DATE:

Root or building block of linear algebra is Vector

What a vector is

There are 3 related ideas in terms of

physics Student perspective | Mathematician perspective | CS student perspective

↓ vectors are arrows pointing in space
it has length & direction

↓ ordered lists of nos

$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 5 \\ 0 \\ 0 \\ -5 \end{bmatrix}$ $\begin{bmatrix} 23 \\ -7 \\ 01 \end{bmatrix}$

$\begin{bmatrix} 2{,}600\ ft^2 \\ \$300{,}000 \end{bmatrix}$

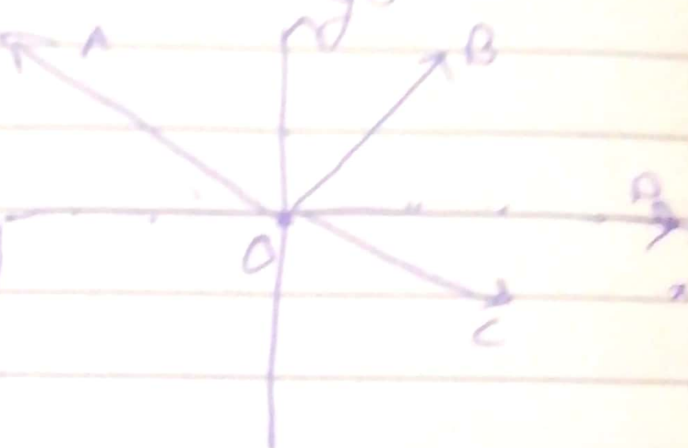
Mathematicians

$\vec{v} + \vec{w}$

$2\vec{v}$

DATE:

In Linear algebra, vectors always starts from origin.



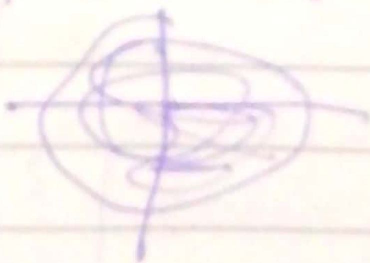
$$\begin{array}{cccc} A & B & C & D \\ \begin{bmatrix} -2 \\ 3 \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \end{bmatrix} & \begin{bmatrix} 2 \\ -1 \end{bmatrix} & \begin{bmatrix} 4 \\ 0 \end{bmatrix} \end{array}$$

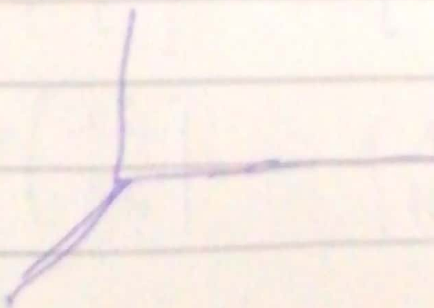
Co-ordinates of a vector explains how far is it the vector from origin to its tail point can be written as (-4, 2)

* but vectors will be written with Square brackets $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

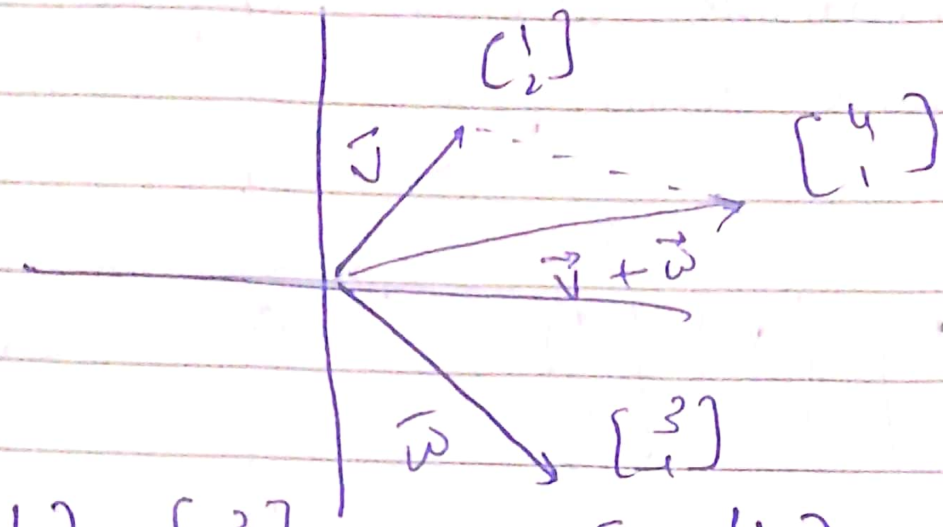
In a 3 dimensional Space, each vector will be represented by triplet of nos

for eg $\begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$





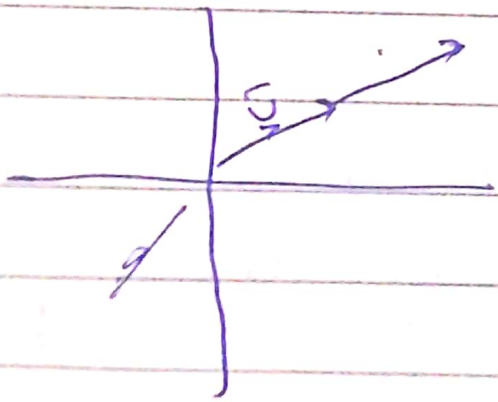
$\vec{v} + \vec{w}$



$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}$$

$2\vec{v}$



Scaling a Vector

$2, \frac{1}{3}, -1.3$

"Scalars"

$\frac{1}{3}\vec{v}$

$-\frac{1}{3}\vec{v}$

$$2 \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$$2 \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

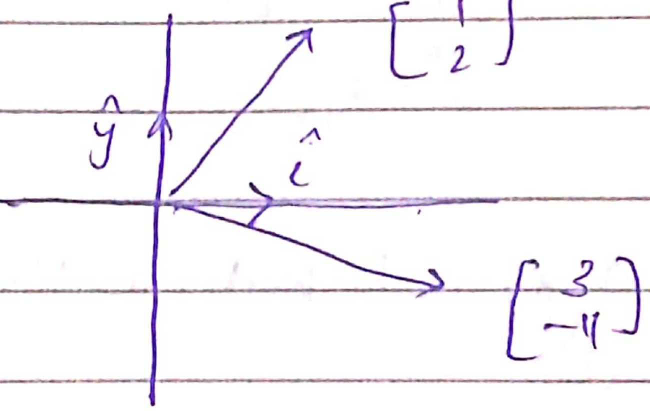
Linear algebra ~~follows~~ topics te: Vector addition & Scalar multiplication

Linear algebra topics tend to revolve around two operations Vector addition

Scal multiplication

Linear Combinations:-

Span and Basis Vectors

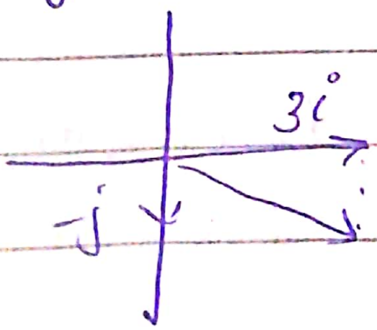


Vector Co-ordinates :-

Think of each as scalars [3] [-1]

$\hat{i}$ → unit vector in the x-direction

$\hat{j}$ - unit vector in the y-direction



$(3)i + (-2)j$

adding together two scaled vectors

$\hat{i}$ and $\hat{j}$ are basis vectors of xy-cordinate system

What if we chose different basis vectors?
We get a different co-ordinate system
We can reach all the points in
xy co-ordinate system by using new
cordinate system as well.
But the scalar multiplication will differ

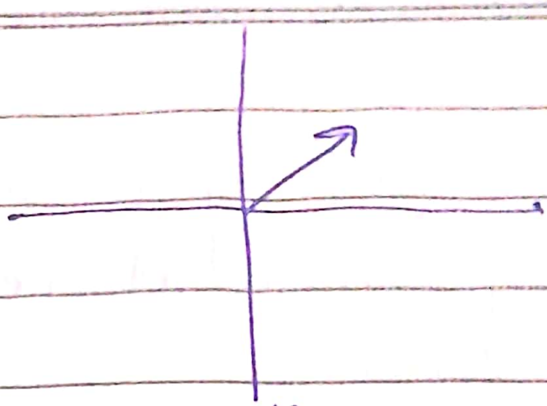
Linear Combination of
vectors :- $\vec{v} + \vec{w}$
$= a\vec{v} + b\vec{w}$
Scalars

~~Span :- set of all post possible linear combinations of vectors~~

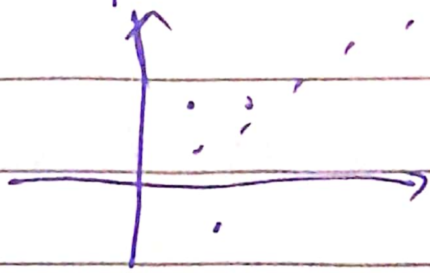
Span :- set of all possible vectors we can reach with a linear combination of given pair of vectors is called span of those two vectors.
Span — 2d space
— line if both lie on same line
— zero if both are zero

Vectors Vs points

in conceptual, think if only vectors as arrow



if there are mutiple Vectors think of them as points

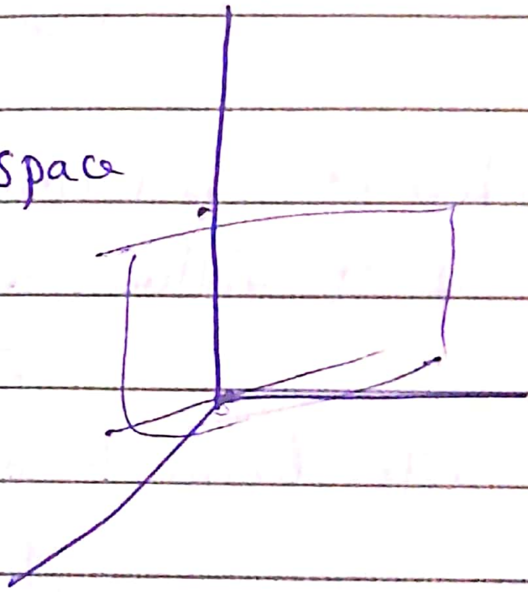


Span of 2vectors in 3d-space is a plane cutting through origin.

Span of 3 vectors in 3dspace is

Linear combination of 3.

$a\vec{v} + b\vec{w} + c$



Linearly dependent:- if one of the vector can be represented by a linear combination of other, those two vectors are linearly dependent (or)

One vector lies in the span of the other

Linearly Independent :- Is vice versa.

Basis:-

The basis of a vector space is a set of linearly independent vectors that span the full space

Matrices as Linear transformations:

Linear transformation:
function.

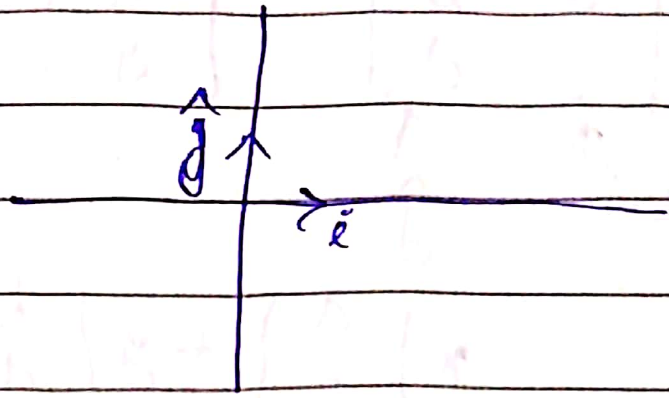
Transformation is linear transformation,
if it follows:- * all lines remain lines
* origin remains fixed

after linear transformation
grid lines are still parallel
and evenly spaced

how do you describe these transformations numerically?

$$\begin{bmatrix} x_{in} \\ y_{in} \end{bmatrix} \rightarrow ??? \rightarrow \begin{bmatrix} x_{out} \\ y_{out} \end{bmatrix}$$

$\vec{v} = -1\hat{i} + 2\hat{j}$



Transformed $\vec{v} = -1$ (Transformed $\hat{i}$) $+ 2$ (Transformed $\hat{j}$)

$$= -1\begin{bmatrix} 1 \\ -2 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\hat{i} \rightarrow \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad \hat{j} \rightarrow \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow x\begin{bmatrix} 1 \\ -2 \end{bmatrix} + y\begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1x + 3y \\ -2x + 0y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

where $\hat{i}$ lands   where $\hat{j}$ lands   Any vector

$$-1\begin{bmatrix} 1 \\ -2 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{array}{c} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ \hat{i} \qquad \hat{j} \end{array} \qquad \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

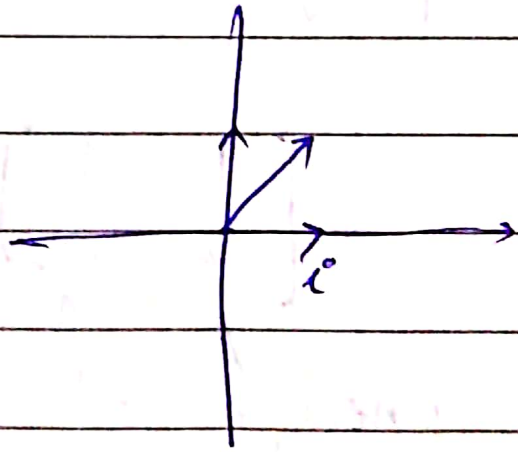
can be represented as

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}}_{\text{matrix multiplication}} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

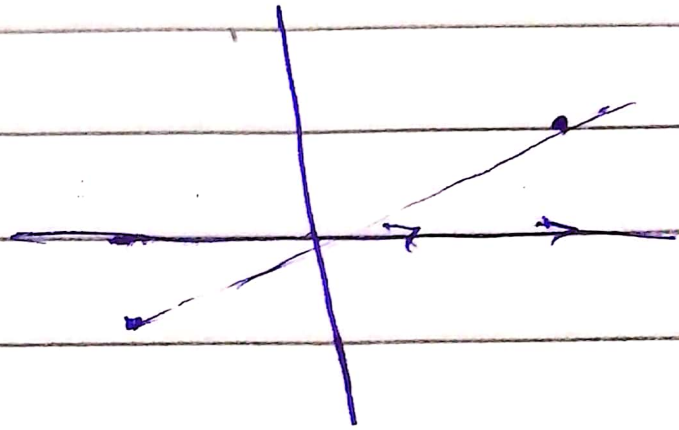
Shear

$$\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$



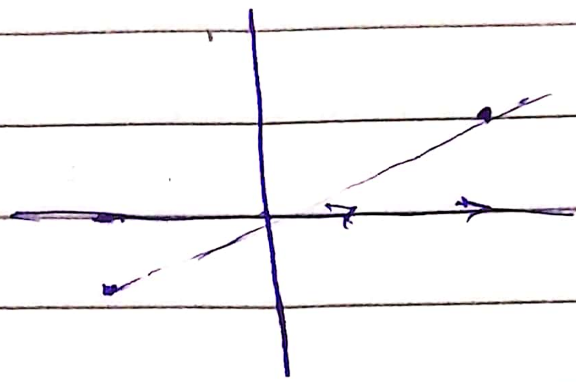
$$\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$



Linearly dependent columns

$$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$



Linearly dependent columns