Divide a fraction by a like-denominator fraction with a whole number result.

Fraction Decomposition and Division

Description

This is a set of progressive prompts that will allow students to construct understanding of division of a fraction by a fraction. Decomposing fractions with like denominators enhances student's ability to create meaning of the operation of division. Encourage students to build models and then a visual representation to deepen conceptual understanding. This series of prompts can be used for a variety of grades and purposes. The time required will vary depending on the grade level and student readiness.

Mathematics

Often, symbolic manipulation, such as 'invert and multiply' is presented as the way to divide fractions. However, division involving fractions is more meaningful with models as it helps students understand the impact of dividing a fraction by a fraction. These prompts support student understanding of the language and mathematical concepts associated with the division symbol. Students explore that $\frac{a}{b} \div \frac{c}{b}$ means "how many $\frac{c}{b}$ are there in $\frac{a}{b}$". Since the denominator is the fractional unit and the denominators are the same, the mathematical statement becomes 'how many *c* are there in a', which students can come to understand through these prompts. This is the same as 'how many *c* kg are in *a* kg', where the answer is $c \div a$. Once students recognize this they can more easily divide fractions with like denominators.

Curriculum Connections

Students will:

- represent the division of fractions with like-denominators using a variety of models;
- solve problems involving division of fractions using models and symbols;
- decompose simple fractions into unit fractions.

Instructional Sequence

1. Partner students and introduce the task. Post the first prompt, or distribute on a handout.
2. Provide students with time to complete the task by first having them use concrete (e.g., relational rods) or virtual (e.g., mathies.ca) manipulatives to visually represent only the prompt.
3. Next, have students demonstrate their thinking about division as decomposition of the fraction to answer the prompt using number lines or rectangular models (array) and explain the strategy used.
4. Debrief selected student responses with whole class. Use the key questions to guide the discussion.
5. Follow the same process with the next prompt.

Prompt #1

How many $\frac{1}{4}$ are there in $\frac{7}{4}$?

Prompt #2

How many $\frac{3}{4}$ are there in $\frac{9}{4}$?

Prompt #3

How many $\frac{2}{3}$ are there in $\frac{16}{3}$?

Highlights of Student Thinking	Key Questions
Students may: • represent $\frac{7}{4}$ as the mixed fraction $1\frac{3}{4}$ without saying how many $\frac{1}{4}$ are in the total; • label the segments cumulatively (e.g., $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, …); • not make the connection between decomposing the fraction into unit fractions and division; • need encouragement to label each unit fraction in their model; • more easily recognize individual unit fractions in their model if they use different colours; and • convert the improper fraction into a mixed fraction to recognize it is greater than a whole.	1. What did you do first to break the fraction into units? 2. How many partitions do you need in your model to represent $\frac{7}{4}$? 3. a) How much does each segment represent? b) What is your whole? How many wholes do you have?

Materials

- concrete manipulatives such as relational rods
- devices that support virtual manipulatives
- paper and markers