Modelling a Hypothetical CATV Communication Scheme

Siva Sundar, EE23B151

December 6, 2024

Problem Statement

The goal is to model a hypothetical CATV (Cable Television) communication scheme. The system begins with a **Broadcast Channel Tower (A)**, which transmits channel data via an uplink to a **Satellite (B)**. The satellite relays this data to a **Multiple-System Operator (MSO, C)**, which processes and distributes the channel content to its **Subscribers (D)**—those households that have subscribed to the respective channel—through the cable network.

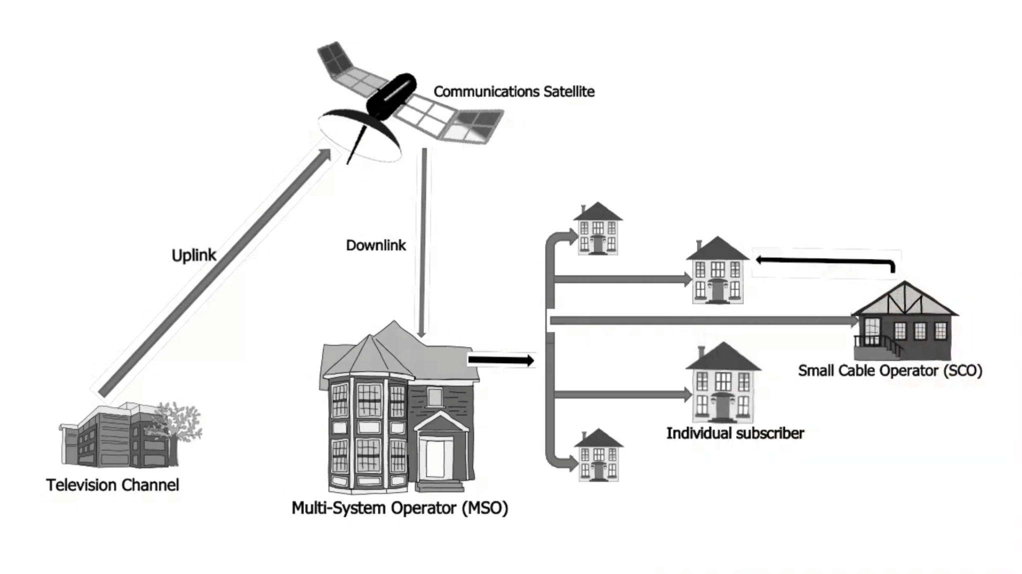

Figure 1: Network diagram of Cable TV Network

Communication Between Components

The communication between each component in the CATV system occurs as follows:

1. **Transmission from Broadcast Tower to Satellite:** The Broadcast Channel Tower (A) transmits the channel data to the Satellite (B) via an uplink process. For this, half-wave dipole antennas are chosen to operate at the **uplink frequency**, f_t, and medium between A and B is **vacuum** (with impedence η_0).
2. **Data Relay from Satellite to MSO:** The Satellite (B) receives the uplinked data and relays it to the Multiple-System Operator (MSO, C) via a downlink process. This communication uses half-wave dipole antennas operating at the **downlink frequency**, f_r, and medium between B and C is **vacuum**.
3. **MSO Data Processing and Distribution:** Once the MSO (C) receives the data, it processes it and then distributes the channel content to the relevant subscribers

(D) through the cable network. The transmission is carried out using coaxial cables, where Z_0 represents the **characteristic impedance** of the transmission line and γ represents its **propagation constant**.

4. **Subscriber Access:** Subscribers (D) who have subscribed to specific channels receive the content at their homes. A key element in this communication is **impedance matching** between the set-top box (the receiver, with Z_L impedence) in the subscriber's home and the transmission line from the MSO (the transmitter). This ensures minimal signal reflection and optimal power transfer, which is essential for maintaining high-quality content delivery.

1 Transmission from Broadcast Tower to Satellite

In this section, we will perform the calculations for the signal generation by the transmitting antenna at A and the signal reception by the receiving antenna at B, both of which are modeled as half-wave dipole antennas. Let us assume the satellite uses the open circuit voltage V_{oc} across its recieving antenna for signal processing.

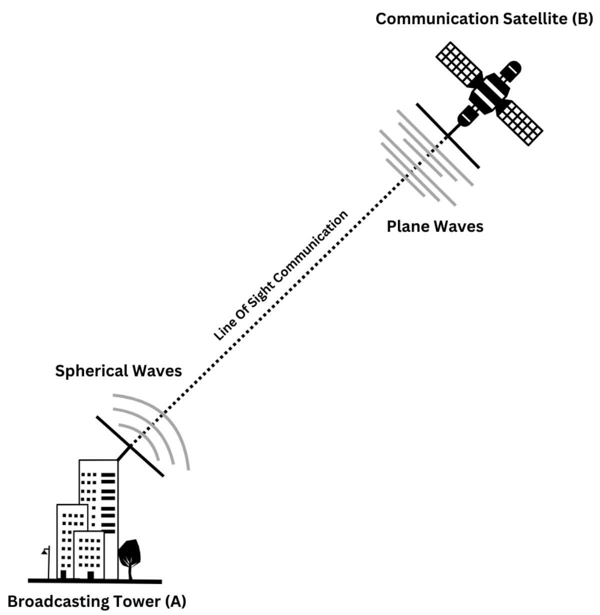

Figure 2: Uplink Process, data is transmitted from ground to satellite

To begin, we derive the necessary equations. Starting from Maxwell's equations, we apply the Lorenz gauge condition to simplify the analysis. This leads to the **Helmholtz equation**, which governs the propagation of electromagnetic waves in free space.

$$\left.\begin{aligned}\nabla\cdot\mathbf{E} &= \frac{\rho}{\epsilon_0}\\ \nabla\cdot\mathbf{B}&=0\\ \nabla\times\mathbf{E}&=-\frac{\partial\mathbf{B}}{\partial t}\\ \nabla\times\mathbf{H}&=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\end{aligned}\right\} \xrightarrow[\underbrace{\frac{\partial \mathbf{V}}{\partial t}+\nabla\cdot\mathbf{A}=0}_{\text{Lorenz Gauge}}]{\mathbf{B}=\nabla\times\mathbf{A}\quad \mathbf{E}=-\nabla V-\frac{\partial\mathbf{A}}{\partial t}} \underbrace{\Box V=-\frac{\rho}{\epsilon_0}\quad \Box\mathbf{A}=-\mu_0\mathbf{J}}_{\Box=\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)}$$

Using the Helmholtz equation, we can derive the vector potential for a **Hertz dipole**—an infinitesimally small antenna of length dl, excited by a sinusoidal source—by employing the *Green's function method.*

$$\mathbf{A} = \mu_0\mathbf{J} * G \approx \mu_0 I dl\frac{e^{-jk_t r}}{4\pi r}\hat{z} \qquad \text{(Phasor form, } k_t = \frac{2\pi}{\lambda_t}\text{)}$$

Our primary focus is on the vector potential associated with the half-wave dipole antenna, which can be conceptualized as an *integral summation* of infinitesimally small Hertz dipole antennas. The current distribution across the antenna length ($2h = \lambda/2$) exhibits a sinusoidal variation.

As there is no current at the ends, we use the following distribution:

$$I(z) = \begin{cases} I_m \sin(k_t(h - |z|)), & \text{for } |z| \leqslant h, \\ 0, & \text{otherwise.} \end{cases}$$

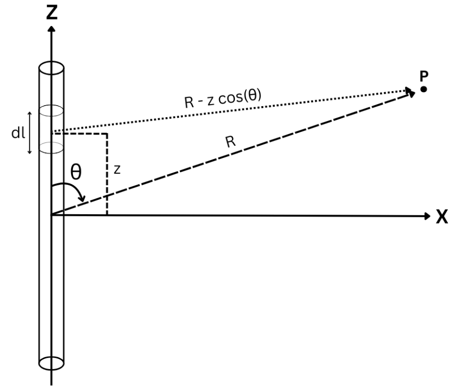

Figure 3: Half-wave dipole antenna as a collection of elementary Hertz dipole antennas

Hence, the Vector Potential of a Half-wave dipole antenna is given by:

$$\begin{aligned}
\mathbf{A} &= \mu_0 I_m \frac{e^{-jk_tR}}{4\pi R}\hat{z}\int_{-h}^{+h} sin(k_t(h-|z|))e^{jk_tzcos\theta}dz \\
&= \mu_0 I_m \frac{e^{-jk_tR}}{4\pi R}\hat{z}\left(\int_{0}^{+h} sin(k_t(h-z))e^{jk_tzcos\theta}dz + \int_{-h}^{0} sin(k_t(h+z))e^{jk_tzcos\theta}dz\right) \\
&= \mu_0 I_m \frac{e^{-jk_tR}}{4\pi R}\hat{z}\left(\int_{0}^{+h} 2\,sin(k_t(h-z))\left(\frac{e^{jk_tzcos\theta}+e^{-jk_tzcos\theta}}{2}\right)dz\right) \\
&= \mu_0 I_m \frac{e^{-jk_tR}}{4\pi R}\hat{z}\left(\int_{0}^{+h} 2\,sin(k_t(h-z))\,cos(k_tz\,cos\theta)dz\right)
\end{aligned}$$

$$\Rightarrow \boxed{\mathbf{A} = \mu_0 I_m \frac{e^{-jk_tR}}{4\pi k_t R} \times \underbrace{\frac{2(cos(\frac{\pi}{2}cos\theta))}{sin^2\theta}}_{\text{space factor, } \alpha}\hat{z}} \qquad (\alpha = 2, \text{ for } \underbrace{\textbf{LOS}}_{\theta=\pi/2} \text{ communication})$$

This vector potential, which is **approximately constant** throughout the recieving antenna, serves as a foundational parameter for characterizing the signal transmitted by the antenna located at point A. Utilizing the findings from $\mathbf{A}$, the potential at the receiver end, denoted as point B, can be determined through the application of the *Lorenz gauge condition*[1].

$$j\omega_t V = -\nabla\cdot\mathbf{A} = \mu_0 I_m \alpha\frac{e^{-jk_tR}}{4\pi k_t R}\times\frac{z}{R}\times[1+jk_t]$$

$$\therefore V(z) = \mu_0 I_m \alpha \frac{e^{-jk_tR}}{4\pi\omega_t k_t R^2} z(k_t - j) \qquad \text{(Phasor form)}$$

The voltage drop across the reciever end is given by:

$$\begin{aligned}
V_{oc} &= -\int_C \mathbf{E}\cdot\vec{dl} = \int_C \left(\nabla V + j\omega_t\mathbf{A}\right)\cdot\vec{dl} \\
&= \Delta V + \int_{z=-h}^{z=h} j\omega_t A_z dz \qquad \text{(Fundamental Theorem of Calculus)}
\end{aligned}$$

[1] In [1], the author used this trick to find the potential across the terminals V_{oc} of a short, center-fed linear dipole antenna.

$$= V(h) - V(-h) + j\omega_t A_z\,(2h)$$

$$V_{oc} = \underbrace{\mu_0 I_m \alpha \frac{e^{-jk_tR}}{4\pi\omega_t k_t R^2}\, 2h(k_t - j)}_{\propto \frac{h}{\omega_t},\ \frac{h}{\omega_t^2}} + \underbrace{j\mu_0 c I_m \alpha \frac{e^{-jk_tR}}{4\pi R} 2h}_{\propto h}$$

To get the real valued voltage we do the following and using $2h = \frac{\lambda}{2}$:

$$V_{oc}(t) = Re(V_{oc}e^{j\omega t})$$

$$\boxed{V_{oc}(t) = \frac{\mu_0 I_m \alpha \lambda_t}{8\pi\omega_t k R^2}(k_t cos(\omega_t t - k_t R) + sin(\omega_t t - k_t R)) + \frac{\mu_0 c I_m \lambda_t \alpha}{8\pi R} sin(k_t R - \omega_t t)}$$

Hence, we got the voltage recieved at time t by the satellite (B) which is caused due to the EM waves transmitted from the antenna of the Channel tower (A).

2 Data Relay from Satellite to MSO

Data from the satellite is transmitted to the MSO building (C) via a half-wave dipole antenna operating at a frequency f_r, with reception at the MSO building also performed by a half-wave dipole antenna. The distance between them is R'. Here, we perform

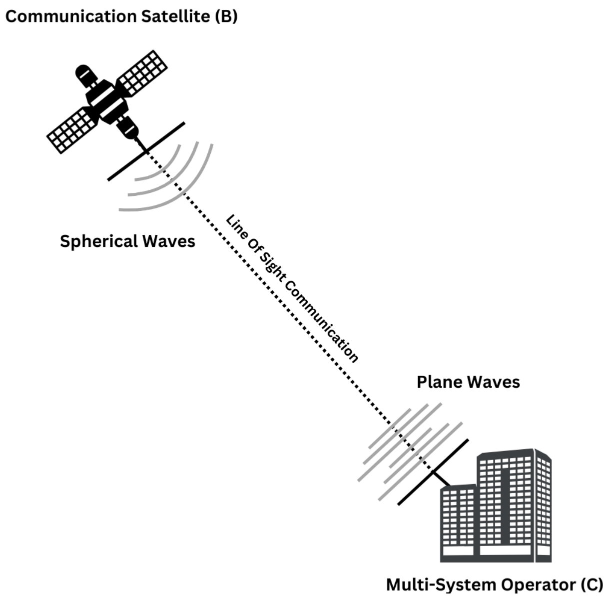

Figure 4: Down-link Process, data is transmitted from satellite to ground

similar calculations as before but assume the MSO processes signals via current through a signal analyzer with variable impedance Z_L. To find the current in the receiver circuit, we determine the antenna impedance. For maximum power transfer ($Z_L = Z^*_{\text{antenna}}$), the current is:

$$I = \frac{V}{Z_L + Z_{\text{antenna}}} = \frac{V}{2R_{\text{rad}}},$$

where only the radiation resistance R_{rad} is needed, which is easier to calculate than the antenna reactance [2].

We know the vector potential $\mathbf{A}$ at a distance r from the center of the antenna:

$$\mathbf{A} = \frac{\mu_0 I_m e^{-jk_r r}}{4\pi k_r r}\alpha(\theta)\left(\cos\theta\hat{r} - \sin\theta\hat{\theta}\right) \quad (\hat{z} = \cos\theta\hat{r} - \sin\theta\hat{\theta})$$

Hence, the magnetic field $\mathbf{H}$ is given by:

$$\begin{aligned}
\mathbf{H} &= \frac{1}{\mu_0}\nabla \times \mathbf{A} \\
&= \frac{I_m}{4\pi k_r}\nabla \times \left(\frac{e^{-jk_r r}}{r}\alpha(\theta)(\cos\theta\hat{r} - \sin\theta\hat{\theta})\right) \\
&= \frac{I_m}{4\pi k_r} \times \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \partial_r & \partial_\theta & \partial_\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix} \\
&= \frac{I_m}{4\pi k_r} \times \frac{1}{r}\hat{\phi}\left[\sin\theta\alpha(\theta)jk_r e^{-jk_r r} - \underbrace{\frac{e^{-jk_r r}}{r}\partial_\theta\left(\cos\theta\alpha(\theta)\right)}_{\propto\frac{1}{r}}\right]
\end{aligned}$$

Hence, the **Radiation** fields ($\propto \frac{1}{r}$) are given by:

$$\mathbf{H} = \frac{jI_m e^{-jk_r r}}{4\pi r}\frac{2\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\hat{\phi} \qquad \& \qquad \mathbf{E} = \eta_0\,\mathbf{H}\times\hat{r}$$

Radiated Power density due to these fields is given by the real part of poynting vector $\mathbf{S}$:

$$P_{rad} = \frac{1}{2}Re(\mathbf{E}\times\mathbf{H}^*) = \frac{\eta_0}{2}\left|\mathbf{H}\right|^2\hat{r}$$

$$\Rightarrow P_{rad} = \frac{\eta_0 I_m^2}{8\pi^2 r^2}\frac{cos^2(\frac{\pi}{2}cos\theta)}{sin^2\theta}\hat{r}$$

Hence, the total power radiated over a sphere of radius ‘r’ is given by the surface integral:

$$\begin{aligned}
W_{rad} &= \int_S P_{rad}\cdot\hat{r}\,dA \\
&= \frac{\eta_0 I_m^2}{8\pi^2 r^2}\int_0^{2\phi}\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}r^2\sin\theta\,d\theta\,d\phi \\
&= \frac{\eta_0 I_m^2}{4\pi}\underbrace{\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}d\theta}_{\beta\approx 1.2188}
\end{aligned}$$

$$\therefore W_{rad} = \frac{\eta_0 I_m^2\beta}{4\pi} = \frac{I_m^2}{2}R_{rad} \quad \text{(See fig.5)}$$

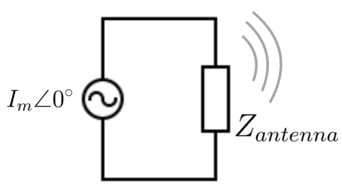

Figure 5: Lumped circuit analysis
$W_{avg} = \frac{I_m^2}{2}R_{rad}$

Hence, Radiation resistance of the half-wave dipole antenna is given by:

$$R_{rad} = \frac{\eta_0\beta}{2\pi} \approx 73\Omega \qquad \text{(Same result as in [3])}$$

From the previous section, we found open circuit voltage V_{oc} at reciever end:

$$V_{oc}(t) = \frac{\mu_0 I\alpha\lambda}{8\pi\omega kr^2}(kcos(\omega t - kr) + sin(\omega t - kr)) + \frac{\mu_0 cI_m\lambda\alpha}{8\pi r}sin(kr - \omega t)$$

Now, current in the reciever circuit for maximum power transfer will be:

$$I(t) = \frac{V_{oc}(t)}{2R_{rad}}$$

$$\boxed{I(t) = \frac{\mu_0 I'_m \alpha \lambda_r}{16\pi\omega_r R_{rad} k_r R'^2}(k_r cos(\omega_r t - k_r R') + sin(\omega_r t - k_r R')) + \frac{\mu_0 c I_m \lambda_r \alpha}{16\pi R_{rad} R'} sin(k_r R' - \omega_r t)}$$

Used I'_m to show the current used here is different from that of the previous section. Using appropriate signal processing techniques, MSO (C) decodes the data sent by satellite (B) which is then sent to the channel's subscribers (D).

3 MSO to Subscriber Communication

We consider the MSO to generate a sinusoidal voltage signal and transmit it via a transmission line of propagation constant γ and characteristic impedence Z_0, to the subscriber's set top box (load, with impedence Z_L). We assume ideal generator at MSO.

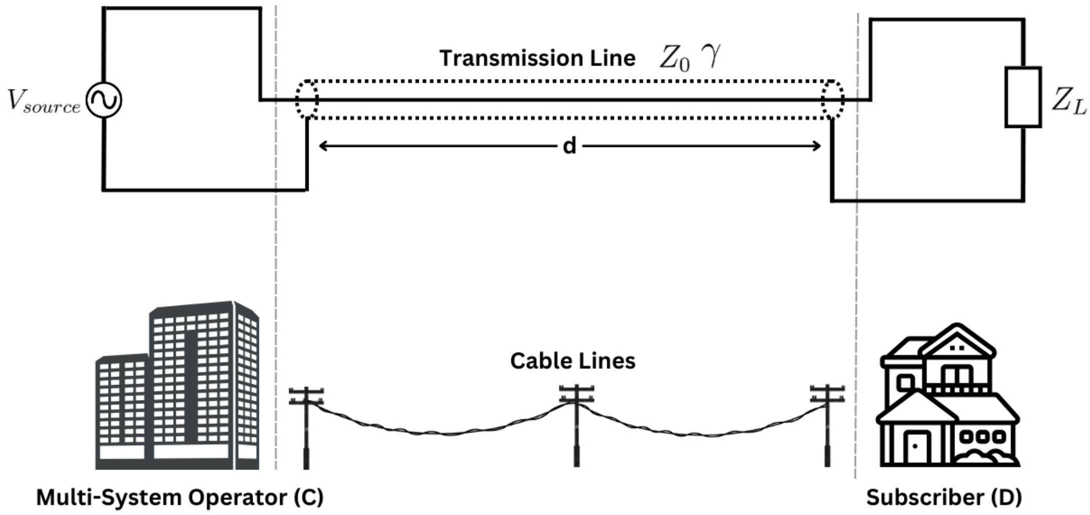

Figure 6: MSO to Subscriber Co-axial cable connection model

To begin, we need to understand **transmission line theory**. As the length of a wire increases, the finite velocity of signal propagation causes a potential difference to arise between two points along the wire. This phenomenon is known as the *transmission line effect*. It can be modeled using an infinitesimal segment of the wire, dx, which consists of a combination of **RLC components**. A general representation of this circuit is shown below:

Here, R, G, C, and L are the parameters defined per unit length of the transmission line, and x represents the distance measured from the source end. Using Kirchhoff's Voltage Law (KVL), we obtain:

$$\begin{aligned} dV &= (I - dI)[R + j\omega L]dx && \text{(Between } V \text{ and } V + dV\text{)} \\ dI &= (V)[G + j\omega C]dx && \text{(Between } V \text{ and } \texttt{GND}\text{)} \end{aligned}$$

The two *coupled* differential equations can be solved by differentiating one and substituting in the other, which will yield:

$$\frac{d^2}{dx^2}\{V, I\} = \underbrace{(R + j\omega L)(G + j\omega C)}_{= \gamma^2}\{V, I\} \qquad \text{(Telegrapher's Equation)}$$

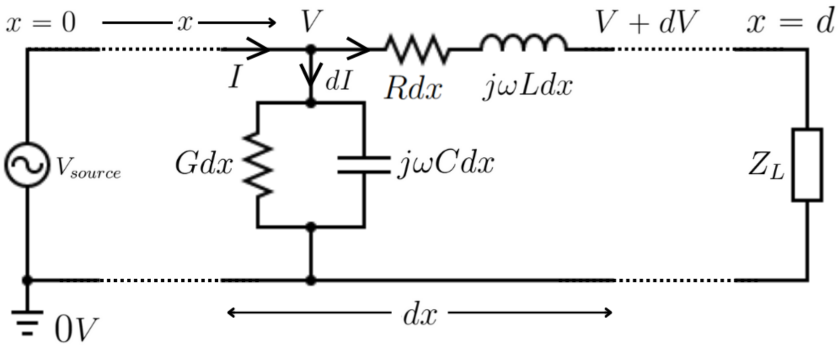

Figure 7: Transmission line effect modelled using a Lumped Circuit

where, $\gamma = \alpha + j\beta$ is the **propagation constant**, α is the **attenuation constant**, β is the **phase constant**. Using $V \propto e^{\pm\gamma x}$, we get:

$$V(x) = \underbrace{U_+ e^{-\gamma x}}_{\text{Source to Load}} + \underbrace{U_- e^{\gamma x}}_{\text{Load to Source}}$$

But, we commonly use these equations with **load end as origin** ($l = d - x$), hence:

$$\begin{aligned} V(l) &= \underbrace{U_+ e^{-\gamma x}}_{V_+} e^{\gamma l} + \underbrace{U_- e^{\gamma x}}_{V_-} e^{-\gamma l} \\ &= \underbrace{V_+ e^{\gamma l}}_{\text{Source to Load}} + \underbrace{V_- e^{-\gamma l}}_{\text{Load to Source}} \end{aligned}$$

Let us define the **reflection coefficient** at a given l as the ratio of backward to forward voltage waves:

$$\Gamma(l) = \frac{V_- e^{-\gamma l}}{V_+ e^{+\gamma l}} = \frac{V_-}{V_+} e^{-2\gamma l}$$

For current, we get the following equations:

$$\begin{aligned} I(l) &= I_+ e^{\gamma l} + I_- e^{-\gamma l} && \text{(From Telegrapher equation)} \\ &= \frac{1}{R + j\omega L}(\gamma V_+ e^{\gamma l} - \gamma V_- e^{-\gamma l}) && \text{(By using coupled DE of } V\text{)} \end{aligned}$$

Comparing them, we get the following:

$$\frac{V_+}{I_+} = \frac{-V_-}{I_-} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = Z_0 \qquad \text{(Characteristic Impedence of Transmission Line)}$$

This tempts us to take the ratio of voltage and current at a given l, which gives the impedence between the point on the transmission line at l and `GND`:

$$Z(l) = \frac{V(l)}{I(l)} = \frac{V_+ e^{\gamma l} + V_- e^{-\gamma l}}{\frac{1}{Z_0}(V_+ e^{\gamma l} - V_- e^{-\gamma l})} = Z_0 \frac{1 + \frac{V_- e^{-\gamma l}}{V_+ e^{+\gamma l}}}{1 - \frac{V_- e^{-\gamma l}}{V_+ e^{+\gamma l}}}$$

$$Z(l) = Z_0 \frac{1 + \Gamma(l)}{1 - \Gamma(l)} \implies \left[\Gamma(l) = \frac{Z(l) - Z_0}{Z(l) + Z_0} \right]$$

At $l = 0$, the impedence $Z(l)$ will be equal to the **load impedence** Z_L. Also, $\Gamma(0) = \frac{V_-}{V_+}$, which allows me to write:

$$\Gamma(0) = \frac{V_-}{V_+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Hence, we can relate $Z(l)$ to load impedence as:

$$Z(l) = Z_0 \frac{1 + \frac{V_- e^{-\gamma l}}{V_+ e^{+\gamma l}}}{1 - \frac{V_- e^{-\gamma l}}{V_+ e^{+\gamma l}}} = Z_0 \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma l}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma l}}$$

$$= Z_0 \frac{Z_L + Z_0 + (Z_L - Z_0)e^{-2\gamma l}}{Z_L + Z_0 - (Z_L - Z_0)e^{-2\gamma l}} = Z_0 \frac{Z_L(1 + e^{-2\gamma l}) + Z_0(1 - e^{-2\gamma l})}{Z_0(1 + e^{-2\gamma l}) + Z_L(1 - e^{-2\gamma l})}$$

Multiplying top and bottom by $\frac{e^{\gamma l}}{2}$, we get:

$$\boxed{Z(l) = Z_0 \frac{Z_L\, cosh(\gamma l) + Z_0\, sinh(\gamma l)}{Z_0\, cosh(\gamma l) + Z_L\, sinh(\gamma l)}} \qquad \text{(Impedence Transformation Equation)}$$

Now, let's understand how power depends on length l,

$$P = \frac{1}{2} Re(V \cdot I^*)$$
$$= \frac{1}{2} Re\left(V_+ e^{\gamma l}(1 + \Gamma(l)) \cdot \frac{V_+^*}{Z_0^*} e^{\gamma^* l}(1 - \Gamma(l)) \right)$$
$$\Rightarrow \boxed{P_{avg} = \frac{|V_+|^2 e^{2\alpha l}}{2Re(Z_0)} (1 - |\Gamma(0)|^2)} \qquad (\, |\Gamma(l)| = |\Gamma(0)|\,)$$

This indicates that the **power remains constant** throughout the transmission line **if** $\alpha = 0$ **(Loss-less transmission line)**. However, at the load, some power may be lost due to reflections. When there are no reflections, **maximum power** is transferred to the load.

Now, we are ready to address the question. Using the previously derived impedance transformation equation, we can represent the load at the subscriber's house and the entire transmission line as a **single lumped impedance box** (see fig.9).

$$Z_L' = Z_0 \frac{Z_L\, cosh(\gamma d) + Z_0\, sinh(\gamma d)}{Z_0\, cosh(\gamma d) + Z_L\, sinh(\gamma d)}$$

Now from the lumped circuit in fig., we find that for to maximize power transfer to load:

$$\Gamma(0) = 0 \Longrightarrow Z_L = Z_0 \qquad \text{(Reflectionless Matching)}$$

To achieve this, we need to transform the load impedance Z_L to match the characteristic impedance Z_0. This is accomplished using a lossless ($\alpha = 0$) transmission line (with impedence Z_T) of quarter-wave length. This technique is known as the **Quarter-Wave Transform (QWT)**[2]:

[2] There are other methods as well, like single-stub matching and double-stub matching, see [4]

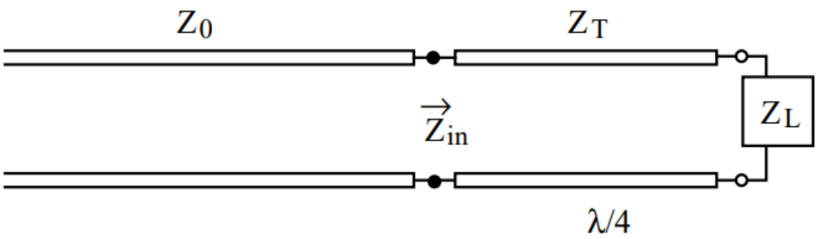

Figure 8: Quarter-Wave Transform

$$Z_{in} = Z\left(\frac{\lambda}{4}\right) \implies Z_0 = Z_T \frac{Z_L\, cosh(j\frac{\pi}{2}) + Z_T\, sinh(j\frac{\pi}{2})}{Z_T\, cosh(j\frac{\pi}{2}) + Z_L\, sinh(j\frac{\pi}{2})}$$

$$Z_0 = Z_T \times \frac{Z_T}{Z_L} \implies \left[Z_T = \sqrt{Z_0\, Z_L}\right]$$

Hence, Z'_L is given by:

$$Z'_L = Z_0 \frac{Z_{in}\, cosh(\gamma d) + Z_0\, sinh(\gamma d)}{Z_0\, cosh(\gamma d) + Z_{in}\, sinh(\gamma d)} = Z_0 \qquad (Z_{in} = Z_0)$$

Now, let us analyze the voltage across the set-top box (Z_L) and the power it receives. For this purpose, we use the two circuits shown below.

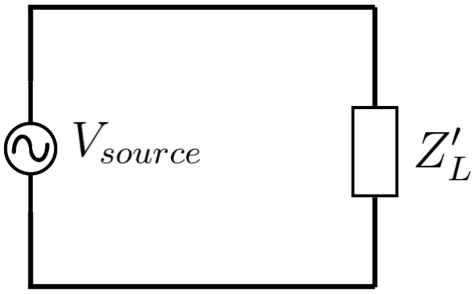

Figure 9: Lumped circuit after transformation

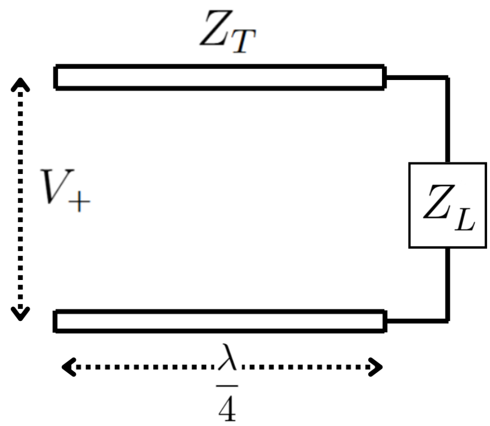

Figure 10: Load end circuit before transformation

At source end of transmission line (fig.8,9), voltage is given by:

$$V(d) = V_{source} = V_+ e^{\gamma d}$$

Hence, voltage at the junction of the two transmission lines ($l = 0$):

$$V(l = 0) = V_+ = V_{source} e^{-\gamma d}$$

This voltage is the source-end voltage for the quarter-wave transformer line:

$$\text{ie, } V_T\left(\frac{\lambda}{4}\right) = V_{T+} e^{j\beta\frac{\lambda}{4}} \left(1 + \Gamma_T\left(\frac{\lambda}{4}\right)\right)$$

$$V_{source} e^{-\gamma d} = 2j V_{T+} \frac{\sqrt{Z_0}}{\sqrt{Z_L} + \sqrt{Z_0}} \qquad \left(\Gamma_T\left(\frac{\lambda}{4}\right) = \underbrace{\frac{Z_L - Z_T}{Z_L + Z_T}}_{=\Gamma(0)} e^{-2j\beta\frac{\lambda}{4}}\right).$$

Hence, voltage across the load end of this transmission line is given by:

$$\begin{aligned} V_{load} &= V_T(0) = V_{T+}(1+\Gamma(0)) \\ &= \frac{V_{source}e^{-\gamma d}}{2j}\frac{\sqrt{Z_L}+\sqrt{Z_0}}{\sqrt{Z_0}} \times \frac{2\sqrt{Z_L}}{\sqrt{Z_L}+\sqrt{Z_0}} \end{aligned}$$

$$\therefore \boxed{V_{load} = V_{source}e^{-\gamma d - j\frac{\pi}{2}}\sqrt{\frac{Z_L}{Z_0}}}$$

Similarly, using $I = \frac{V_{T+}}{Z_T}e^{j\beta l}(1-\Gamma(l))$ or $V = IZ_L$, we get the current in the load Z_L as:

$$\boxed{I_{load} = \frac{V_{source}e^{-\gamma d - j\frac{\pi}{2}}}{\sqrt{Z_0 Z_L}}}$$

For power calculation:

$$P_{avg} = \frac{1}{2}Re(V\cdot I^*) = \boxed{\frac{|V_{source}|^2 e^{-2\alpha d}}{2|Z_0 Z_L|}Re(Z_L)}$$

The final wiring diagram between MSO and Subscriber is shown below:

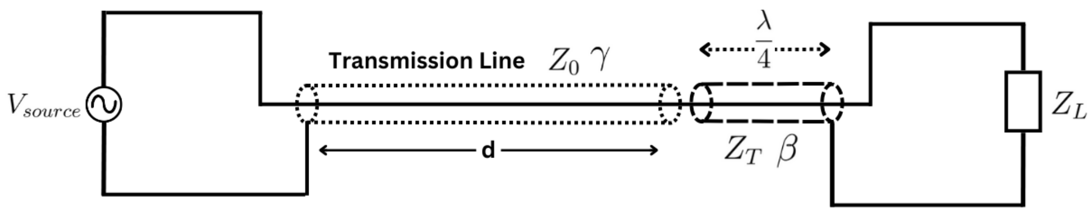

Figure 11: Only difference from fig.6 is the extra transmission line used for QWT

4 Feasibility of using Half-Wave Dipole antennas for SatCom

In this section, I will analyze the Canadian **geostationary** satellite **Anik-F1**, which operates at a distance of approximately $R \approx 36,000\,\text{km}$ from the Earth's surface. The satellite is used by **Shaw Direct G.P.**, a leading distributor of **direct broadcast satellite television** in Canada.

The **Anik-F1** satellite operates in the **C-band** and **Ku-band**, but for this analysis, we will consider the **C-band** with **Uplink frequency** $f_{up} = 6\,\text{GHz}$, **Downlink frequency** $f_{down} = 4\,\text{GHz}$. The **power consumed** by this satellite is $P = 17.5\,\text{kW}$.

Assuming the satellite employs a half-wave dipole antenna and operates under these conditions, let us compute the current amplitude I_m generated at the MSO building due to the fields produced by the satellite.

$$\begin{aligned} I_m &\approx \left(\frac{\mu_0 I'_m \alpha \lambda_{down}}{16\pi\omega_{down}R_{rad}kR'^2}\times k\right) + \left(\frac{\mu_0 c I_m \lambda_{down}\alpha}{16\pi R_{rad}R'}\right) \\ &\approx \underbrace{\frac{\mu_0\sqrt{\frac{P}{R_{rad}}}\,c\times 2}{16\pi^2 R_{rad}\,f_{down}^2 R^2}}_{\approx 10^{-35}\text{A}} + \underbrace{\frac{\mu_0 c^2\sqrt{\frac{P}{R_{rad}}}\times 2}{16\pi f_{down}R_{rad}R'}}_{26.5\,\text{nA}} = 26.5\,\text{nA} \end{aligned}$$

The first term we calculated is much smaller than the second term, so it can be ignored. Since nano-ampere currents can be detected, the half-wave dipole can be used for satellite communication (SatCom).

The **Anik-F1** satellite uses an *omni-directional* (See [5]) antenna with a radiation pattern similar to a half-wave dipole, making it a great choice for reliable communication.

5 Results

- The potential difference $V_{oc}(t)$ generated at the receiver end of the satellite due to fields emitted by a broadcasting tower (at a distance R from the satellite) operating at an uplink frequency f_t with a transmitting circuit current I_m is given by:

$$V_{oc}(t) = \frac{\mu_0 c I_m \lambda_t \alpha}{8\pi R} sin(k_t R - \omega_t t)$$

- The current $I(t)$ (for maximum power) generated in the receiver circuit at the MSO building (at a distance R' from the satellite) due to the fields produced by the satellite (operating at a downlink frequency f_r and transmitting with current I'_m) is given by:

$$I(t) = \frac{\mu_0 c I_m \lambda_r \alpha}{16\pi R_{rad} R'} sin(k_r R' - \omega_r t)$$

- Voltage across the set-top-box with impedence Z_L is given by:

$$V_{load} = V_{source} e^{-\gamma d - j\frac{\pi}{2}} \sqrt{\frac{Z_L}{Z_0}}$$

- Maximum Power delivered to the set-up-box is given by:

$$P_{avg} = \frac{|V_{source}|^2 e^{-2\alpha d}}{2|Z_0 Z_L|} Re(Z_L)$$

References

1. Kirk McDonald, *Receiver*, Princeton University, 2024. Available at: `http://kirkmcd.princeton.edu/examples/receiver.pdf`.

2. Kirk McDonald, *Transmitter*, Princeton University, 2024. Available at: `http://kirkmcd.princeton.edu/examples/transmitter.pdf`.

3. UCSD, *Half-Wave Antennas and Arrays*, Physics 100C Lecture Notes, Spring 2016. Available at: `https://courses.physics.ucsd.edu/2016/Spring/physics100c/Lectures/HalfwaveAntennasAndArrays.pdf`.

4. W.C. Chew, *Transmission Line Theory*, ECE 350, University of Illinois, 2010. Available at: `https://wcchew.ece.illinois.edu/chew/ece350/ee350-10.pdf`.

5. R.J. Tinley, E. Bertenyi, *Telesat Canada's Anik F1 Spacecraft*, Acta Astronautica, Volume 50, Issue 5, 2002, Pages 315-323. Available at: `https://www.sciencedirect.com/science/article/pii/S0094576501001722`.