Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Types of Damping

1.1. Match the transfer functions of the second-order systems with the nature of the systems given below

Transfer functions	Systems
P : $\frac{15}{s^2+5s+15}$	1:Overdamped
Q : $\frac{25}{s^2+10s+25}$	2:critically damped
R : $\frac{35}{s^2+18s+35}$	3 : *Underdamped*

$$(A)P-1, Q-2, R-3 \tag{1.1.1}$$

$$(B)P-2, Q-1, R-3 \tag{1.1.2}$$

$$(C)P-3, Q-2, R-1 \tag{1.1.3}$$

$$(D)P-3, Q-1, R-2 \tag{1.1.4}$$

Solution: The standard transfer function is H(s)=$\frac{\omega^2}{s^2+2\zeta\omega+\omega^2}$
where
"ω" is natural frequency
and"ζ" is damping factor

then compare the given functions with this we get

1. For Transfer function H(s)=$\frac{15}{s^2+5s+15}$,

$$\omega^2 = 15$$
$$2\zeta\omega = 5$$
$$\text{then we get } \zeta = \sqrt{\frac{5}{12}}<1 \qquad ref$$

2. For Transfer function H(s)=$\frac{25}{s^2+10s+25}$,

$$\omega^2 = 25$$
$$2\zeta\omega = 10$$
$$\text{then we get } \zeta = \sqrt{\frac{5}{5}} = 1 \qquad ref$$

3. For Transfer function H(s)=$\frac{35}{s^2+18s+35}$,

$$\omega^2 = 35$$
$$2\zeta\omega = 18$$
$$\text{then we get } \zeta = \sqrt{\frac{81}{35}}>1 \qquad ref$$

The damping of a system can be described as being one of the following:
Overdamped : The system returns to equilibrium without oscillating.For this

$$\zeta>1. \tag{1.1.5}$$

Critically damped: The system returns to equilibrium as quickly as possible without oscillating.
For this

$$\zeta = 1 \tag{1.1.6}$$

Underdamped: The system oscillates(at reduced frequency compared to the undamped case) with the amplitude gradually decreasing to zero.
For this

$$0<\zeta<1 \tag{1.1.7}$$

Undamped : The system oscillates at its natural

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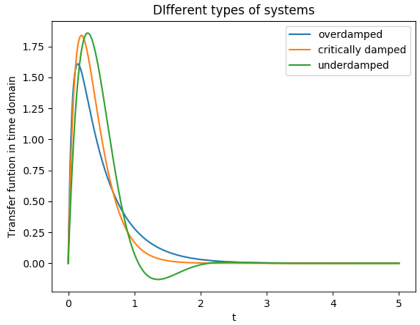

Fig. 1.1: Different systems based on ζ

resonant frequency($\omega 0$).
For this

$$\zeta = 0 \tag{1.1.8}$$

Relation between damping and ζ	
Types of Damping	$\zeta(dampingratio)$
Overdamped	$\zeta > 1$
Criticallydamped	$\zeta = 1$
Underdamped	$0 < \zeta < 1$
Undamped	$\zeta = 0$

Final Analysis

- As for P : $\zeta < 1$
  $It is Underdamped system$
- As for Q : $\zeta = 1$
  $It is critically damped system.$
- As for R : $\zeta > 1$
  $It is an overdamped system.$

So,P-3,Q-2,R-1. Option (C) is correct.

1.2. Python code for the above damping graph :

codes/ee18btech110012/damping.py