CHAPTER-11
TRIANGLES

1 Exercise 11.2

Q2.Construct a triangle ABC in which $BC = 8cm$,$\angle B = 45^0$ and $AB - AC = 3.5cm$.

Solution:

Let **A**,**B** and **C** are the vertices of the triangle with coordinates. Given $BC = 8cm$.So the coordinates of vertices **B**,**C** are:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \ \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

Also given $\angle B = 45^0$,so by finding the coordinates of the other side we can form a required triangle.

The input parameters for this construction are

Symbol	Value	Description
a	8cm	BC
θ	45^0	$\angle BC$ in ΔABC
k	3.5	AB-AC i.e(c-b)
$\mathbf{e}_2$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	basis vector

Table 1: Parameters

Caluclating Other Coordinate:

Let $\mathbf{A}$ = c $\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

Using the Cosine formula in ΔABC,

$$b^2 = a^2 + c^2 - 2accos\mathbf{B} \tag{1}$$

$$(b+c)(b-c) = a^2 - 2accos\mathbf{B} \tag{2}$$

Given

$$c - b = k \tag{3}$$

$$b - c = -k \tag{4}$$

Upon Simplifaction we get:-

$$(b+c)(-k) = a^2 - 2accos\mathbf{B} \tag{5}$$

From equations (4) and (5) , we obtain the matrix equation:-

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} \frac{-a^2+2accos\mathbf{B}}{k} \\ -k \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} b \\ c \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \frac{-a^2+2accos\mathbf{B}}{k} \\ -k \end{pmatrix} \tag{7}$$

From the above equation

$$c = \frac{1}{2}\mathbf{e}_2^T \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \frac{-a^2+2accos\mathbf{B}}{k} \\ -k \end{pmatrix} \tag{8}$$

$$c = \frac{1}{2(1-\frac{acos\mathbf{B}}{k})}\mathbf{e}_2^T \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \frac{-a^2}{k} \\ -k \end{pmatrix} \tag{9}$$

$$c = \frac{1}{2(1-\frac{acos\mathbf{B}}{k})}\mathbf{e}_2^T \begin{pmatrix} \frac{-a^2}{k}-k \\ \frac{-a^2}{k}+k \end{pmatrix} \tag{10}$$

$$c = \frac{1}{2(1-\frac{8cos45^0}{3.5})} \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-64}{3.5}-3.5 \\ \frac{-64}{3.5}+3.5 \end{pmatrix} \tag{11}$$

$$c = \frac{1}{2\left(\frac{3.5-5.65}{3.5}\right)} \begin{pmatrix} \frac{-64+12.25}{3.5} \end{pmatrix} \tag{12}$$

$$c = 12 \tag{13}$$

The vertices of Δ ABC are

$\mathbf{A} = 12\begin{pmatrix} cos45 \\ sin45 \end{pmatrix} = \begin{pmatrix} 8.48 \\ 8.48 \end{pmatrix}$

$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$

Construction:

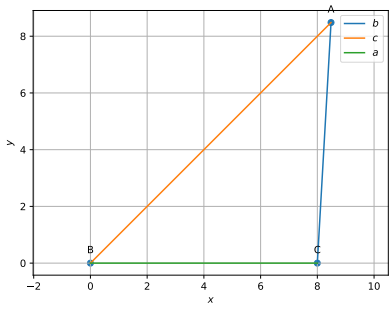

Figure 1: Triangle ABC