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Abstract—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.

Download python codes using

svn co https://github.com/gadepall/school/trunk/control/codes

1 STABILITY

- 2 ROUTH HURWITZ CRITERION
 - 3 Compensators
 - 4 Nyquist Plot
- 4.1 Polar plot
- 4.1. Sketch the Polar Plot for

$$G(s) = \frac{1}{(1+s)(1+2s)} \tag{4.1.1}$$

Solution: Then the given open loop Transfer Function is

$$G(s) = \frac{1}{(1+s)(1+2s)} \tag{4.1.2}$$

Now we have to substitute $s=j\omega$

$$G(j\omega) = \frac{1}{(1+j\omega)(1+2j\omega)}$$
(4.1.3)

4.2. Then find the Magnitude of the Transfer Function

Solution:

$$|G(j\omega)| = \frac{1}{\sqrt{(1+(\omega^2))(1+(2\omega)^2}}$$
 (4.2.1)

4.3. Next find the Phase of Transfer Function **Solution:**

$$\angle G(1\omega) = \angle G(1\omega)_{num} - \angle G(1\omega)_{den} \qquad (4.3.1)$$

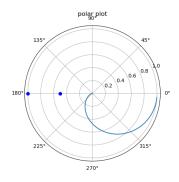


Fig. 4.4: Polar plot of given transfer function

$$\angle G(1\omega) = -\tan^{-1}(\omega) - \tan^{-1}(2\omega)$$
 (4.3.2)

4.4. Polar plot is drawn based on this magnitude and phase of transfer function

Solution:

For $\omega = 0$

$$|G(1\omega)| = 1 \tag{4.4.1}$$

$$\angle G(1\omega) = 0 \tag{4.4.2}$$

For $w = \infty$

$$|G(j\omega)| = 0 \tag{4.4.3}$$

$$\angle G(1\omega) = -\pi \tag{4.4.4}$$

Next Polar Plot is drawn by varying ω from 0 to ∞ .

4.5. Verify the Polar Plot by running the following Code

4.6. What actually is a polar plot

The polar plot of the frequency response of a system is the line traced out as the frequency is changed from 0 to infinity by the tips of the phasors whose lengths represent the magnitude, i.e. amplitude gain, of the system and which are drawn at angles corresponding to their phase

4.7. Uses of Polar plots in control systems:

1. This is half of Nyquist plot running over positive frequencies.

2.If we know type and order of a transfer function,we can easily draw its magnitude versus phase plot using polar plot.

For example, In the given transfer function

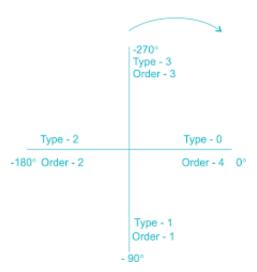


Fig. 4.7: Polar plot shape based on order and type

$$G(s) = \frac{1}{(1+s)(1+2s)}$$
 (4.7.1)

Order is highest power of the polynomial in denominator of G(s), Type of system is number of poles at origin.

Here order=2,type=0

So this polar plot lies in third and fourth quardrant. These order and type of a system determines shape of polar plot as shown in fig.4.7

4.8. Stability of transfer function

Polar plot also determines stability of a system as

In the given transfer function

$$G(s) = \frac{1}{(1+s)(1+2s)}$$
 (4.8.1)

Poles of G(s) are (-1,0),(-1/2,0) which lies on the left side of s-plane.

As these poles (-1,0),(-1/2,0) are not encountered inside the polar plot as shown in fig.4.4. The given transfer function G(s) is stable.