrac{17.5}{37}$

$= 0.47$.

2) Calculate the coefficient of Mean deviation (Discrete).

x	f	fx	$\lvert D\rvert = \lvert x-\bar{x}\rvert$	$f\lvert D\rvert$
10	3	30	$\lvert 10-12\rvert = 2$	$3\times 2=6$
11	12	132	$\lvert 11-12\rvert = 1$	$12\times 1=12$
12	18	216	$\lvert 12-12\rvert = 0$	$18\times 0=18$
13	12	156	$\lvert 13-12\rvert = 1$	$12\times 1=12$
14	3	42	$\lvert 14-12\rvert = 2$	$3\times 2=6$
	$\Sigma f=48$	$\Sigma fx=576$		$\Sigma f\lvert D\rvert=36$

$$\boxed{MD = \frac{\Sigma f\lvert D\rvert}{\Sigma f}}$$

$\lvert D\rvert = \lvert x-\bar{x}\rvert$

$$\boxed{\bar{x} = \frac{\Sigma fx}{\Sigma f}} \Rightarrow \frac{576}{48} = 12$$

$$\boxed{MD = \frac{\Sigma f\lvert D\rvert}{\Sigma f}} = \frac{36}{48} = 0.75$$

$$\text{Coeffient of } \boxed{MD = \frac{MD}{\bar{x}}} \Rightarrow \frac{0.75}{12}$$

$$= 0.0625$$