Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Types of Damping

1.1. Match the transfer functions of the second-order systems with the nature of the systems given below

Transfer function	Systems
P: $\frac{15}{s^2+5s+15}$	1:Overdamped
Q: $\frac{25}{s^2+10s+25}$	2:Criticallydamped
R: $\frac{35}{s^2+18s+35}$	3:Underdamped

$$(A)P-1, Q-2, R-3 \tag{1.1.1}$$

$$(B)P-2, Q-1, R-3 \tag{1.1.2}$$

$$(C)P-3, Q-2, R-1 \tag{1.1.3}$$

$$(D)P-3, Q-1, R-2 \tag{1.1.4}$$

Solution: The standard transfer function is H(s)=$\frac{\omega^2}{s^2+2\zeta\omega+\omega^2}$
where
$"\omega"$ *is natural frequency*
and $"\zeta"$ *is damping factor*

then compare the given functions with this we get

1. For Transfer function H(s)=$\frac{15}{s^2+5s+15}$,

$$\omega^2 = 15$$
$$2\zeta\omega = 5$$
$$\text{then we get } \zeta = \sqrt{\frac{5}{12}}<1 \qquad ref$$

2. For Transfer function H(s)=$\frac{25}{s^2+10s+25}$,

$$\omega^2 = 25$$
$$2\zeta\omega = 10$$
$$\text{then we get } \zeta = \sqrt{\frac{5}{5}} = 1 \qquad ref$$

3. For Transfer function H(s)=$\frac{35}{s^2+18s+35}$,

$$\omega^2 = 35$$
$$2\zeta\omega = 18$$
$$\text{then we get } \zeta = \sqrt{\frac{81}{35}}>1 \qquad ref$$

The damping of a system can be described as being one of the following:
Overdamped : The system returns to equilibrium without oscillating.For this

$$\zeta>1. \tag{1.1.5}$$

Critically damped: The system returns to equilibrium as quickly as possible without oscillating.
For this

$$\zeta = 1 \tag{1.1.6}$$

Underdamped: The system oscillates(at reduced frequency compared to the undamped case) with the amplitude gradually decreasing to zero.
For this

$$0<\zeta<1 \tag{1.1.7}$$

Undamped : The system oscillates at its natural

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resonant frequency($\omega 0$).
Forthis

$$\zeta = 0 \tag{1.1.8}$$

Relation between damping and ζ	
Types of Damping	$\zeta(dampingratio)$
Overdamped	$\zeta>1$
Criticallydamped	$\zeta = 1$
Underdamped	$0 <\zeta<1$
Undamped	$\zeta = 0$

Final Analysis

- As for P : $\zeta<1$
  It is Underdamped system
- As for Q : $\zeta = 1$
  It is critically damped system.
- As for R : $\zeta>1$
  It is an overdamped system.
  So,P-3,Q-2,R-1. Option (C) is correct.

1.2. Python code for the above damping graph :

```
codes/ee18btech110012/damping.py
```

1.3. Python code for damped outputs taking unit step function as input:

```
codes/ee18btech110012/unitstepdamping.py
```

1. For Transfer function H(s)=$\frac{15}{s^2+5s+15}$
the damped output for step input is

$$y(t) = 25te^{-5t}u(t) \tag{1.3.1}$$

2. For Transfer function H(s)=$\frac{25}{s^2+10s+25}$

$$y(t) = \frac{30}{\sqrt{35}}e^{\frac{-5t}{2}}sin(\frac{\sqrt{35}}{2}t)u(t) \tag{1.3.2}$$

3. For Transfer function H(s)=$\frac{35}{s^2+18s+35}$,

$$y(t) = \frac{35}{2\sqrt{46}}(e^{(-9+\sqrt{46})t} - e^{(-9-\sqrt{46})t})u(t) \tag{1.3.3}$$

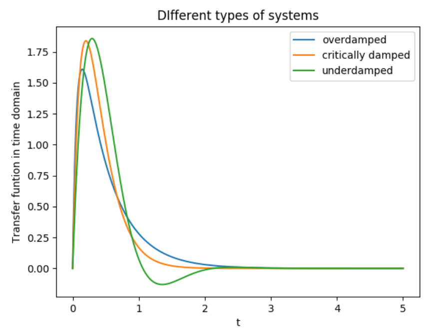

Fig. 1.3: Different systems based on ζ

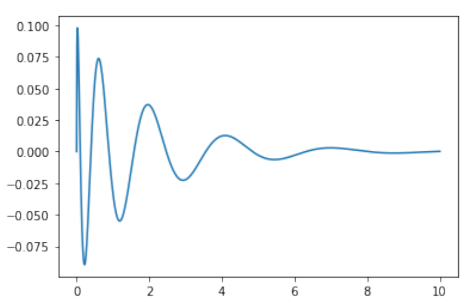

Fig. 1.3: Damped output for step input