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Basic probability

1. Two dies are rolled at once. Find out the probability for sum of numbers being even and one of the die shows 6.

Analysis: It is single event, the total no of possible combinations are : 36 (as there are two dice) Total no of favourable outcomes = (sum is even and one of the die should show 6) possible combinations are : (6,2) (6,4), (6,6), (2,6), (4,6)

Answer: probability = (No of favourable outcomes / Total no of possible outcomes) = (5/36)

2. Two dies are rolled at once. Find out the probability for sum of numbers being less than 7.

Analysis: No of favourable outcomes are : (1,5)(1,4),(1,3),(1,2),(1,1) ,(2,4),(2,3),(2,2),(2,1),(3,3),(3,2),(3,1), (4,2),(4,1),(5,1)

Answer: probability = (No of favourable outcomes / Total no of possible outcomes) = (15/36)

3. You toss a fair coin three times :Given that you have observed atleast one heads , what is the probability that you observe atleast two heads?

Analysis: Total possible outcomes = 7, as it was observed that , atleast one head No of favourable outcomes = 4, (which had atleast two heads)(THH)(HTH)(HHT)(HHH)

Answer: $p(\text{atleastTwoHeads}) = 4/7$

4. A and B are a married couple with two kids. One of them is a girl. What is the probability that their other kid is also a girl?

Analysis: The second kid probability is not dependent on first kid, so it is normal probability

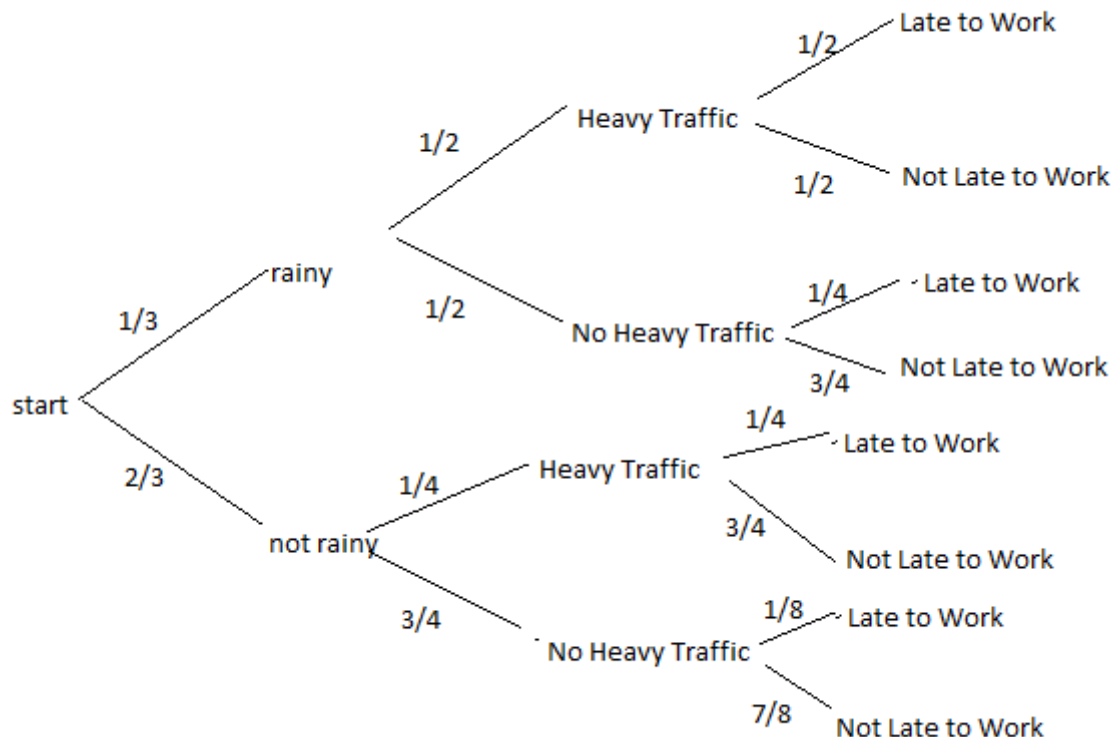
Answer: $p(\text{girl}) = 1/2$

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Conditional, Joint and Marginal Probability

5. In my town, it's rainy for one third of the days. Given that it is rainy, there will be heavy traffic with

probability $1/2$, and given that it is not rainy, there will be heavy traffic with probability $1/4$. If it's rainy and there is heavy traffic, I arrive late for work with probability $1/2$. On the other hand, the probability of being late is $1/8$ if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25 , 0.25 . You pick a random day.



a) What is the probability that it's not raining and there is heavy traffic and I am not late?

Analysis:

$$p(\text{NotRaining} \wedge \text{HeavyTraffic} \wedge \text{Not Late}) = p(\text{NotLate} | (\text{HeavyTraffic} \& \text{Not rainy})) \times p(\text{HeavyTraffic} | \text{not rainy}) \times p(\text{not rainy})$$

Answer: $p(\text{NotRaining} \wedge \text{HeavyTraffic} \wedge \text{Not Late}) = (3/4) \times (1/4) \times (2/3) = (1/8)$

b) What is the probability that I am late?

Analysis:

$$p(\text{Late}) = p(\text{Late} | (\text{HT} \& \text{Rainy})) + p(\text{Late} | (\text{NHT} \& \text{Rainy})) + p(\text{Late} | (\text{HT} \& \text{Not Rainy})) + p(\text{NHT} \& \text{Not Rainy})$$

Answer: $p(\text{Late}) = (1/2 \times 1/2 \times 1/3) + (1/4 \times 1/2 \times 1/3) + (1/4 \times 1/4 \times 2/3) + (1/8 \times 3/4 \times 2/3) = 11/48$

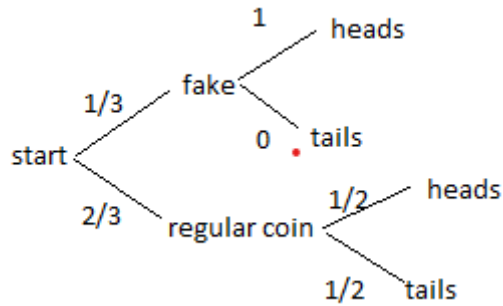
c) Given that I arrived late at work, what is the probability that it rained that day?

Analysis: $p(\text{rainy} | \text{Late}) = p(\text{Late} | \text{rainy}) \times p(\text{Late}) / p(\text{rainy})$

$$= [p(\text{Late} | (\text{HT} \& \text{rainy})) + p(\text{Late} | (\text{NHT} \& \text{rainy}))] \times p(\text{Late}) / p(\text{rainy})$$

Answer: $p(\text{rainy} | \text{Late}) = [(1/2 \times 1/2) + (1/4 \times 1/2)] \times 11/48 / (1/3) = 33/128$

6. A box contains three coins: two regular coins and one fake two-headed coin ($P(\text{Heads})=1$), You pick a coin at random and toss it.



6a) What is the probability that it lands heads up?

Analysis: and Answer:

$$p(\text{Heads}) = p(H|\text{fake}) \times p(\text{fake}) + p(H|\text{regular}) \times p(\text{regular}) = (1 \times 1/3) + (1/2 \times 2/3) = 2/3$$

6b) You pick a coin at random and toss it and get heads. What is the probability that it is the two-headed coin?

Analysis: and Answer:

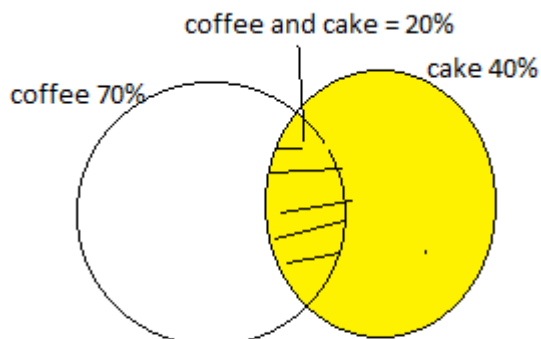
$$p(\text{Heads}) = 2/3 \text{ (from previous question)}$$

$$p(\text{fake}|\text{Heads}) = p(\text{Heads}|\text{fake}) \times p(\text{fake}) / p(\text{Heads}) = (1 \times 1/3) / (2/3) = 1/2$$

7. Suppose that, of all the customers at a coffee shop

- 70% purchase a cup of coffee
- 40% purchase a piece of cake
- 20% purchase both a cup of coffee and a piece of cake.

Given that a randomly chosen customer has purchased a piece of cake, what is the probability that he/she also purchased a cup of coffee.



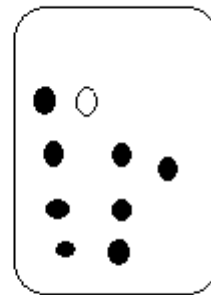
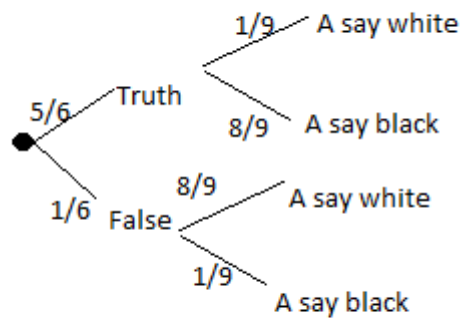
$$P(\text{coffee}|\text{cake}) = (\text{No of favourable outcomes} / \text{Total no of possible outcomes}) \\ = P(\text{coffee and cake}) / p(\text{cake})$$

Analysis: and Answer:

$$p(\text{coffee}) = 0.7, p(\text{cake}) = 0.4, p(\text{coffee and cake}) = 0.2$$

$$p(\text{coffee}|\text{cake}) = p(\text{coffee and cake}) / p(\text{cake}) = 0.2 / 0.4 = 0.5$$

8. A is known to tell the truth in 5 cases out of 6 and he states that a white ball was drawn from a bag containing 8 blacks and 1 white ball. Find the probability that the white ball was drawn.



$$p(\text{black}) = 8/9$$

$$p(\text{white}) = 1/9$$

Analysis: and Answer:

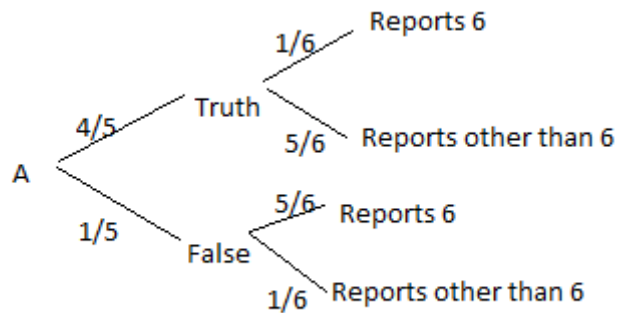
$$p(\text{Truth}|\text{white}) = p(\text{white}|\text{Truth}) \times p(\text{Truth}) / p(\text{white})$$

$$p(\text{white}) = p(\text{white}|\text{Truth}) \times p(\text{Truth}) + p(\text{white}|\text{False}) \times p(\text{False})$$

$$= (1/9 \times 5/6) + (8/9 \times 1/6) = 13 / 54$$

$$p(\text{Truth}|\text{white}) = (1/9 \times 5/6) / (13/54) = 5/13$$

9 A speaks the truth 4 out of 5 times. A die is tossed. A report that it is a 6. What are the chances that there actually was a 6?



$$p(\text{truth}) = 4/5, p(\text{false}) = 1/5,$$

$$p(\text{truth} | 6) = p(6 | \text{truth}) \times p(\text{truth}) / p(6)$$

$$\begin{aligned} p(6) &= p(6 | \text{truth}) \times p(\text{truth}) + p(6 | \text{false}) \times p(\text{false}) \\ &= 1/6 \times 4/5 + 5/6 \times 1/5 \\ &= 9/30 \end{aligned}$$

$$\begin{aligned} p(\text{truth} | 6) &= (1/6) \times 4/5 / (9/30) \\ &= 4/9 \end{aligned}$$

Analysis: and Answer:

$$p(\text{truth}) = 4/5, p(\text{false}) = 1/5,$$

$$p(\text{truth} | 6) = p(6 | \text{truth}) \times p(\text{truth}) / p(6) = 4/9$$

10. In a class, 40% of the student's study math and science. 60% of the student's study math. What is the probability of a student studying science given he/she is already studying math?

Analysis: and Answer:

$$p(\text{Maths and Science}) = 0.4, p(\text{Maths}) = 0.6$$

$$P(\text{science} | \text{Maths}) = p(\text{ Maths and Science}) / p(\text{Maths})$$

$$= 0.4 / 0.6 = 2/3$$

11. Below is a table of graduates and post graduates

	Graduate	Post Graduate	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

a) What is the probability that a randomly selected individual is a male and a graduate? What kind of probability is it (Marginal / Joint / Conditional)

$$p(\text{male and graduate}) = 19/100 = 0.19$$

It is Joint Probability

b) What is the probability that a randomly selected individual is a male?

$$p(\text{male}) = 60/100 = 0.6$$

c) What is the probability of a randomly selected individual being a graduate ? What kind of probability is this?

$$p(\text{graduate}) = 31/100 = 0.31$$

It is marginal probability

d) What is the probability that a randomly selected person is a female given that the selected person is a post graduate ? What kind of probability is this?

$$p(\text{female}|\text{PG}) = 28/69 \text{ (out of 69 PG, 28 are female)}$$

It is conditional probability

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Bayes Theorem

12. You need to figure out whether a company is fraud based on the legal charges they filed. We have the knowledge that, the chances a company submitting fraudulent fillings is 0.1. There exists an algorithm that can predict fraud. This algorithm returns a correct positive result in 92% of the cases in which the fraud is present and correct negative results in 90% of the cases where the fraud is not present. Suppose we observe a company for whom the algorithm test returns a fraud result. Calculate the posterior probability that this company truly did fraud in their filings.

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