EE3025 IDP

Assignment-1

Download all python codes from

https://github.com/Sivanidevarapalli26/EE3025_IDP/tree/main/Assignment−1/Codes

and latex-tikz codes from

https://github.com/Sivanidevarapalli26/EE3025_IDP/tree/main/Assignment−1

1 Question

1.1. Let

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{1.1.1}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.1.2}$$

Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{1.1.3}$$

and $H(k)$ using $h(n)$.

2 Solution

2.1. When Unit impulse signal is given as input to the LTI system then its impulse response is the ouput of the system. So,from equation (1.1.2) we can say that the Impulse response of the system is,

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2) \tag{2.1.1}$$

2.2. Given that DFT of a Input Signal $x(n)$ is :

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{2.1.2}$$

2.3. Similarly, DFT of Impulse Response $h(n)$ is,

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{2.1.3}$$

2.4. Python Code to compute the DFT of $x(n)$ and $h(n)$ is given below

https://github.com/Sivanidevarapalli26/EE3025_IDP/blob/main/Assignment−1/Codes/ee18btech11012.py

2.5. From the above code we get the following plots.

https://github.com/Sivanidevarapalli26/EE3025_IDP/tree/main/Assignment−1/figs

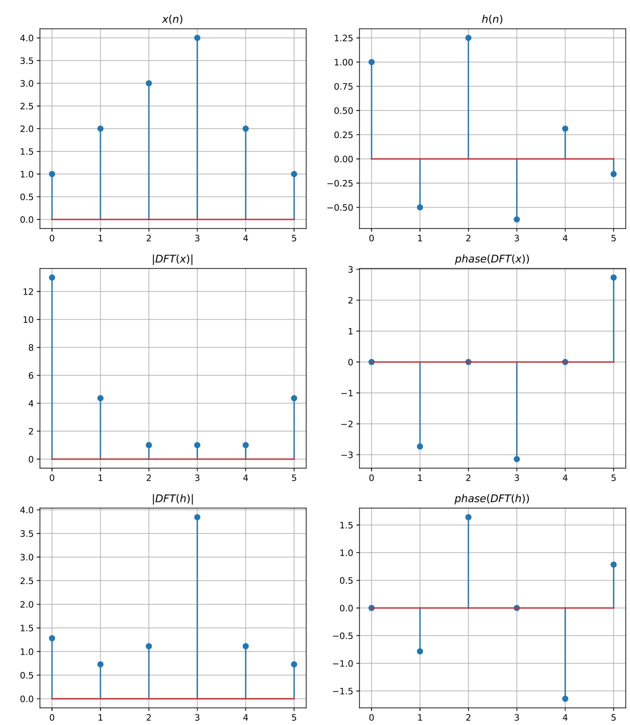

Fig. 2.1: Plots of x(n) and h(n),their DFTs

2.6. Soving DFT of x(n) and h(n) using Matrix multiplication method
we know that

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.1.4}$$

Let $W_N^{nk} = e^{-j2\pi kn/N}$ then this can be expressed interms of matrices as:

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1\\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5\\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10}\\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15}\\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20}\\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0)\\ x(1)\\ x(2)\\ x(3)\\ x(4)\\ x(5) \end{bmatrix}$$

(2.1.5)

2.7. Given that x(n) = $\left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\}$ and As,N=6 then above equation on multiplying matrices becomes

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5) \end{bmatrix} = \begin{bmatrix} 1+2+3+4+2+1\\ 1+(2)e^{-j\pi/3}+...+(1)e^{-j5\pi/3}\\ 1+(2)e^{-2j\pi/3}+...+(1)(e^{-2j5\pi/3}\\ 1+(2)e^{-3j\pi/3}+...+(1)e^{-3j5\pi/3}\\ 1+(2)e^{-4j\pi/3}+...+(1)e^{-4j5\pi/3}\\ 1+(2)e^{-5j\pi/3}+...+(1)e^{-5j5\pi/3} \end{bmatrix} \tag{2.1.6}$$

2.8. On solving we get,

$$\implies X(0) = 13 + 0j, \tag{2.1.7}$$
$$X(1) = -4 - 1.732j, \tag{2.1.8}$$
$$X(2) = 1 + 0j, \tag{2.1.9}$$
$$X(3) = -1 + 0j, \tag{2.1.10}$$
$$X(4) = 1 + 0j, \tag{2.1.11}$$
$$X(5) = -4 + 1.732j \tag{2.1.12}$$

2.9. Now to find $H(k)$ we need to know h(n) first. So we will first calculate h(n). For that we need to first find the Y(z) by applying Z-transform on equation (1.1.2) i.e.,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{2.1.13}$$

$$\implies Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{2.1.14}$$

Now we can find H(z) using Y(z) i.e.,

$$H(z) = \frac{Y(z)}{X(z)} \tag{2.1.15}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{2.1.16}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{2.1.17}$$

From this we can say that h(n) is,

$$h(n) = Z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{2.1.18}$$

$$h(n) = \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{2.1.19}$$

Now for the calculations we can assume that length of h(n) is same as length of x(n) i.e., N = 6.
Similarly Now on solving equation (2.1.3) using matrix method for each value of k we get,

$$\begin{bmatrix} H(0)\\ H(1)\\ H(2)\\ H(3)\\ H(4)\\ H(5) \end{bmatrix} = \begin{bmatrix} h(0)+h(1)+h(2)+h(3)+h(4)+h(5)\\ h(0)+h(1)e^{-j\pi/3}+...+h(5)e^{-j5\pi/3}\\ h(0)+h(1)e^{-2j\pi/3}+...+h(5)e^{-2j5\pi/3}\\ h(0)+h(1)e^{-3j\pi/3}+...+h(5)e^{-3j5\pi/3}\\ h(0)+h(1)e^{-4j\pi/3}+...+h(5)e^{-4j5\pi/3}\\ h(0)+h(1)e^{-5j\pi/3}+...+h(5)e^{-5j5\pi/3} \end{bmatrix} \quad (2.1$$

2.10. On solving we get,

$$\implies H(0) = 1.28125 + 0j, \tag{2.1.21}$$
$$H(1) = 0.51625 - 0.5141875j, \tag{2.1.22}$$
$$H(2) = -0.078125 + 1.1095625j, \tag{2.1.23}$$
$$H(3) = 3.84375 + 0j, \tag{2.1.24}$$
$$H(4) = -0.071825 - 1.1095625j. \tag{2.1.25}$$
$$H(5) = 0.515625 + 0.5141875j \tag{2.1.26}$$

So,These values which we got are same as that of from the plots.