CHAPTER-11
TRIANGLES

1 Exercise 11.2

Q2.Construct a triangle ABC in which $BC = 7cm$,$\angle B = 45^0$ and $AB - AC = 3.5cm$.

Solution:

Let **A**,**B** and **C** are the vertices of the triangle with coordinates. Given $BC = 8cm$.So the coordinates of vertices **B**,**C** are:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \ \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

Also given $\angle B = 45^0$,so by finding the coordinates of the other side we can form a required triangle.

The input parameters for this construction are

Symbol	Value	Description
a	8cm	BC
θ	45^0	$\angle BC$ in ΔABC
k	3.5	AB-AC i.e(c-b)

Table 1: Parameters

Caluclating Other Coordinate:

Let $\mathbf{A}$ = b$\times \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

Using the Cosine formula in ΔABC,

$$b^2 = a^2 + c^2 - 2accos\mathbf{B} \tag{1}$$

$$(b+c)(b-c) = a^2 - 2accos\mathbf{B} \tag{2}$$

Given

$$c - b = k \tag{3}$$

$$b - c = -k \tag{4}$$

Upon Simplifaction we get:-

$$(b+c)(-k) = a^2 - 2accos\mathbf{B} \tag{5}$$

$$-kc - kb + 2accos\mathbf{B} = a^2 \tag{6}$$

$$-kb - c(k - 2acos\mathbf{B}) = a^2 \tag{7}$$

From equations (4) and (7) , we obtain the matrix equation:-

$$\begin{pmatrix} 1 & -1 \\ -k & -k+2acos\mathbf{B} \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} -k \\ a^2 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} 1 & -1 \\ -3.5 & -3.5+2(8)cos45^0 \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} -3.5 \\ 64 \end{pmatrix} \tag{9}$$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 1 & -1 & -3.5 \\ -3.5 & 7.81 & 64 \end{array}\right) \tag{10}$$

Reducing to echelon form:-

$$\left(\begin{pmatrix} 1 & -1 & -3.5 \\ 0 & \frac{4.31}{3.5} & \frac{51.75}{3.5} \end{pmatrix} \xleftarrow{R_2 \leftarrow \frac{1}{3.5}R_2 + R_1} \right)$$

$$\left(\begin{pmatrix} 1 & -1 & -3.5 \\ 0 & 1 & \frac{51.75}{4.31} \end{pmatrix} \xleftarrow{R_2 \leftarrow \frac{3.5}{4.31}R_2} \right)$$

$$\left(\begin{pmatrix} 1 & 0 & \frac{36.67}{4.31} \\ 0 & 1 & \frac{51.75}{4.31} \end{pmatrix} \xleftarrow{R1 \leftarrow R_1 + R_2} \right)$$

Reduced Echelon Form:

$$\begin{pmatrix} 1 & 0 & 8.5 \\ 0 & 1 & 12 \end{pmatrix}$$

$$\begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 8.5 \\ 12 \end{pmatrix}$$

The vertices of Δ ABC are

$$\mathbf{A} = 8.5\begin{pmatrix} cos45 \\ sin45 \end{pmatrix} = \begin{pmatrix} 6.01 \\ 6.01 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

Construction:

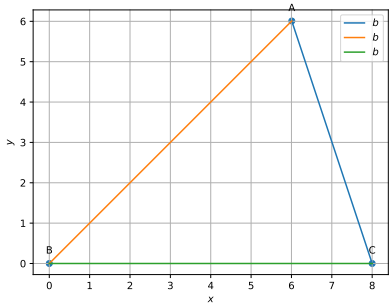

Figure 1: Triangle ABC