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I Basic Probability.II. Conditional Probability. III.Bayes Theorem

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Basic probability

1. Two dies are rolled at once. Find out the probability for sum of numbers being even and one of the die shows 6.

Analysis: It is single event, the total no of possible combinations are : 36 (as there are two dice) Total no of favourable outcomes = (sum is even and one of the die should show 6) possible combinations are : (6,2) (6,4), (6,6),(2,6),(4,6)

Answer: probability = (No of favourable outcomes / Total no of possible outcomes) = (5/36)

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2. Two dies are rolled at once. Find out the probability for sum of numbers being less than 7.

Analysis: No of favourable outcomes are : (1,5)(1,4),(1,3),(1,2),(1,1) ,(2,4),(2,3),(2,2),(2,1),(3,3),(3,2),(3,1), (4,2),(4,1),(5,1)

Answer: probability = (No of favourable outcomes / Total no of possible outcomes) = (15/36)

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3. You toss a fair coin three times :Given that you have observed atleast one heads , what is the probability that you observe atleast two heads?

Analysis: Total possible outcomes = 7, as it was observed that , ateast one head No of favouraable outcomes = 4,(which had atleast two heads)(THH)(HTH)(HHT)(HHH)

Answer: p(atleastTwoHeads) = 4/7

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4. A and B are a married couple with two kids. One of them is a girl. What is the probability that their other kid is also a girl?

Analysis: The second kid probability is not dependent on first kid, so it is normal probability

Answer: p(girl) = 1/2

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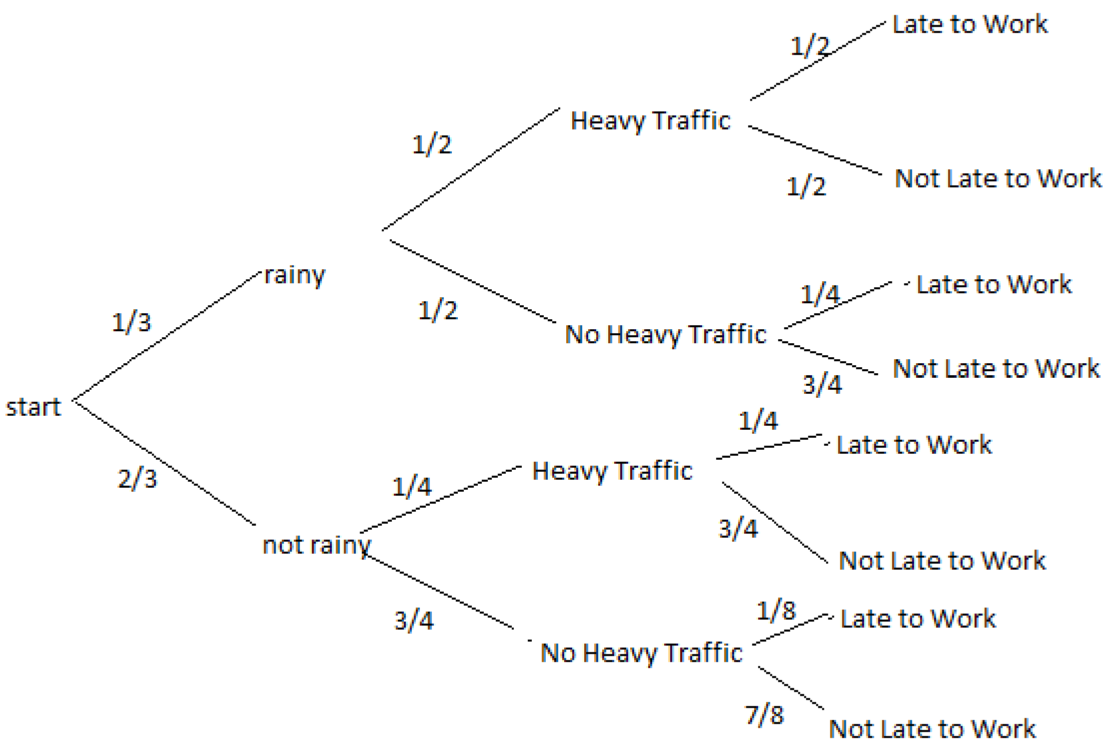
Conditional, Joint and Marginal Probability

5. In my town, it's rainy for one third of the days. Given that it is rainy, there will be heavy traffic with

probability 1/2, and given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, I arrive late for work with probability 1/2. On the other hand, the probability of being late is 1/8 if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25, 0.25. You pick a random day.



a) What is the probability that it's not raining and there is heavy traffic and I am not late?

Analysis:

p(NotRaining ^ HeavyTraffic ^ Not Late) = p(NotLate|(HeavyTraffic & Not rainy)) X p(HeavyTraffic|not rainy) X p(not rainy)

Answer: p(NotRaining ^ HeavyTraffic ^ Not Late) = (3/4) X (1/4) X (2/3) = (1/8)

b) What is the probability that I am late?

Analysis:

p(Late) = p(Late| (HT & Rainy)) + p(Late| (NHT & Rainy) + p(Late| (HT & Not Rainy)) + p(NHT & Not Rainy)

Answer: p(Late) = (1/2 X 1/2 X 1/3) + (1/4 X 1/2 X 1/3) + (1/4 X 1/4 X 2/3) + (1/8 X 3/4 X 2/3) = 11/48

c) Given that I arrived late at work, what is the probability that it rained that day?

Analysis: p(rainy|Late) =p(Late|rainy) X p(Late) / p(rainy)

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             = [p(Late|(HT&rainy)) + p(Late|(NHT & rainy))] X p(Late) / p(rainy)
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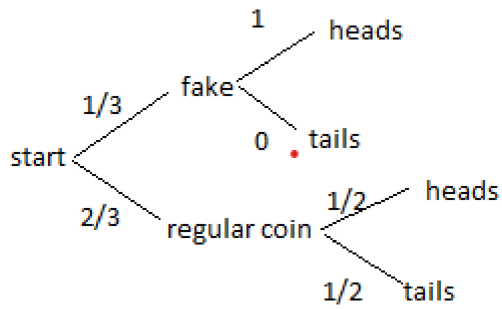
Answer: p(rainy|Late) = [(1/2 X 1/2) + (1/4 X 1/2)] X 11/48 /(1/3) = 33/128

6. A box contains three coins: two regular coins and one fake two-headed coin (P(Heads)=1), You pick a coin at random and toss it.



6a) What is the probability that it lands heads up?

Analysis: and Answer:

p(Heads) = p(H|fake)X p(fake) +p(H|regular) X p(regular) = (1 X 1/3) + (1/2 X 2/3) = 2/3

6b) You pick a coin at random and toss it and get heads. What is the probability that it is the two-headed coin?

Analysis: and Answer:

p(Heads) = 2/3 (from previous question)

p(fake|Heads) = p(Heads|fake) X p(fake) / p(Heads) = (1 X 1/3) / (2/3)= 1/2

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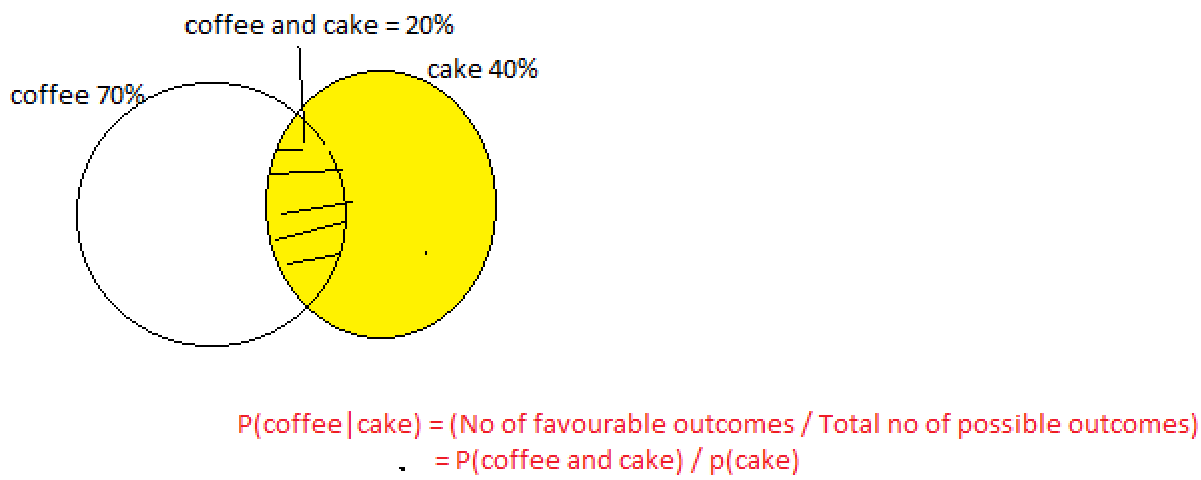
7. Suppose that, of all the customers at a coffee shop

- 70% purchase a cup of coffee
- 40% purchase a piece of cake
- 20% purchase both a cup of coffee and a piece of cake.

Given that a randomly chosen customer has purchased a piece of cake, what is the probability that he/she also purchased a cup of coffee.



Analysis: and Answer:

p(coffee) = 0.7 , p(cake) = 0.4, p(coffe and cake) = 0.2

p(coffee|cake) = p(coffe and cake) / p(cake) = 0.2 / 0.4 = 0.5

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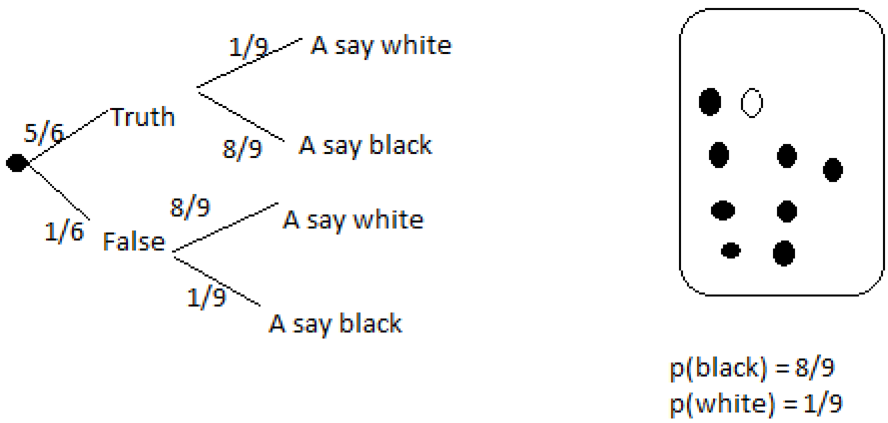
8. A is known to tell the truth in 5 cases out of 6 and he states that a white ball was drawn from a bag containing 8 blacks and 1 white ball. Find the probability that the white ball was drawn.



Analysis: and Answer:

p(Truth|white) = p(white|Truth) X p(Truth) / p(white)

p(white) = p(white|Truth) X p(Truth) + p(white|False) X p(False)

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        = (1/9 X 5/6) + (8/9 X 1/6) = 13 /54
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p(Truth|white) = (1/9 X 5/6) / (13/54) = 5/13

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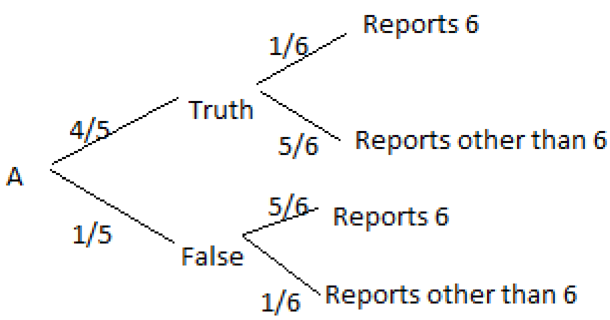
9 A speaks the truth 4 out of 5 times. A die is tossed. A report that it is a 6. What are the chances that there actually was a 6?



p(truth)=4/5, p(false)=1/5,

p(truth|6) = p(6|truth) X p(truth) / p(6)

p(6) = p(6|truth) X p(truth) + p(6|false) X p(false)
= 1/6 X 4/5 + 5/6 X 1/5
= 9/30

p(truth |6) = (1/6) X 4/5 / (9/30)
= 4/9

Analysis: and Answer:

p(truth)= 4/5, p(false)=1/5,

p(truth|6) = p(6|truth) X p(truth) / p(6) = 4/9

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10. In a class, 40% of the student’s study math and science. 60% of the student’s study math. What is the probability of a student studying science given he/she is already studying math?

Analysis: and Answer:

p(Maths and Science) = 0.4, p(Maths) = 0.6

P(science|Maths) = p(Maths and Science) / p(Maths)

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              = 0.4/0.6 = 2/3
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11. Below is a table of graduates and post graduates

	Graduate	Post Graduate	Total
Male	19	41	60
Female	12	28	40
Total	31	69	100

a)What is the probability that a randomly selected individual is a male and a graduate? What kind of probability is it (Marginal / Joint / Conditional)

p(male and graduate) = 19/100 = 0.19

It is Joint Probability

b)What is the probability that a randomly selected individual is a male?

p(male) = 60/100 = 0.6

c) What is the probability of a randomly selected individual being a graduate ? What kind of probability is this?

p(graduate) = 31/100 = 0.31

It is marginal probability

d) What is the probability that a randomly selected person is a female given that the selected person is a post graduate ? What kind of probability is this?

p(female|PG) = 28/69 (out of 69 PG, 28 are female)

It is conditional probability

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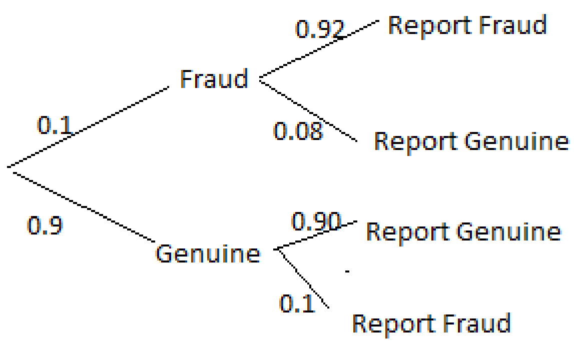
Bayes Theorem

12. You need to figure out whether a company is fraud based on the legal charges they filed. We have the knowledge that, the chances a company submitting fraudulent fillings is 0.1. There exists an algorithm that can predict fraud. This algorithm returns a correct positive result in 92% of the cases in which the fraud is present and correct negative results in 90% of the cases where the fraud is not present. Suppose we observe a company for whom the algorithm test returns a fraud result. Calculate the posterior probability that this company truly did fraud in their filings.

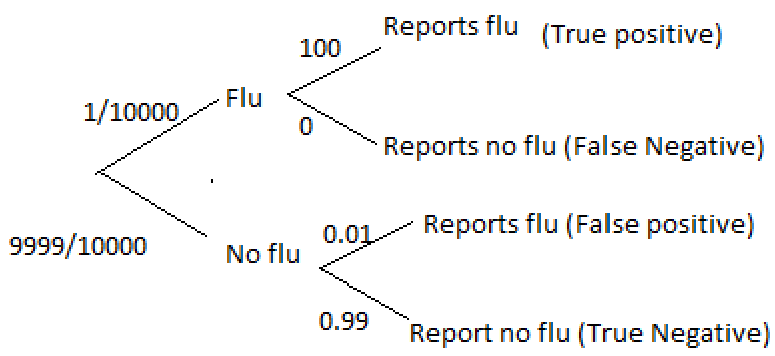


p(Fraud) = 0.1,
p(Genuine) = 0.9

p(Fraud | ReportFraud) = p(ReportFraud | Fraud) * p(Fraud) / (P(Report Fraud)
= 0.92*0.1/(0.92*0.1+ 0.1 * 0.9)

p(Fraud|ReportFraud) - 0.092/(0.182)=0.5054~= 0.51

14. You go to see the doctor about an ingrowing toe nail. The doctor selects you at random to have a blood test for swine flu, which for the purposes of this exercise we will say is currently suspected to affect 1 in 10,000 people in Australia. The test is 99% accurate, in the sense that the probability of a false positive is 1%. The probability of a false negative is zero. What is the new probability that you have swine flu?



p(flu) = p(ReportFlu|Flu) X p(Flu)+p(reportsFlu|No flu)X p(No flu) = 1 * 0.0001 + 0.01 * 0.9999 = 0.01

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