Concept 6: Probability

Probability is the science of predicting the likelihood of occurrences. *Probability* is a fraction between 0 and 1 that represents the likelihood of occurrence of an event in an experiment. An *outcome* is one of the possible events that can occur as a result of an experiment. The set of all possible outcomes for an experiment is called its *sample space*, and is denoted by 'S'. Any subset of the sample space is called an *event*, and is denoted by 'E'. The probability of an event E is denoted by P(E).

$P(A) = n(A) / n(S)$

If A and B are two different events then,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Independent events: Two events are said to be independent, if the outcome of one doesn't affect the outcome of the other. **Example:** When tossing a coin twice, the result of the first toss doesn't affect the probability of the outcome of the second toss.

Remember

$P(S) = 1$

$0 \leq P(E) \leq 1$

P (Impossible event) = 0

P (Definite event) = 1

Collectively exhaustive events: Two or more events are said to be collectively exhaustive if at least one of the events must occur. In other words, their union must cover all the events within the entire sample space. **Example:** When rolling a six-sided die, the outcomes getting an odd number and getting an even number are collectively exhaustive, because they encompass the entire range of possible outcomes.

Mutually exclusive events: Two or more events are said to be mutually exclusive if they cannot occur at the same time. **Example:** When tossing a coin, getting a head and getting a tail are mutually exclusive as they can't happen simultaneously.

Odds in favour/against: The odds in favour of an event are the ratio of probability that the event will happen to the probability that it will not happen. If P is the probability of the event, then the probability against the event is $1 - P$ and the odds in favour of the event are **P/(1 - P).** The odds against an event are the inverse ratio of odds in favour of the event and are**(1 - P)/P.**

Drill 6

a. Problems on Sample Space

i. If a coin is tossed 4 times, how many different outcomes are possible?
Number of outcomes per toss of a coin = _____
Total number of outcomes for 4 tosses = _____
If 4 coins are tossed at once, will the number of outcomes be the same? Yes/No

Hint
Use Fundamental Counting Principle

ii. If a die is rolled 3 times, how many different outcomes are possible?
Number of outcomes per roll of a die = _____
Total number of outcomes for rolling the die thrice = _____

iii. If 2 cards are drawn at random from a pack of cards, how many different outcomes are possible?

- With replacement _____
- Without replacement _____

b. Problems on Probability

i. If two different letters are selected at random from the English alphabet, what is the probability that

- Both of them are vowels?

$$\text{Probability} = \frac{\text{Number of ways of selecting 2 vowels}}{\text{Number of ways of selecting 2 alphabets}} = \_\_\_\_ = \_\_\_\_$$

- At least one of them is a vowel?

Total Probability = Probability of selecting 2 vowels + Probability of selecting 1 vowel = _____ + _____ = _____

ii. If 2 dice are rolled, what is the probability that the sum of the values on them is a prime number? _____

iii. If 3 coins are tossed simultaneously, what is the probability that exactly 2 of them will show tails? _____

iv. If 4 coins are tossed simultaneously, what is the probability of getting 2 or more tails? _____

v. If two cards are drawn at random from a pack of cards, what is the probability that

- Both of them are spades? _____
- Both of them are red or both of them are kings? _____

c. Problems on odds in favour / odds against

i. Two fair coins are tossed. What are the odds in favour of getting heads on both the coins?

Probability of getting heads on both the coins = _____

Odds in favour = Probability of getting heads on both: Probability of not getting heads on both = _____ : _____

ii. If 3 dice are rolled simultaneously, what are the odds against getting a prime number on each of the 3 faces? _____