Concept 2: Permutation and Combination

Permutation: Permutation means arrangement of things. The word arrangement is used if the order of things is considered.

Combination: Combination means selection of things. The word selection is used when the order of things has no importance.

Example:

Permutation or Combination

Is it correct to call this a combination lock?

or

Should this be called a permutation lock?

Fruit Basket

A fruit basket has an apple, a peach, grapes and bananas. Do we care about the order of the fruits in the basket? Yes/No

Identify whether the following are permutations or combinations.

Drill 2

Remember

- If the order doesn't matter, it is **Combination**.
- If the order does matter, it is **Permutation**

a. The number of ways a cricket team of 11 can be selected from a 16 - member squad. P/C

b. The number of ways 7 dignitaries can seat themselves in seven chairs kept on the stage. P/C

c. The number of ways a panel of 4 judges can be formed from 6 retired judges. P/C

d. The number of ways 5 friends can occupy 7 empty chairs in a theatre. P/C

e. The number of ways Raghav can invite 3 out of 7 friends to his house for a party. P/C

Concept 3: Computation of Permutation

Permutations with repetition: Consider '*n*' items to be arranged in '*r*' spaces.

If the same item can go into multiple spaces, there will always be '*n*' items available for each of the '*r*' slots. (In other words, there are '*n*' possibilities for the first choice, same '*n*' possibilities for the second choice, and so on).

This means that each of the '*r*' slots can be filled in '*n*' ways. Therefore, the number of ways of filling the '*r*' slots is **n x n ... (r times) =** $\mathbf{n^r}$

Permutations without repetition: Consider '*n*' items to be arranged in '*r*' spaces.

If the same item cannot go into multiple spaces, there will be '*n*' items available for filling the first slot. But there will be only *(n - 1)* items available for the second slot (leaving out the item that would have been used in the first slot), *(n - 2)* items available for the third slot and so on. So the number of ways of filling the '*r*' slots is **n (n - 1) (n - 2).... *(n - r + 1)***

> **Remember**
>
> $^nP_n = n!$
>
> $^nP_1 = n$

$$= \frac{n \times (n-1) \times (n-2) \ldots \times (n-r+1) \times (n-r) \times (n-r-1) \ldots \times 1}{(n-r) \times (n-r-1) \ldots \times 1} = \frac{n!}{(n-r)!}$$

The number of ways of permuting ***'r'*** items out of ***'n'*** items $= {}^nP_r = n!/(n-r)!$

Drill 3

a. For the following questions, identify whether repetition is allowed (A) or not allowed (NA).

   i. How many 3 - digit numbers can be formed using single digit prime numbers?
   A / NA

   ii. In how many ways can 15 friends sit in 20 chairs in a movie theatre?
   A / NA

   iii. In how many ways can 4 letters be posted in 6 post boxes? *A / NA*

b. In how many ways can 5 friends be seated in three chairs?

   Number of ways of filling the 1st chair = ________

   Number of ways of filling the 2nd chair = ________

   Number of ways of filling the 3rd chair = ________

   Total number of ways of filling the 3 chairs = ________

c. What is the maximum number of attempts required to open a 3 - slot number lock where each slot can have any digit between 0 and 9?

	1st Slot	2nd Slot	3rd Slot	Total
Total number of possibilities = x =				

d. How many 5 letter words (with or without meaning) can be formed using the letters of the word 'GREAT'?

   i. If repetition of alphabets is allowed

   _____ x _____ x _____ x _____ x _____ = __________

   ii. Without repetition of alphabets

   _____ x _____ x _____ x _____ x _____ = __________

iii. Such that all the vowels are together

_____ x _____ x _____ x _____ x _____ = __________

iv. Such that the vowels are together and the consonants are together

__________ x __________ = __________

v. No two vowels are together

_____ $\square$ _____ _____ $\square$ _____ = __________ $\square$

e. Making use of the five digits 0, 2, 6, 7, and 9,

i. How many 4-digit numbers can you make without repetition of digits?

_____ x _____ x _____ x _____ = __________

ii. How many 4-digit even numbers can you form?

_____ x _____ x _____ x _____ = __________

iii. How many 4-digit numbers divisible by four can you form?

_____ x _____ x _____________ = __________

f. In how many ways can three boys and three girls sit in six chairs?

i. If no 2 boys should sit together __________

ii. If no 2 boys or girls should sit together __________

g. How many 4-letter words can be formed using all the letters of the word 'TEST'? __________

h. How many words can be formed using all the letters of the word 'ELEPHANT'? __________

i. In how many ways can 3 red balls and 2 blue balls be arranged in a straight line? __________

j. In how many ways can 3 red and 2 blue balls be arranged in a straight line? __________

k. In how many ways can 5 letters be posted in 4 post - boxes?__________