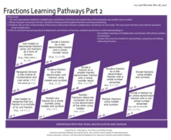

Fractions Learning Pathways Part 2

Math Teaching for Learning:
Multiplication and Division of Fractions across the K-12 Mathematics Curriculum

Within the *Fractions Learning Pathways*, twelve cells specify the foundational actions that support student understanding of multiplication and division of fractions. This support document is intended to provide a K-12 perspective on the role of multiplying and dividing fractions in students' learning. *Studies have shown "if children are given the time to develop their own reasoning for at least three years without being taught standard algorithms for operations with fractions and ratios, then a dramatic increase in their reasoning abilities occurred, including their proportional thinking" (Brown & Quinn, 2006, p. 39, citing Lamon, 1999).*

Multiplication as the Inverse of Division and vice versa

Students have experience with the inverse relationship between division and multiplication with whole numbers. They use this knowledge when solving questions such as $4 \times ? = 12$ by determining that $12 \div 4 = 3$. As students compose and decompose fractions, they often intuitively use multiplication and division interchangeably. For example, a young student may state that 'One whole is equal to two halves' or that 'One whole divided into two equals half.' Older students use the inverse relationship to solve fractions tasks involving multiplication and division. When presented with the question $\frac{6}{4} \div \frac{3}{8}$, many students considered 'how many $\frac{3}{8}$ are in $\frac{6}{4}$?' (or $? \times \frac{3}{8} = \frac{6}{4}$).

What does this look like in the Ontario Mathematics Curriculum (Specific Expectations)?
Students will:
Grade 2: regroup fractional parts into wholes, using concrete materials.
Grade 3: divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half; three thirds; two fourths or two quarters), without using numbers in standard fractional notation.
Grade 7: use estimation when solving problems involving operations with whole numbers, decimals, percents, integers, and fractions, to help judge the reasonableness of a solution;
Grade 9• solve problems requiring the expression of percents, fractions, and decimals in their equivalent forms (MPM 1D)

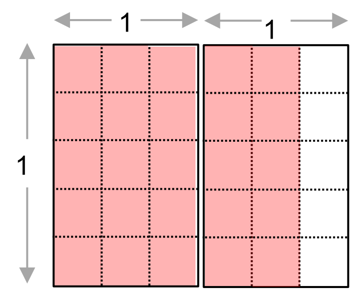
Connecting Multiplication and Division to Counting by Unit Fractions and Composing/Decomposing Fractions

Unit fractions are foundational to all fractions understanding. Any fraction can be expressed as the product of a whole number and a unit fraction. For example, $\frac{2}{5} = 2 \times \frac{1}{5}$. As students intuitively record their thinking using multiplication or division, they will develop an understanding of the connection between the count (numerator) and the multiplier (whole number). They will begin to understand that multiplication of a fraction by a whole number affects the count but not the fractional unit (denominator). Later, when answering questions such as $2 \times \frac{3}{5}$, students may consider this as '2 x 3 one-fifth units = 6 one-fifth units'. This is parallel to questions such as 2 × 3 cm = 6 cm.

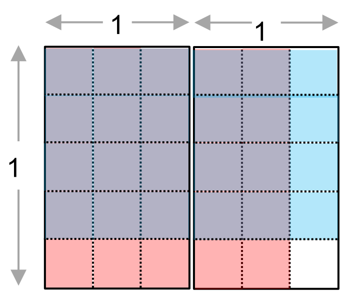
What does this look like in the Ontario Mathematics Curriculum (Specific Expectations)?
Students will:
Grade 1: divide whole objects into parts and identify and describe, through investigation, equal-sized parts of the whole, using fractional names (e.g., halves; fourths or quarters).
Grade 6: determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions, decimal numbers, and percents.
Grade 8: translate between equivalent forms of a number;
Grade 11: express the equation of a quadratic function in the vertex form $\mathrm{f(x)} = a(x-h)^2 + k$, *given the standard form* $\mathrm{f(x) = ax^2 + bx + c}$, *by completing the square, including cases where* $\frac{\mathrm{b}}{\mathrm{a}}$ *is a simple rational number (MCF 3M)*

Multiplication and Division of Fractions Using Models

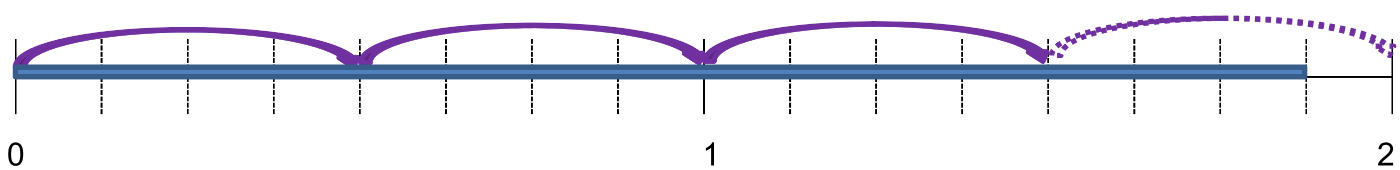
Number lines and arrays are powerful models for understanding multiplication and division of fractions. Use of models aids students in making sense of their answers. An array can be used to easily calculate the answer to a multiplication question, such as $1\frac{2}{3} \times \frac{4}{5}$. Each array represents 1 whole. One dimension is partitioned into thirds and $1\frac{2}{3}$ is shaded red.



The other is partitioned into fifths, with $\frac{4}{5}$ shaded blue. Notice that it is four-fifths of the vertical dimension so both areas are coloured. Another way to think about this is by considering it as a connection to the Distributive Property – four-fifths of both one and of two-thirds. The answer is indicated by the overlapping area (purple)– a total of 20 regions, each region representing $\frac{1}{15}$ – equal to $\frac{20}{15}$.



Number lines are particularly helpful for division questions with a remainder. For example, consider $1\frac{7}{8} \div \frac{1}{2}$. Students can see that each hop is $\frac{1}{2}$ and there is a remainder of $\frac{3}{8}$. They could see that the remainder of $\frac{3}{8}$ is equivalent to $\frac{3}{4}$ of one hop. They can reason that $1\frac{7}{8} \div \frac{1}{2} = 3\frac{3}{4}$. Students may phrase this as 'there are three halves and three-fourths of a half'.



What does this look like in the Ontario Mathematics Curriculum (Specific Expectations)?
Students will:
Grade 5: describe multiplicative relationships between quantities by using simple fractions and decimals;
Grade 8: represent the multiplication and division of fractions, using a variety of tools and strategies;
Grade 10: use the imperial system when solving measurement problems. (MFM 2P)

For further information, visit fractionsteaching.ca

- *Fractions Learning Pathways*
- *Fractions Operations: Multiplication and Division Literature Review*