1. A random variable 'X' has the following probability function.

value of X	x	0	1	2	3	4	5	6	7
	$P(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(i) Find K (ii) Evaluate $P(X<6)$, $P(X\geq 6)$ and $P(0<X<5)$ (iii) If $P(X\leq K)>\frac{1}{2}$, find the minimum value of k and determine the distribution function of X.

(i) $\sum_{x=0}^{7} P(x)=1$

$K+2K+2K+3K+K^2+2K^2+7K^2+K=1$

$10K^2+9K-1=0$

$K=-1, 1/10.$

-1 is not possible.

$\therefore K=1/10.$

(ii) $P(X<6)=P(X=0)+P(X=1)+\dots+P(X=5)$

$=\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}+\frac{1}{100}=\frac{81}{100}$

$P(X\geq 6)=1-P(X<6)=\frac{19}{100}$

$P(0<X<5)=P(X=1)+\dots+P(X=4)=8K=\frac{8}{10}=\frac{4}{5}.$

(iii) $P(X\leq K)\geq\frac{1}{2}$

we get trail by $K=4$.

(iv) Distribution function of X.

X	$F(x)=P(X\leq x)$
0	0
1	$K=1/10$
2	$3K=3/10$
3	$5K=5/10$

X	$F(x) = P(X \leq x)$
4	$8K = 4/5$
5	$8K + K^2 = 81/100$
6	$8K + 3K^2 = 83/100$
7	$9K + 10K^2 = 1$

2. After a coin is tossed 2 times, if X is the no: of heads find the probability distubution of X.

Let a coin be tossed 2 times

The sample space is, $S = \{HH, HT, TH, TT\} \rightarrow ①$.

$P(X=0) = \frac{1}{4}$

$P(X=1) = \frac{2}{4}$

$P(X=2) = \frac{1}{4}$

The probability distribution of X is,

X	0	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

3(i) Is the function defined as follows a density function?

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

(ii) If so determine the probability that the variate having the density will fall in the interval (1, 2).

Solution:-

In the interval $(0, \infty)$, e^{-x} is (+ve)

$f(x) \geq 0$ in $(0, \infty)$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} e^{-x}\,dx$$

$$= \left[-e^{-x}\right]_{0}^{\infty} = -e^{-\infty} + 1 = 1$$

$f(x)$ satisfies the condition of density function.

(ii) $P(1 \le x \le 2) = \int_{1}^{2} f(x)\,dx = \int_{1}^{2} e^{-x}\,dx = \left[-e^{-x}\right]_{1}^{2}$

$$\Rightarrow -e^{-2} + e^{-1} = 0.368 - 0.135 = 0.233.$$

4. The amount of time in hours, that a computer functions before breaking down is a continuous random variable with probability density function given by,

$$f(x) = \begin{cases} \lambda e^{-x/100}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

What is the probability that (a) a computer will function between 50 and 150 hours before breaking down.

(b) it will function less than 500 hours.

Solution:-

$f(x)$ is a p.d.f of 'x'.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{\infty} \lambda \cdot e^{-x/100}\,dx = 1$$

$$\lambda\left[\frac{e^{-x/100}}{-1/100}\right]_0^{\infty} = 1$$

$$-100\lambda\left[e^{-\infty} - e^{0}\right] = 1. \qquad \left[e^{-\infty} = 0,\ e^{0} = 1\right]$$

$$100\lambda = 1$$

$$\lambda = 1/100$$

(a) $$P(50 < X < 150) = \int_{50}^{150} \frac{1}{100} \cdot e^{-x/100}\, dx$$

$$= \frac{1}{100}\left[\frac{e^{-x/100}}{-1/100}\right]_{50}^{150} = -\left[e^{-3/2} - e^{-1/2}\right]$$

$$= e^{-1/2} - e^{-3/2}$$

(b) $$P(X < 100) = \int_{0}^{100} \frac{1}{100} \cdot e^{-x/100}\, dx$$

$$= \frac{1}{100}\left[\frac{e^{-x/100}}{-1/100}\right]_0^{100} = -\left[e^{-1} - 1\right] = 1 - e^{-1}.$$

5. If a random variable 'x' has the pdf.

$$f(x) = \begin{cases} 1/2\,(x+1), & \text{if } -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance of X.

Solution:-

$$\text{Mean} = \int_{-1}^{1} x f(x)\, dx = \frac{1}{2}\int_{-1}^{1} x(x+1)\, dx$$

$$= \frac{1}{2}\left[\int_{-1}^{1} (x^2 + x)\, dx\right] = \frac{1}{2}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{1}$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{3} + \frac{1}{2} + \frac{1}{3} - \frac{1}{2}\right] = 1/3$$

Mean = 1/3.

Variance $\mu_2 = \int_{-1}^{} (x-1/3)^2 \frac{(x+1)}{2} dx$

$$\left[\mu_2 = \int_a^b (x-\text{mean})^2 f(x)dx\right]$$

$$= \frac{1}{18}\int_{-1}^{1} (9x^2+1-6x)(x+1)dx$$

$$= \frac{1}{18}\int_{-1}^{1} (9x^3+3x^2-5x+1)dx$$

$$= \frac{1}{18}\left[\frac{9x^4}{4}+x^3-\frac{5x^2}{2}+x\right]_{-1}^{1}$$

variance $= 2/9$.

6. Given that the pdf of a R.V 'x' is $f(x) = Kx$, $0<x<1$, find K and $P(X>0.5)$.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 Kx\,dx = 1 \Rightarrow K\left[\frac{x^2}{2}\right]_0^1 = 1$$

$$K\left[\frac{1}{2}-0\right] = 1$$

$$K = 2.$$

$$P(X>0.5) = \int_{0.5}^{\infty} f(x)\,dx = \int_{0.5}^{1} 2x\,dx = 2\left[\frac{x^2}{2}\right]_{1/2}^{1}$$

$$= \left[1-\frac{1}{4}\right] = \frac{3}{4}.$$

7. Find the value of 'K' and hence find mean and variance of distribution,

$$dF = Kx^2e^{-x}dx,\ 0<x<\infty$$

Solution :-

$$\frac{dF}{dx} = Kx^2 e^{-x}$$

The P.D.F is given by, $f(x) = Kx^2 e^{-x}$.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^{\infty} Kx^2 e^{-x} dx = 1 \Rightarrow K\int_0^{\infty} x^2 e^{-x} dx = 1$$

$$K\left[x^2(-e^{-x}) - 2x(e^{-x}) + 2(-e^{-x})\right]_0^{\infty} = 1$$

$$K(0+2) = 1 \quad [e^{-\infty} = 0 ; e^0 = 1]$$

$$K = 1/2.$$

$$\text{Mean} = \int_0^{\infty} x f(x)dx$$

$$= \int_0^{\infty} x \cdot \frac{1}{2} x^2 e^{-x} dx = \frac{1}{2}\left[\int_0^{\infty} x^3 (e^{-x})dx\right]$$

$$\Rightarrow \frac{1}{2}\left[x^3(-e^{-x}) - 3x^2(e^{-x}) + 6x(-e^{-x}) - 6(e^{-x})\right]_0^{\infty} = 3$$

$$\text{Variance } \mu_2 = \int_0^{\infty} (x-3)^2 f(x)dx$$

$$= \int_0^{\infty} (x-3)^2 \, 1/2 \, x^2 e^{-x} dx$$

$$= \frac{1}{2}\int_0^{\infty} (x^2 - 6x + 9) x^2 e^{-x} dx$$

$$= \frac{1}{2}\int_0^{\infty} (x^4 - 6x^3 + 9x^2) e^{-x} dx$$

$$= \frac{1}{2}\Big[(x^4 - 6x^3 + 9x^2)(-e^{-x}) - (4x^3 - 18x^2 + 18x)(e^{-x}) + (12x^2 - 36x + 18)(-e^{-x}) - ((24x) - 36)(e^{-x}) + (24)(-e^{-x})\Big]_0^{\infty}$$

$$= \frac{1}{2}[18 - 36 + 24] = \frac{6}{2} = \underline{\underline{3}}.$$

8. Experience has shown that while walking in a certain park, the time 'x' (in mins) between seeing two people smoking has a density function ~~of~~ of the form,

$$f(x) = \begin{cases} \lambda x e^{-x}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Calculate the value of λ

(b) Find the distribution function of 'x'

(c) What is the probability that Jeff, who has just seen a person smoking will see another persons smoking in ~~just~~ 2 to 5 minutes? In atleast 7 minutes.

Solution:-

$$f(x) = \begin{cases} \lambda x e^{-x}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^{\infty} \lambda x e^{-x}\,dx = 1$$

$$\lambda\left[-x(e^{-x}) - e^{-x}\right]_0^{\infty} = 1$$

$$\lambda[0+1] = 1$$

$$\lambda = 1$$

(b) $F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_0^{x} x e^{-x}\,dx = \left[-x(e^{-x}) - e^{-x}\right]_0^{x}$

$$\Rightarrow -xe^{-x} - e^{-x} + 1 = 1 - (x+1)e^{-x}.$$

(c) $P(2 < X < 5) = F(5) - F(2) = 3e^{-2} - 6e^{-5}$

$P(X \geq 7) = 1 - P(X < 7) = 1 - F(7) = 8e^{-7}$

9. A random variable 'X' has the p.d.f

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find,

(i) $P\left(X < \frac{1}{2}\right)$ (ii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$ (iii) $P\left(X > \frac{3}{4} / X > \frac{1}{2}\right)$

Solution:-

(i) $P\left(X < \frac{1}{2}\right) = \int_0^{1/2} f(x)\,dx = \int_0^{1/2} 2x\,dx = 2\left(\frac{x^2}{2}\right)_0^{1/2} = 2\left(\frac{1}{8}\right) = \frac{1}{4}.$

(ii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right) = \int_{1/4}^{1/2} f(x)\,dx = \int_{1/4}^{1/2} 2x\,dx = 2\left(\frac{x^2}{2}\right)_{1/4}^{1/2}$

$= \left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3}{16}$

(iii) $P\left(X > \frac{3}{4} / X > \frac{1}{2}\right) = \frac{P\left(X > \frac{3}{4} \text{ and } X > \frac{1}{2}\right)}{P\left(X > \frac{1}{2}\right)} = \frac{P\left(X > \frac{3}{4}\right)}{P\left(X > \frac{1}{2}\right)} \rightarrow (1)$

$P\left(X > \frac{3}{4}\right) = \int_{3/4}^{1} f(x)\,dx = \int_{3/4}^{1} 2x\,dx = 2\left(\frac{x^2}{2}\right)_{3/4}^{1}$

$= 1 - \frac{9}{16} = \frac{16-9}{16} = \frac{7}{16} \rightarrow (2)$

$P\left(X > \frac{1}{2}\right) = \int_{1/2}^{1} f(x)\,dx = \int_{1/2}^{1} 2x\,dx = 2\left[\frac{x^2}{2}\right]_{1/2}^{1}$

$= 1 - 1/4 = 3/4 \rightarrow (3)$

Substitute (2) and (3) in (1), we get

$P\left(X > 3/4 \mid X > 1/2\right) = \frac{P(X > 3/4)}{P(X > 1/2)} = \frac{7/16}{3/4} = 7/12.$

10. A continuous random variable 'X' has a pdf $f(x) = 6x(1-x)$, $0 \le x \le 1$. Determine 'b' if $P(X<b) = P(X>b)$

Solution:-

$f(x)$ is a pdf of X.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$P(X<b) = P(X>b) = 1 - P(X \le b)$$

$$\int_{-\infty}^{b} f(x)dx = 1 - \int_{-\infty}^{b} f(x)dx$$

$$2\int_{-\infty}^{b} f(x)dx = 1 \quad \Rightarrow 2\int_{0}^{b} f(x)dx = 1$$

(i.e) $2\int_{0}^{b} f(x)dx = 1 \Rightarrow 2\int_{0}^{b} 6x(1-x)dx = 1 \Rightarrow 12\int_{0}^{b}(x-x^2)dx = 1$

$$12\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^b = 1$$

$$12\left[\frac{b^2}{2} - \frac{b^3}{3}\right] = 1$$

$$12\left[\frac{3b^2 - 2b^3}{6}\right] = 1$$

$$2\left[3b^2 - 2b^3\right] = 1$$

$$6b^2 - 4b^3 - 1 = 0$$

$$4b^3 - 6b^2 + 1 = 0.$$

Here $b = 1/2$ is a root and also lies between $(0,1)$

11. Let 'X' be a continuous random variable with pdf,

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$ Find c.d.f of the R.V

Solution:-

$$F(x)=\int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{x} dx = 0+[x]_0^x = x.$$

When $x>1$, then $F(x)=\int_{-\infty}^{x} f(x)dx$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{1} dx + \int_{1}^{x} 0\,dx$$

$$= 0+[x]_0^1 + 0 = 1$$

$$\therefore F(x) = \begin{cases} 0, & x<0 \\ x, & 0<x<1 \\ 1, & x>1 \end{cases}$$

12. The pdf of a continuous random variable 'x' is $f(x)=Ke^{-|x|}$. Find k and c.d.f $F(x)$.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_{-\infty}^{\infty} Ke^{-|x|}dx = 1 \qquad \left[e^{-|x|} \text{ is an even function}\right]$$

$$2k\int_{0}^{\infty} e^{-x}dx = 1$$

$$2k\left[-e^{-x}\right]_0^{\infty} = 1$$

$$2k\left[0-(-1)\right] = 4$$

$$2k = 1$$

$$k = \frac{1}{2}.$$

When $x \le 0$, $F(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \frac{1}{2}e^{x}dx.$

$$\left[ke^{-|x|} = \begin{cases} ke^{x} & , -\infty < x < 0 \\ ke^{-x} , & 0 < x < \infty \end{cases} \right]$$

$$= 1/2 \int_{-\infty}^{x} \left[e^{x}\right]_{-\infty}^{x} = \frac{1}{2}\left[e^{x} - e^{-\infty}\right] = \frac{1}{2}e^{x}.$$

When $x > 0$, $F(x) = \int_{-\infty}^{x} f(x)dx$

$$= \int_{-\infty}^{0} \frac{1}{2} e^{x} dx + \int_{0}^{x} \frac{1}{2} e^{-x} dx$$

$$= \frac{1}{2}\left[e^{x}\right]_{-\infty}^{0} + \frac{1}{2}\left[-e^{-x}\right]_{0}^{x}$$

$$= \frac{1}{2}\left[1 - e^{-\infty}\right] + \frac{1}{2}\left[-e^{-x} + e^{0}\right]$$

$$= \frac{1}{2} - \frac{1}{2}e^{-x} + \frac{1}{2}$$

$$\left[\because e^{0} = 1, e^{-\infty} = 0\right]$$

$$= 1 - \frac{1}{2}e^{-x}$$

$$\therefore F(x) = \begin{cases} e^{x}/2 & , x \le 0 \\ 1 - 1/2 e^{-x} , & x > 0 \end{cases}$$

Moment About Origin:- [Property 1]

$$E(x^{r}) = \int_{-\infty}^{\infty} x^{r} f(x)dx$$

$$E\{X - E(x)\}^{r} = \int_{-\infty}^{\infty} \{x - E(X)\}^{r} f(x)dx$$

$$= \int_{-\infty}^{\infty} \{x - \bar{x}\}^{r} f(x)dx$$

$$\mu_r = \int_{-\infty}^{\infty} (x-\bar{X})^r f(x)dx.$$

The r^{th} moment about mean and it is denoted by μ_r.

$$\mu_1 = \int_{-\infty}^{\infty} (x-\bar{X}) f(x)dx$$

$$= \int_{-\infty}^{\infty} x f(x)dx - \int_{-\infty}^{\infty} \bar{X} f(x)dx$$

$$= \bar{X} - \bar{X}\int_{-\infty}^{\infty} f(x)dx \qquad \left[\int_{-\infty}^{\infty} f(x)dx = 1\right]$$

$$= \bar{X} - \bar{X} = 0$$

Put $r=2$,

$$\text{Variance} = \int_{-\infty}^{\infty} (x-\bar{X})^2 f(x)dx$$

$$\text{Variance} = \mu_2 = E\left[\{X - E(X)\}^2\right]$$

Property 2:-

Let $g(X) = aX+b$

$$E[g(X)] = E(aX+b)$$

$$E[aX+b] = aE(X)+b$$

$$E[g(X)] = aE(X)+b.$$

Formula:- $Var(aX+b) = a^2 \, Var(X)$ where a and b are consonants.

13. Give the following probability distribution of X compute (i) $E(X)$ (ii) $E(X^2)$ (iii) $E[2X \pm 3]$ (iv) $Var(2X \pm 3)$

X	-3	-2	-1	0	1	2	3
P(x)	0.05	0.10	0.30	0	0.30	0.15	0.10

(i) $E(x) = \sum_{i=1}^{n} x_i P(x_i)$

$= (-3)(0.05) - 2(0.1) - 1(0.30) + 0 + 1(0.30) + 2(0.15) + 3(0.10)$

$E(x) = 0.25$

(ii) $E(x^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$

$= (-3)^2(0.05) + (-2)^2(0.1) + (-1)^2(0.30) + 0 + (1)^2(0.3)$

$+ (2)^2(0.15) + 3^2(0.10) = 2.95$

(iii) $E(2x \pm 3) = 2E(x) \pm 3$

$[\because E[ax \pm b] = aE(x) \pm b]$

$= 2(0.25) \pm 3 = 0.5 \pm 3$

(iv) $Var(2x \pm 3) = 2^2 \, var(x)$

$Var(x) = E(x^2) - [E(x)]^2$

$= 2.95 - (0.25)^2 = 2.8875$

$\therefore Var(2x \pm 3) = 4(2.8875) = 11.55.$

14. The cumulative distribution function (cdf) of a random variable 'x' is $F(x) = 1-(1+x)e^{-x}, x>0$. Find the pdf of 'x', mean and variance.

Solution:- $F(x) = 1-(1+x)e^{-x}, 0<x<\infty$

pdf is $f(x) = \frac{d}{dx}(F(x))$

$$= \frac{d}{dx}\left[1-e^{-x}-xe^{-x}\right]$$

$$= e^{-x}-(-xe^{-x}+e^{-x})$$

$$= e^{-x}+xe^{-x}-e^{-x} = xe^{-x},\ x>0$$

$$\text{Mean} = E(x) = \int_0^\infty x\, f(x)dx = \int_0^\infty x\cdot xe^{-x}dx = \int_0^\infty x^2e^{-x}dx$$

$$= \left[x^2(-e^{-x})-(2x)(e^{-x})+(2)(-e^{-x})\right]_0^\infty$$

$$\left[e^{-\infty}=0\right]$$

$$= 0+2=2.$$

$$E\left[X^2\right] = \int_0^\infty x^2 f(x)dx = \int_0^\infty x^2\cdot xe^{-x}dx$$

$$= \int_0^\infty x^3e^{-x}dx = \left[x^3(-e^{-x})-(3x^2)(e^{-x})+(6x)(-e^{-x})-(6)(e^{-x})\right]_0^\infty$$

$$= 6$$

$$\therefore \text{Var}(X) = E\left[X^2\right]-\left[E(X)\right]^2 = 6-4 = \underline{2}.$$

Theorem 1:-

If $X_1, X_2, \ldots, X_n$ are independent random variables, then

$$M_{X_1+X_2+\ldots X_n}(t) = M_{X_1}(t)\, M_{X_2}(t) \ldots M_{X_n}(t).$$

Proof:- By definition,

$$M_{X_1+X_2+\ldots+X_n}(t) = E\left[e^{t(X_1+X_2+\ldots+X_n)}\right]$$

$$= E\left[e^{tX_1}\cdot e^{tX_2}\ldots e^{tX_n}\right]$$

$$= E\left[(e^{tX_1}) \cdot E(e^{tX_2}) \ldots E(e^{tX_n})\right]$$

$[\because X_1, X_2, \ldots, X_n$ are independent$]$

$$= M_{X_1}(t) \cdot M_{X_2}(t) \ldots M_{X_n}(t)$$

Note:-

$a = E(X) = \mu$ and $h = \sigma_x = \sigma$ then $U = \frac{X - E(X)}{\sigma_x} = \frac{X-\mu}{\sigma} = Z$ is known as standard normal variate. Thus the mgf of a Standard normal variate Z is given by,

$$M_Z(t) = e^{\frac{-\mu t}{\sigma}} \cdot M_X\left(\frac{t}{\sigma}\right)$$

$$E(Z) = E\left[\frac{X-\mu}{\sigma}\right]$$

$$= \frac{1}{\sigma}\left[E(X) - \mu\right] = \frac{1}{\sigma}\left[\mu - \mu\right] = 0$$

$$Var(Z) = Var\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2} Var(X-\mu) = \frac{1}{\sigma^2} Var(X)$$

$$= \frac{\sigma^2}{\sigma^2} = 1$$

$$E(Z) = 0$$

$$Var(Z) = 1$$

15. A random variable X has density function given by.

$$f(x) = \begin{cases} 1/k, & \text{for } 0 < x < k \\ 0, & \text{otherwise} \end{cases}$$

Find, (i) mgf (ii) rth moment (iii) mean (iv) variance

$$M_X(t) = E\left[e^{tx}\right]$$

$$= \int_0^k \frac{1}{k} e^{tx}\, dx$$

$$= \frac{1}{k}\left(\frac{e^{tx}}{t}\right)_0^k$$

$$= \frac{1}{kt}\left(e^{kt} - 1\right)$$

$$= \frac{1}{tk}\left[1 + \frac{(kt)}{1!} + \frac{(kt)^2}{2!} + \dots - 1\right]$$

$$= \frac{1}{kt}\left[\frac{kt}{1!} + \frac{(kt)^2}{2!} + \dots\right]$$

$$= 1 + \frac{kt}{2!} + \dots + \frac{(kt)^r}{(r+1)!} + \dots$$

μ'_r = coefficient of t^r.

$$= \frac{k^r}{(r+1)!}$$

When $r = 1$,

$$\mu'_r = \text{Mean} = k/2$$

When $r = 2$,

$$\mu'_r = \frac{2k^2}{3!} = \frac{k^2}{3}$$

Variance $= \mu'_2 - (\mu'_1)^2 = \frac{k^2}{3} - \frac{k^2}{4} = \underline{\underline{\frac{k^2}{12}}}.$