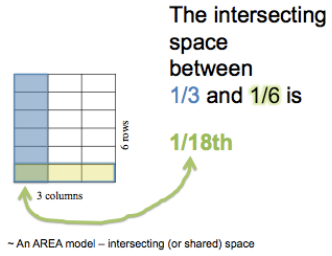
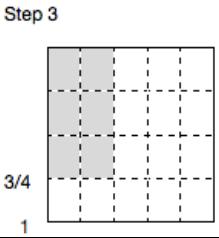
Models or Strategies for Multiplication and Division Involving Fractions

From *Fractions Operations: Multiplication and Division Literature Review*, pages 13-15

Three multiplication strategies (meaning, ways of thinking about multiplication) applied to both fractions and other number systems are outlined in the table below: (See Empson, page 189)

	Multiplication Strategy	Description	Example
Applied to other number systems	Measurement multiplication	When thinking about equal groups, the known values are usually the number of groups and the size of the groups. We use these to determine the total quantity.	A recipe calls for $\frac{3}{4}$ of a cup of flour. How much flour is needed to make $\frac{1}{2}$ of the recipe? "What is one-half of three-fourths?" $\frac{3}{4} \times \frac{1}{2} = \square$
	"Partial groups" multiplication	In this example we are multiplying one fraction quantity with another fraction quantity.	Each bag of candy has $\frac{1}{2}$ a pound. There are $3\frac{1}{2}$ bags of candy. How much candy do I have all together?
	Cartesian product	This model considers multiplication as the shared space of two numbers.	

It is also important to understand what happens when we divide two fractions. The complexity of this operation is apparent when we consider the many interpretations for the division of fractions…. Sinicrope, Mick and Kolb (2002) explain that we may "divide to determine how many times one quantity is contained in a given quantity, to share, to determine what the unit is, to determine the original amount, and to determine a dimension for an array" (p. 161). As with multiplication of fractions, it is helpful to relate our knowledge of whole number operations to fraction operations, and, therefore, to consider models that can be used for both division of whole numbers and division of fractions. The following table outlines division strategies as they apply to other number systems and as they relate more specifically to fractions:

	Division Strategy	Description	Example
Applied to other number systems	Measurement division (Quotative)	This model involves determining the number of groups, or how many times x goes in to y.	Consider using pattern blocks and thinking about how many blue rhombuses fit into 3 yellow hexagons – what fraction would one rhombus represent?
	Partitive division (Fair Share)	This model involves sharing something equally among friends. It involves determining the size of the group. Paper folding is a helpful way for children to understand partitive division.	If three friends share $\frac{1}{3}$ kilogram of chocolate, how much chocolate does each friend get?
	Division as the inverse of a Cartesian product (product-and-factors division)	This model is similar to the area model interpretation of multiplication described above (finding a Cartesian product). It involves determining the dimension of a rectangular area.	A rectangle has an area of $\frac{6}{20}$ square units. If one side length is $\frac{3}{4}$ units, what is the other side length? 
As related to fractions	Determination of a unit rate	This model emphasizes the size of one group (the unit rate).	A printer can print 20 pages in two and one-half minutes. How many pages does it print per minute?
	Inverse of multiplication	This model relies on understanding that division is the inverse of multiplication. By inverting a fraction and multiplying, the inverse is applied.	In a seventh-grade survey of lunch preferences, 48 students prefer pizza. This is one and one-half times the number of students who prefers the salad bar. How many prefer the salad bar?

(Sinicrope, Mick & Kolb, 2002)