

PLANNING FOR EFFECTIVE PROFESSIONAL LEARNING IN MATHEMATICS:

Insights from 7 Years of Research on Learning and Teaching Fractions in Ontario

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INTRODUCTION

Dr. Cathy Bruce and her research team from Trent University, in partnership with the Ministry of Education, have been conducting research on fractions teaching and learning in Ontario since the fall of 2011. The goal of this research has been to investigate effective instructional strategies in the difficult-to-teach and difficult-to-learn area of fractions. We wanted to better understand how to support students in the development of their understanding of fractions concepts and procedures. Throughout this process, we have gathered classroom-based evidence by working with teachers and students in a highly collaborative process that has involved nine school boards, 238 educators, and with targeted data collection involving 3235 students, spanning Grades 3-12. This work has led to the development of many resources for teachers and math leaders, including the Fractions Learning Pathways resource. (See p. 8 for more information on these resources and where to find them.)

With the project entering its final phases, the team looks back at the lessons learned through seven years of engaging in teacher professional learning and research at school boards in Ontario. In this document, we share strategies for maximizing the success of teacher professional learning in fractions. This document may be of interest to you if you are: a system or school leader, a mathematics consultant or anyone who is responsible for professional learning in mathematics.

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WHY FOCUS ON FRACTIONS?



Fractions are central to mathematics understanding. They are an essential aspect of number sense, of course, but fractions also cross into many other aspects of mathematics such as measurement. Fractions knowledge is a predictor of success in algebra, is essential to understanding probability, and involves high levels of spatial and proportional reasoning. It is perhaps because of the complexity of the relationship between fractions and this web of connected mathematical ideas and processes that they are notoriously difficult to learn and teach. These difficulties have been researched and documented starting in the early primary years, and are shown to persist through middle, secondary and post-secondary education and into adulthood in North America (Orpwood, Schollen, Leek, Marinelli-Henriques, & Assiri, 2011). The stakes are high for students; a shaky grounding in fractions can prevent individuals from pursuing advanced mathematics and shut students off from a significant number of career opportunities in later life.

For example, the STEM domains (science, technology, engineering and mathematics), as well as fields as broad-ranging as medicine, construction, art and design all demand considerable fractions knowledge. Researchers have noted that there is “a great deal of agreement that learning rational number concepts remains a serious obstacle in the mathematical development of children” (Behr, Harel, Post, & Lesh, 1992, p. 296). Fractions are a worthwhile area of focus for both teachers and students, as a foundational concept in mathematics with many connections to other important areas of mathematics.

CHARACTERISTICS OF EFFECTIVE PROFESSIONAL LEARNING

A recently conducted study, *The State of Educators’ Professional Learning in Canada* (Campbell, Osmond-Johnson, Faubert, Zeichner & Hobbs-Johnson, 2016), comprehensively investigated professional learning in Canada across all 10 provinces and 3 territories. The researchers examined “promising practices” as well as challenges in the Canadian context.

Following an extensive review of the literature, the researchers found that effective professional learning:

- is focused on quality content, that:
 - is evidence-informed;
 - is subject-specific;
 - is focused on pedagogical content knowledge and student outcomes; and
 - balances teacher voice and system coherence (goals are aligned).
- possesses a design for learning and implementation to include:
 - active and variable learning;
 - collaborative learning experiences; and
 - job-embedded learning.
- considers issues of support and sustainability, strengthened by:
 - a sustained duration;
 - the availability of resources; and
 - supportive and engaged leadership.

Their study on the Canadian education context further found that effective professional learning:

- balances evidence, inquiry and professional judgment;
- focuses on diverse student needs;
- maintains an asset-orientation towards both students and teachers;
- provides opportunities for collaboration that are job/classroom-embedded; and
- benefits from the support of system and school leaders.

These findings are consistent with earlier studies such as Hill’s 2004 study which highlighted that effective professional learning must be: inquiry-based, focused on student thinking, collaborative, job-embedded, content- and pedagogy-focused, teacher-driven and sustained over time.

In the Ontario fractions research, the professional learning that has taken place was developed with these characteristics of effective professional learning in place.

THE IMPORTANCE OF CLASSROOM-EMBEDDED COLLABORATIVE LEARNING

The W model, as shown in Figure 1, conceptualizes the process of collaborative, classroom-embedded professional learning (Bruce, Ross, Flynn, Kostuch, & McPherson, 2009; Flynn & Bruce, in press). Developed through intensive research on lesson study and action research for professional learning in mathematics education, this model emphasizes the value of learning that is focused on the classroom experience as situated learning. According to Lave and Wenger's (1991) seminal work, situated learning occurs within communities of practice at the site at which the learner will be performing the activity and is accordingly embedded within that context. The learning is socially co-constructed through mechanisms of observation, discussion and action. This is crucial for effective professional learning to take place, where the "social context of the classroom becomes the primary and legitimate site of teacher professional learning on an ongoing basis" (Bruce, Esmonde, Ross, Dookie, & Beatty, 2010).

The W Model

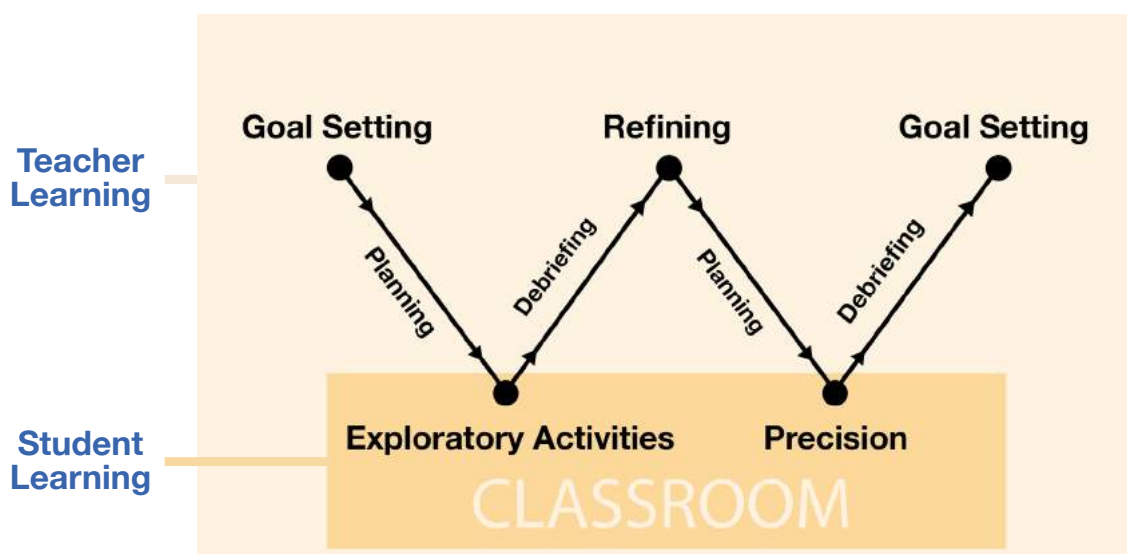


Figure 1: Describing Professional Learning using the W Model

PROFESSIONAL LEARNING WITH A SUSTAINED CONTENT FOCUS

The fractions researchers found that a sustained board-wide focus on one area of mathematics learning not only supported teachers in their learning and collaboration, but also supported student learning of difficult concepts (Ontario Ministry of Education, 2017; Yearley & Bruce, 2014). This is important because it validates school board planning of professional learning that is long-term (3-5 years) and deep, rather than short-term and broad. As such, we recommend that boards select a challenging mathematics content focus and sustain that focus over several years in order to make lasting effects that support students with a solid foundation for later mathematics. To date this research has shown that high quality fractions learning and teaching resources combined with quality professional learning opportunities (approximately 5 to 7 days over the course of a school year) leads to significant increases student achievement (Ontario Ministry of Education, 2015).

This has been my favourite professional learning I've done so far in my career. I feel like I bloomed as a math teacher and my students witnessed it happen. I have always been confident in math, but still insecure about being a teacher, but this has reminded me that I am an amazing math teacher and I can embrace that! A student said to me recently, "I feel like everyone's best subject is math, probably because you are such a good math teacher". Last year, I do not think that comment would have been made. Fractions are something that a lot of students and teachers have feared, so to be able to conquer that learning is huge. The confidence in fractions carries over to every strand of math!

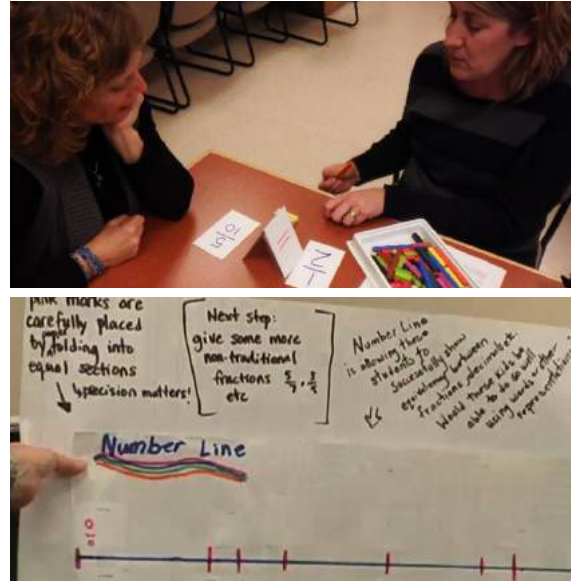
- CAR educator-researcher



APPLYING THE FEATURES OF EFFECTIVE PROFESSIONAL LEARNING: LESSONS FROM THE ONTARIO FRACTIONS PROJECT

Professional learning for teachers is central to supporting the learning and teaching of fractions, given the complexity of the mathematical content and the need for classroom and teacher supports. The fractions research team, along with participating educators, have identified a number of features of the project as central to the success of professional learning in fractions in the Ontario context. The research team recommends that these guidelines be at the heart of professional learning designs and initiatives:

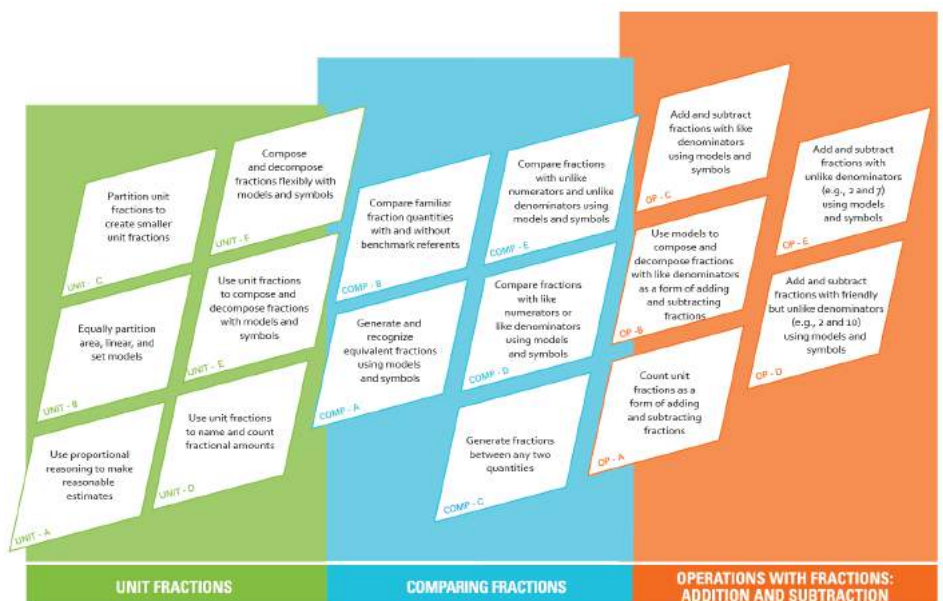
- allow considerable time for learning about the mathematics, including trying out tasks for later classroom use.
- provide opportunities to interpret student thinking (e.g., by analyzing and discussing student responses).
- identify areas where students are struggling and co-develop precise strategies to support students in these areas.
- include a knowledgeable other whenever possible, who can support specific content knowledge in teachable moments.
- refer to research-based resources, such as research articles and summaries, as well as lessons and tasks developed through research processes.
- engage teams of educators in collaborative processes, including classroom teachers as well as administrators, coaches, special education teachers and educational assistants, to draw on a range of insights and expertise on student learning and to ensure common understandings.
- build in structures for accountability by asking educators to return with evidence of student thinking from individual students, small groups or classroom activities and discussions.
- consider a range of learners across ability and grade levels.
- maintain a focus for the professional learning over a long period of time, such as one year or even longer, for deep learning.



FRACTIONS LEARNING PATHWAYS

The Fractions Learning Pathways is an interactive planning tool intended to provide educators with a research informed framework.

» Available on edugains.ca





"The priority area identified by teachers for developing their knowledge and practices is how to support diverse learners' needs."
- Campbell et al., 2016, p. 3

MAKING IT WORK IN YOUR SCHOOL BOARD: THINKING ABOUT OPTIONS FOR PL MODELS

Various models of professional learning at different stages of the project allowed the team to focus on knowledge creation (through collaborative action research) and then knowledge mobilization (by working with school boards using different approaches to scale up the learning opportunities in larger group sessions, or a blend of large- and small-group sessions). These different models are discussed below.

COLLABORATIVE ACTION RESEARCH

The Ontario fractions research project followed a collaborative action research model for professional learning. Collaborative action research involves educators and researchers working together to investigate common areas of interest in cycles of inquiry (Bruce & Flynn, in press; Flynn & Bruce, in press). Within these inquiry cycles, each session involves the collective debriefing of goals, tasks, student work samples and observations of student thinking. By focusing closely on student learning, the team reaches a better understanding of ways to help students with precise and incremental learning goals. The team implements and refines tasks based on these observations and discussions about student needs and assets identified through assessment for, and as, learning. Between-session implementation of tasks and observations, and the sharing of resulting student artefacts, lead to a sense of collective accountability and coherence.

Several participants expressed that participation in the professional learning increased their confidence in their ability to address the wide range of abilities in their classroom. For example, one participant wrote that "knowing where my students are starting out helps me meet my [students'] needs. I had such a range in my classroom, and many of my students had gaps. It was very challenging to fill in those gaps. I have discovered that I need to find tasks that can be modified and extended to meet all the needs in my classroom."

Even though collaborative action research is a time- and resource-intensive model for professional learning and it may be challenging to connect with a researcher to work closely with the team, it is a worthwhile investment. This model has the following benefits: **(i) it supports teacher and student learning; (ii) it builds teacher efficacy; and, (iii) it builds leadership capacity at the board level for facilitating mathematics content knowledge.** Collaborative action research also builds new knowledge and crucial insights that are locally relevant to supporting mathematics learning.

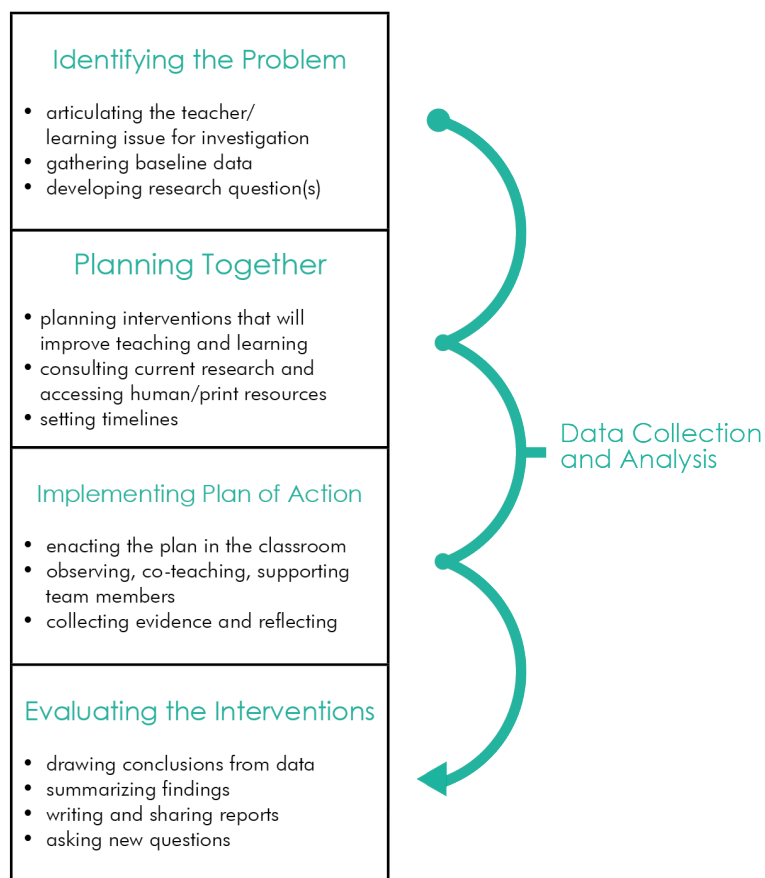
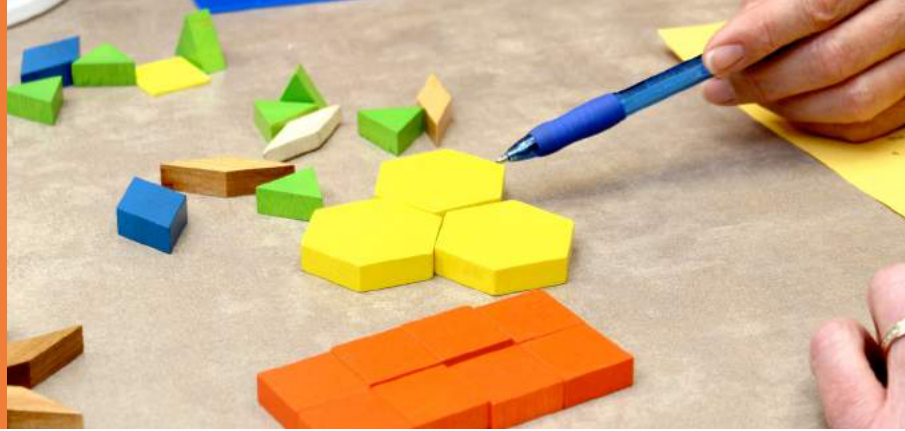


Figure 2: Collaborative Action Research Cycle (Bruce, Flynn & McPherson, 2009).
See tmerc.ca/digitalpapers/car.html for more information.

"Teachers value professional learning that is relevant and practical for work: "job-embedded" should not mean school-based exclusively, as opportunities to engage with external colleagues and learning opportunities matter also."

- Campbell et al., 2016, p. 3



BLENDED LARGE-GROUP AND SMALL-GROUP LEARNING

Another model of professional learning employed to build teacher mathematics knowledge for teaching of fractions is a blend of large group, board-level professional learning and smaller, school-based professional learning supported with individual coaching in-between sessions. In this model, there is alignment between board-level and school-level learning as well as in-class coaching activities. This blend of learning provides educators with multiple opportunities to try out new ideas in the classroom, and to reflect upon and discuss issues of teaching and learning within the context of a specific mathematical content focus.

This model works well with large-scale professional learning that is grounded in the smaller, intensive collaborative teams which provide opportunities for teachers to deeply explore:

- existing research,
- high quality field-tested resources, and
- field-tested frameworks that are grounded in student evidence of learning.

This model is cost intensive and, as a result, may only include a portion of the educators within the board. However, it may strike a balance between whole-group strategies where information and goals can be shared with large groups at once, and classroom-embedded opportunities for teachers to work more intensively and collaboratively. Creative strategies for grouping can also help to maximize the reach and spread; for example, if the professional learning opportunity is offered to a division, educators will be able to discuss learning across grades. This is particularly beneficial to those who are the only teachers of a given grade within their school.

A Model of Integrated Board and School Professional Learning

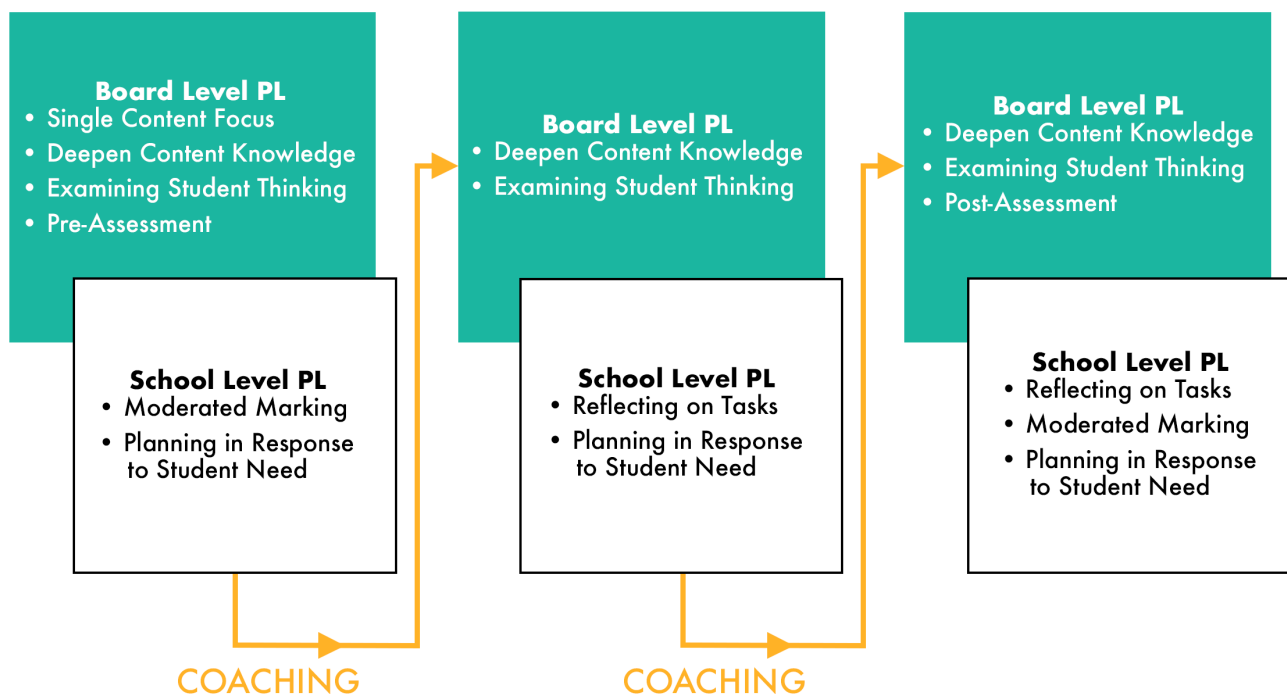


Figure 3. This graphic shows the flow between the board level focus on fractions professional learning and 'in-between' school based learning and coaching activities.

SCHOOL-LEADER FACILITATED PROFESSIONAL LEARNING

School administrators can implement content-based activities into staff learning opportunities, such as staff meetings or lunch-and-learns. Classroom visits are an additional structure in this model that brings the learning back to the classroom, and provides the impetus for school leaders to get directly involved with the teachers in their work. The information gathered during shared classroom observations can form the basis for subsequent staff meeting topics that keep the professional learning focused on the mathematics learning at the school.

A Model of Mathematics Leadership

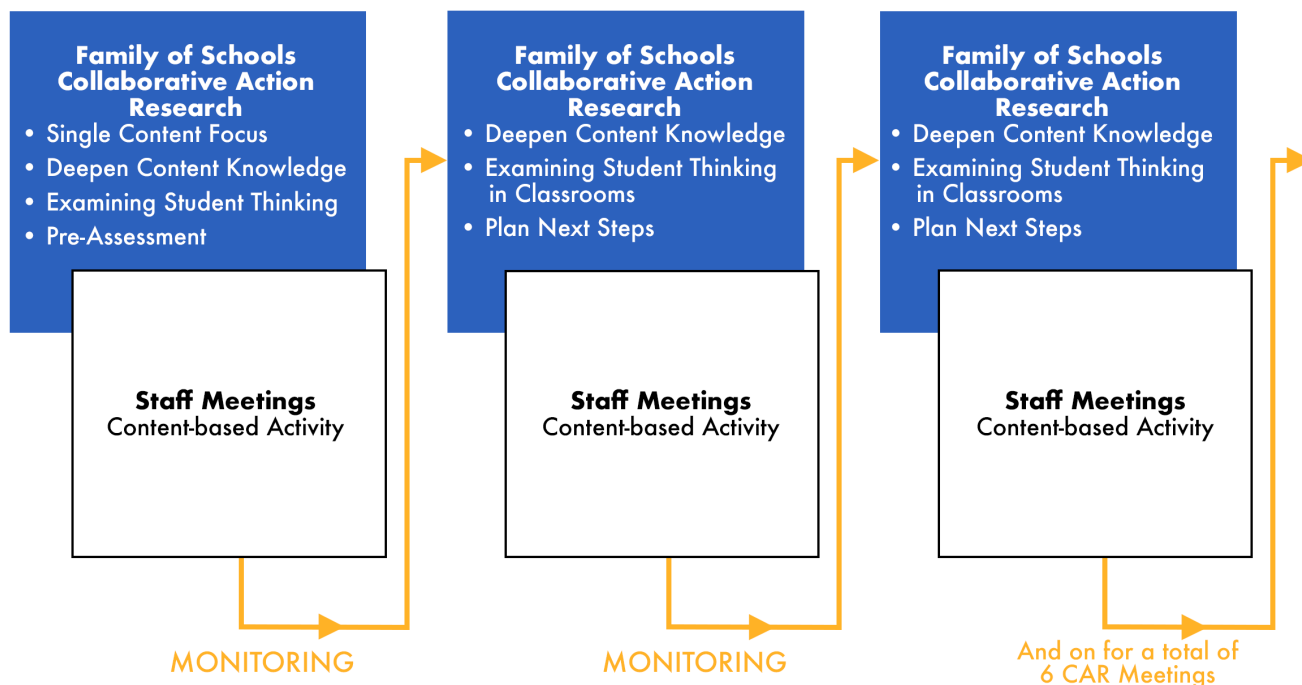


Figure 4. A school leader sustained a focus on fractions by basing staff meeting content learning on the activities explored in a Family of Schools collaborative action research.

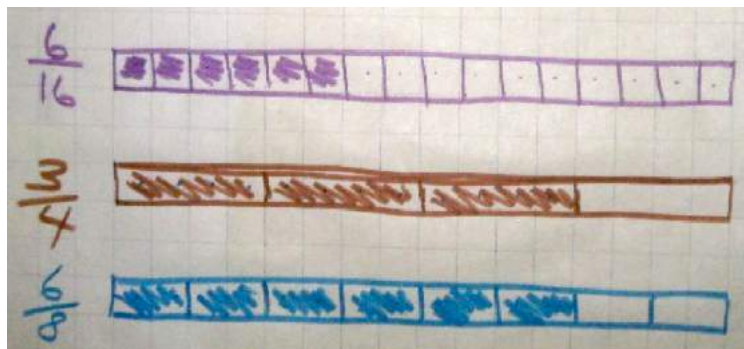


How can I justify a heavy focus on fractions in my school? The answer is quite simple: "Fractions are considered by many to be among the most difficult topics in the elementary school curriculum. As a matter of fact, in a recent national report, mathematicians and mathematics educators alike reported that problems with learning fractions interfere with learning other mathematics topics and continue to plague adults in daily tasks."

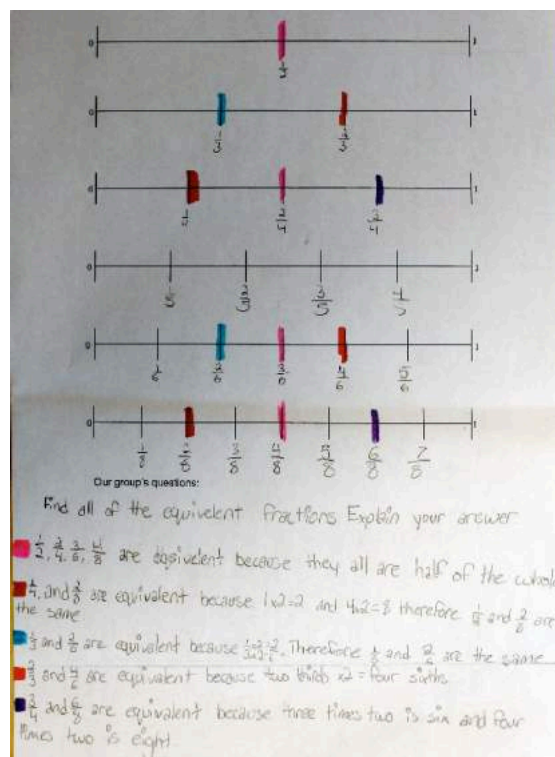
– School Principal (quoting from *A Focus on Fractions: Bringing Research to the Classroom* by Petit, Laird and Marsden, p.xi)

GREATER PRECISION THROUGH TEACHER INQUIRY

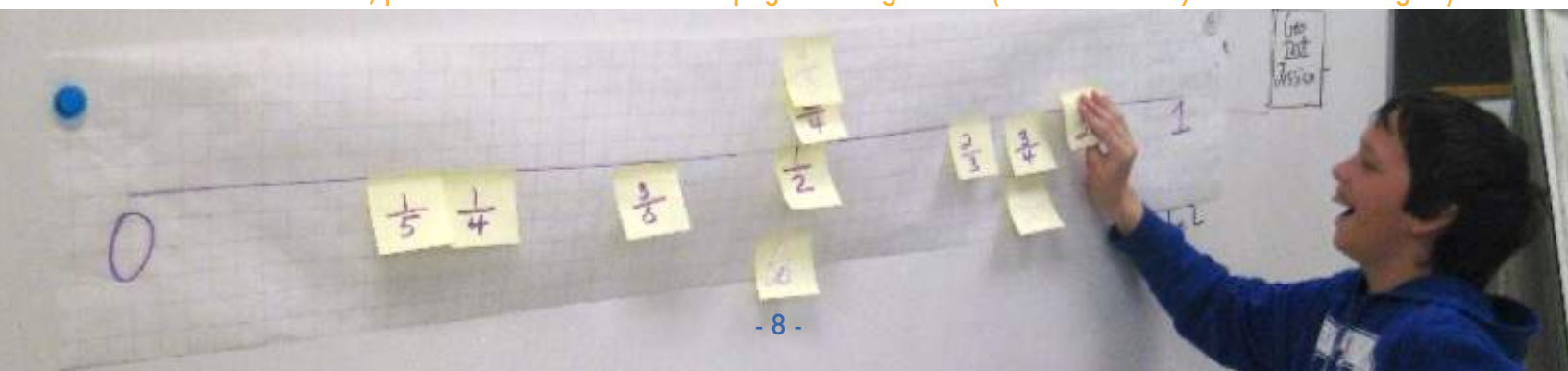
In the six years of active classroom research in the Ontario fractions project, teams focused very closely on student thinking and precision of task design and teacher moves. The observations of the teams led to important findings about powerful instructional strategies that support deep understanding in fractions (e.g., precise use of language, visual representations, active learning strategies). Below are some transformative learnings from the teams:



- use precise language. It is important that language in the classroom consistently reflects mathematical meaning, for example, that we read fractions as numbers, such as $\frac{3}{4}$ is “three sevenths” rather than “three over seven” or “three out of seven”.
- emphasize unit fractions. Opportunities to count by unit fractions, and to compose and decompose quantities using unit fractions, help students build the foundations for more complex ideas and skills in fractions, such as operations.
- use representations carefully and with intention. Students benefit from working with a limited number of representations, such as rectangle area models, number lines, volume models, and sets.
- establish strong connections between representations and symbolic manipulation. This will ensure that students develop strong conceptual foundations alongside an understanding of the symbolic representations. Students will understand, for example, that an equivalent fraction is generated by splitting or merging regions in an area model, and will understand how this translates into the algorithm for determining equivalent fractions symbolically.
- allow all students opportunities to revisit and strengthen foundational understandings, such as returning to unit fractions as a base for operations with fractions (e.g., 2 one-third units plus 3 one-third units equals 5 one-third units).
- punctuate instruction of fractions, with learning opportunities throughout the year for regular practice and continual increases in depth, rather than in a one-time short unit of study.

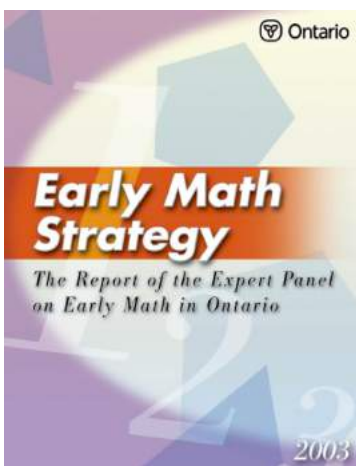


For more information, please see the [Mathematics](https://www.edugains.ca) page of [edugains.ca](https://www.edugains.ca) (also reached by [fractionsteaching.ca](https://www.fractionsteaching.ca)).

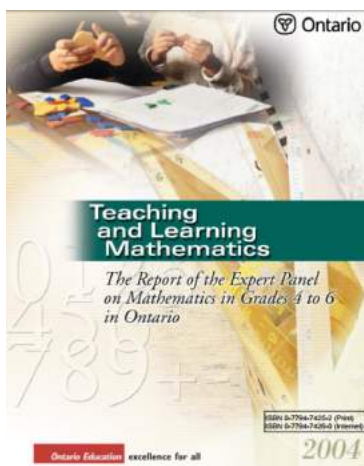


CORE SUPPORT DOCUMENTS FOR MATHEMATICS LEADERS

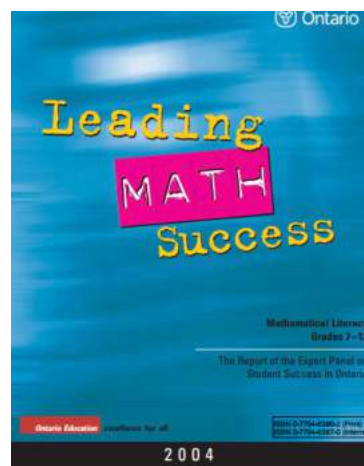
In 2003 and 2004, three expert panel reports were released to inform mathematics learning and teaching in Ontario classrooms.



Early Math Strategy
The Report of the Expert Panel
on Early Math in Ontario

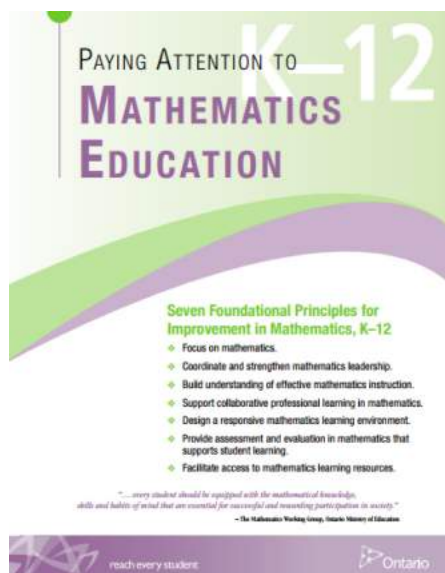


Teaching and Learning Mathematics
The Report of the Expert Panel on
Mathematics in Grades 4 to 6 in Ontario



Leading Math Success
The Report of the Expert Panel
on Student Success in Ontario

In 2011, *Paying Attention to K-12 Mathematics Education* was released. This document, emerging from the Mathematics Working Group, combined the recommendations of the three expert panel reports and supplemented that information with current research. This document has formed a solid foundation for subsequent Ministry resource development and professional learning, with additional documents focused on evidence-based practice in fractions, as well as proportional, spatial and algebraic reasoning.



- Paying Attention to Fractions
- Paying Attention to Proportional Reasoning
- Paying Attention to Spatial Reasoning
- Paying Attention to Algebraic Reasoning

» edugains.ca/newsite/math/payingattentiontomath.html

Given the importance of fractions to the understanding of other areas of mathematics, and given that fractions are an area of difficulty for teachers and students alike, fractions are worthy of an intentional focus for professional learning in mathematics. Both teachers and students are empowered when they are supported in developing a solid understanding built on firm foundations and based on Ontario research. To see all of the resources for supporting fractions teaching and learning developed through this work, please see the Mathematics page of edugains.ca (also available at fractionsteaching.ca).

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