EE3025 EE3015-IDP

Final Presentation

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Assignment-1 Problem

- Let

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\} \tag{1}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2}$$

Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{3}$$

and $H(k)$ using $h(n)$.

Solution

- When Unit impulse signal is given as input to the LTI system then its impulse response is the ouput of the system. So,from equation (2) we can say that the Impulse response of the system is,

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2) \tag{4}$$

- Given that DFT of a Input Signal $x(n)$ is :

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{5}$$

- Similarly, DFT of Impulse Response $h(n)$ is,

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{6}$$

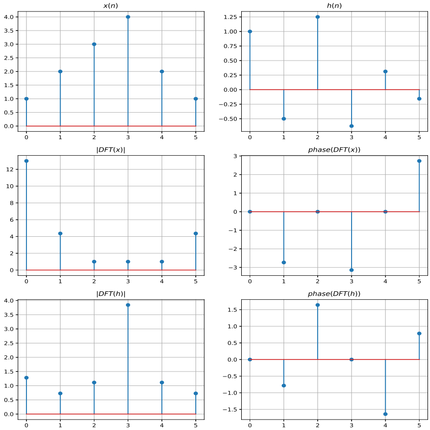

Figure 1: Plots of x(n) and h(n),their DFTs

Using Matrix Multiplication method

- Solving DFT of x(n) and h(n) using Matrix multiplication method we know that

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{7}$$

Let $W_N^{nk} = e^{-j2\pi kn/N}$ then this can be expressed interms of matrices as:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix} \tag{8}$$

- Given that $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\}$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1+2+3+4+2+1 \\ 1+(2)e^{-j\pi/3}+...+(1)e^{-j5\pi/3} \\ 1+(2)e^{-2j\pi/3}+...+(1)(e^{-2j5\pi/3} \\ 1+(2)e^{-3j\pi/3}+...+(1)e^{-3j5\pi/3} \\ 1+(2)e^{-4j\pi/3}+...+(1)e^{-4j5\pi/3} \\ 1+(2)e^{-5j\pi/3}+...+(1)e^{-5j5\pi/3} \end{bmatrix}$$

- On solving we get,

$$\implies X(0) = 13 + 0j,$$

$$X(1) = -4 - 1.732j,$$

$$X(2) = 1 + 0j$$

$$X(3) = -1 + 0j,$$

$$X(4) = 1 + 0j,$$

$$X(5) = -4 + 1.732j$$

Cont..

- Now to find $H(k)$ we need to know h(n) first. So we will first calculate h(n). For that we need to first find the Y(z) by applying Z-transform on equation (2) i.e.,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{9}$$

$$\implies Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{10}$$

Now we can find H(z) using Y(z)
i.e.,

$$H(z) = \frac{Y(z)}{X(z)} \tag{11}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{12}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{13}$$

- From this we can say that h(n) is,

$$h(n) = Z^{-1} \frac{1}{1 + \frac{1}{2}z^{-1} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}} \tag{14}$$

$$h(n) = \frac{-1}{2}^{n} u(n) + \frac{-1}{2}^{n-2} u(n-2) \tag{15}$$

Now for the calculations, we can assume that length of h(n) is same as length of x(n) i.e., N = 6. Similarly Now on solving equation

(6)

using matrix method for each value of k we get,

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0) + h(1) + h(2) + h(3) + h(4) + h(5) \\ h(0) + h(1)e^{-j\pi/3} + ... + h(5)e^{-j5\pi/3} \\ h(0) + h(1)e^{-2j\pi/3} + ... + h(5)e^{-2j5\pi/3} \\ h(0) + h(1)e^{-3j\pi/3} + ... + h(5)e^{-3j5\pi/3} \\ h(0) + h(1)e^{-4j\pi/3} + ... + h(5)e^{-4j5\pi/3} \\ h(0) + h(1)e^{-5j\pi/3} + ... + h(5)e^{-5j5\pi/3} \end{bmatrix} \tag{16}$$

- On solving we get,

$$\implies H(0) = 1.28125 + 0j, \tag{17}$$
$$H(1) = 0.51625 - 0.5141875j, \tag{18}$$
$$H(2) = -0.078125 + 1.1095625j, \tag{19}$$
$$H(3) = 3.84375 + 0j, \tag{20}$$
$$H(4) = -0.071825 - 1.1095625j. \tag{21}$$
$$H(5) = 0.515625 + 0.5141875j \tag{22}$$

So,These values which we got are same as that of from the plots.

- **Tolerances:**
  The magnitude of resonance in Pass-band is called Pass-band Tolerance and similarly magnitude of resonance in Stop-band is called Stop-band Tolerance.

- **Passband:**
  The Frequency band within which signals are transmitted by filter without attenuation.

- **Stopband:**
  The Frequency band within which signals are not transmitted by filter or with a large attenuation.

The Bilinear Transform

- It is a technique in Signal Processing that is used to Continuous-Time System Representations to Discrete-Time System Representations and vice-versa. The bilinear transform is a first-order approximation of the natural logarithm function that is an exact mapping of the z-plane to the s-plane.
- When the Laplace transform is performed on a discrete-time signal (with each element of the discrete-time sequence attached to a correspondingly delayed unit impulse), the result is precisely the $\mathbb{Z}$ transform of the discrete-time sequence with the substitution of $z = e^{st}$.

The FIR Filter

The lowpass filter has a pass frequency ω_l and transition band $\Delta\omega$ with Stop Band Tolerance δ is $h_{lp}(n) = \frac{\sin(n\omega_l)}{n\pi} w(n)$

$$w(n) = \begin{cases} \frac{I\left[\beta N \sqrt{1-\left(\frac{n}{N}\right)^2}\right]}{I_0(\beta N)}, & -N \leq n \leq N, \quad \beta > 0 \\ 0 \text{ otherwise} \end{cases}$$

where $w(n)$ is Kaiser Window and $I_0(x)$ is the modified Bessel function of the first kind of order zero in x, and N are the window shaping factors and they are selected as follows

$N \geq \frac{A-8}{4.57\Delta\omega}$

$A = -20\log_{10}\delta$

$$\beta N = \begin{cases} 0.1102(A-8.7) & A > 50 \\ 0.5849(A-21)^{0.4} + 0.07886(A-21) & 21 \leq A \leq 50 \\ 0 & A < 21 \end{cases}$$

The FIR Filter

The FIR Bandpass Filter: The centre of the passband of the desired bandpass filter was found to be $\omega_c = 0.275\pi$ The impulse response of the desired bandpass filter is obtained from the impulse response of the corresponding lowpass filter as

$$h_{bp}(n) = 2h_{lp}(n)\cos\left(n\omega_c\right)$$

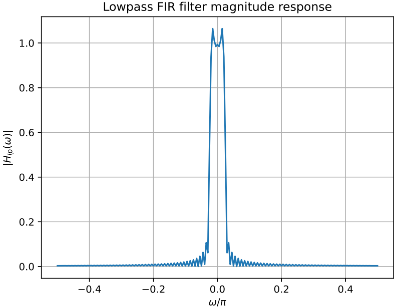

Figure 2: FIR Low Pass Filter

The FIR Filter

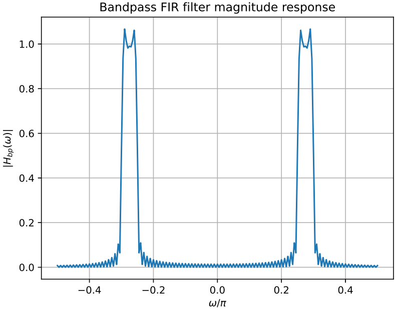

Figure 3: FIR Band Pass Filter

Thank You