

CHAPTER-11
TRIANGLES

1 Exercise 11.2

Q2. Construct a triangle ABC in which $BC = 8cm, \angle B = 45^\circ$ and $AB - AC = 3.5cm$.

Solution:

Let \mathbf{A}, \mathbf{B} and \mathbf{C} are the vertices of the triangle with coordinates. Given $BC = 8cm$. So the coordinates of vertices \mathbf{B}, \mathbf{C} are:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

Also given $\angle B = 45^\circ$, so by finding the coordinates of the other side we can form a required triangle.

The input parameters for this construction are

Symbol	Value	Description
a	8cm	BC
θ	45°	$\angle BC$ in $\triangle ABC$
k	3.5	AB-AC i.e(c-b)
\mathbf{e}_2	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	basis vector

Table 1: Parameters

Calculating Other Coordinate:

$$\mathbf{A} = c \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

We know that

$$c = \frac{1}{2(1 - \frac{a \cos \mathbf{B}}{k})} \mathbf{e}_2^T \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \frac{-a^2}{k} \\ -k \end{pmatrix} \quad (1)$$

$$c = \frac{1}{2(1 - \frac{a \cos \mathbf{B}}{k})} \mathbf{e}_2^T \begin{pmatrix} \frac{-a^2}{k} - k \\ \frac{-a^2}{k} + k \end{pmatrix} \quad (2)$$

$$c = \frac{1}{2(1 - \frac{8 \cos 45^\circ}{3.5})} \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-64}{3.5} - 3.5 \\ \frac{-64}{3.5} + 3.5 \end{pmatrix} \quad (3)$$

$$c = \frac{1}{2 \left(\frac{3.5 - 5.65}{3.5} \right)} \left(\frac{-64 + 12.25}{3.5} \right) \quad (4)$$

$$c = 12 \quad (5)$$

The vertices of ΔABC are

$$\mathbf{A} = 12 \begin{pmatrix} \cos 45 \\ \sin 45 \end{pmatrix} = \begin{pmatrix} 8.48 \\ 8.48 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

Construction:

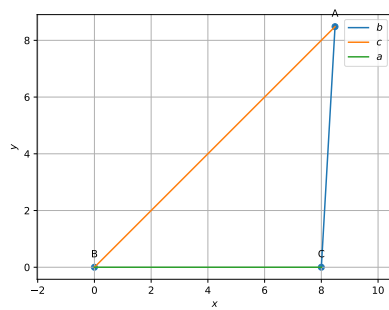


Figure 1: Triangle ABC