

# Control Systems

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### 1 Types of Damping 1

**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

#### 1 TYPES OF DAMPING

1.1. Match the transfer functions of the second-order systems with the nature of the systems given below

Transfer function	Systems
P: $\frac{15}{s^2+5s+15}$	1:Overdamped
Q: $\frac{25}{s^2+10s+25}$	2:Criticallydamped
R: $\frac{35}{s^2+18s+35}$	3:Underdamped

$$(A) P - 1, Q - 2, R - 3 \quad (1.1.1)$$

$$(B) P - 2, Q - 1, R - 3 \quad (1.1.2)$$

$$(C) P - 3, Q - 2, R - 1 \quad (1.1.3)$$

$$(D) P - 3, Q - 1, R - 2 \quad (1.1.4)$$

**Solution:** The standard transfer function is

$$H(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

where

" $\omega$ " is natural frequency

and " $\zeta$ " is damping factor

then compare the given functions with this we get

$$1. \text{ For Transfer function } H(s) = \frac{15}{s^2+5s+15},$$

$$\omega^2 = 15$$

$$2\zeta\omega = 5$$

$$\text{then we get } \zeta = \sqrt{\frac{5}{12}} < 1 \quad \text{ref}$$

$$2. \text{ For Transfer function } H(s) = \frac{25}{s^2+10s+25},$$

$$\omega^2 = 25$$

$$2\zeta\omega = 10$$

$$\text{then we get } \zeta = \sqrt{\frac{5}{5}} = 1 \quad \text{ref}$$

$$3. \text{ For Transfer function } H(s) = \frac{35}{s^2+18s+35},$$

$$\omega^2 = 35$$

$$2\zeta\omega = 18$$

$$\text{then we get } \zeta = \sqrt{\frac{81}{35}} > 1 \quad \text{ref}$$

The damping of a system can be described as being one of the following:

Overdamped : The system returns to equilibrium without oscillating. For this

$$\zeta > 1. \quad (1.1.5)$$

Critically damped: The system returns to equilibrium as quickly as possible without oscillating.

For this

$$\zeta = 1 \quad (1.1.6)$$

Underdamped: The system oscillates (at reduced frequency compared to the undamped case) with the amplitude gradually decreasing to zero.

For this

$$0 < \zeta < 1 \quad (1.1.7)$$

Undamped : The system oscillates at its natural

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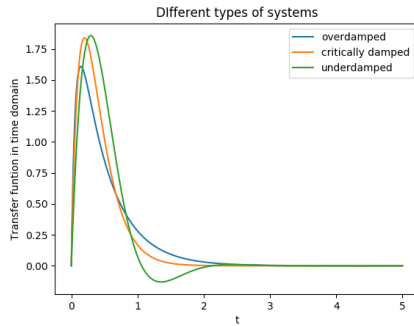


Fig. 1.3: Different systems based on  $\zeta$

resonant frequency( $\omega_0$ ).

*For this*

$$\zeta = 0 \quad (1.1.8)$$

Relation between damping and $\zeta$	
Types of Damping	$\zeta$ (damping ratio)
Overdamped	$\zeta > 1$
Critically damped	$\zeta = 1$
Underdamped	$0 < \zeta < 1$
Undamped	$\zeta = 0$

Final Analysis

- As for P :  $\zeta < 1$   
*It is Underdamped system*
  - As for Q :  $\zeta = 1$   
*It is critically damped system.*
  - As for R :  $\zeta > 1$   
*It is an overdamped system.*
- So, P-3, Q-2, R-1. Option (C) is correct.

1.2. Python code for the above damping graph :

```
codes/ee18btech110012/damping.py
```

1.3. Python code for damped output taking unit step function as input:

```
codes/ee18btech110012/unitstepdamping.py
```