Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback circuits

1.0.1. Consider an op amp having a single pole open loop response $G_o = 10^5$ and $f_p = 10$ Hz.Let op amp be ideal connected in non-inverting terminal with a nominal low frequency of closed loop gain of 100 and wired as a unity gain buffer.

Find the frequency at which $|GH| = 1$ and What is its corresponding phase margin **Solution:** For a single-pole amplifier, open loop transfer function is

$$G(s) = \frac{G_o}{1 + \frac{s}{\omega_p}} \tag{1.0.1.1}$$

Given that $f_p = 10$ Hz and $G_o = 10^5$

$$G(s) = \frac{G_o}{1 + \frac{s}{2\pi f_p}} \implies \frac{10^5}{1 + \frac{s}{2\pi.10}} \tag{1.0.1.2}$$

So,the open-loop gain of the op amp is

$$G(s) = \frac{10^5}{1 + \frac{s}{2\pi.10}} \tag{1.0.1.3}$$

For a unity-gain buffer,the feedback factor is

$$H = 1 \tag{1.0.1.4}$$

Thus,

$$G(\jmath\omega)H = \frac{10^5.1}{1 + \frac{\jmath\omega}{2\pi.10}} \tag{1.0.1.5}$$

To find the frequency at which $|G(j\omega)H| = 1$, we write

$$|\frac{10^5.1}{1 + \frac{\jmath\omega}{2\pi.10}}| = 1 \tag{1.0.1.6}$$

$$1 + \frac{\omega_1^2}{2\pi.10} = 10^{10} \tag{1.0.1.7}$$

Thus

$$\omega_1 = 6.283Mrad/sec \implies f_1 = \frac{\omega_1}{2\pi} = 1MHz \tag{1.0.1.8}$$

From definition of phase margin $\alpha = 180° + \phi$ where ϕ is the phase of $G(j\omega_1)H$

$$\phi = -\tan^{-1}\left(\frac{\omega_1}{2\pi.10}\right) \tag{1.0.1.9}$$

At $\omega_1 = 2\pi.10^6 rad/sec$

$$\phi = -\tan^{-1}\left(2\pi.10^6 2\pi.10\right) \tag{1.0.1.10}$$

$$\implies \phi = -90°(approx) \tag{1.0.1.11}$$

Therefore,the phase margin is

$$\alpha = 180 + \phi \implies \alpha = 180° - 90° \implies \alpha = 90° \tag{1.0.1.12}$$

Hence for frequency $f = 1MHz$ Hz, $|GH| = 1$ and phase margin is 90°

1.0.2. The following is the code for bode plot of the given system

```
codes/ee18btech11012_1/ee18btech11012_1.
    py
```

1.0.3. Verification using Bode plot

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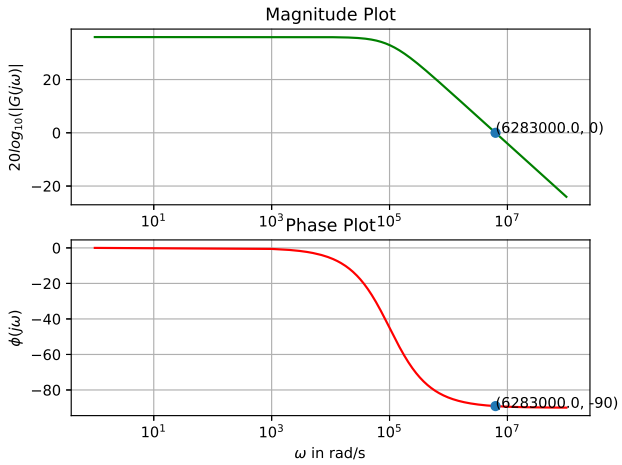
Magnitude Plot
$20log_{10}(|G(j\omega)|$
20
0
−20
(6283000.0, 0)
10^1
10^3
10^5
10^7
Phase Plot
$\phi(j\omega)$
0
−20
−40
−60
−80
(6283000.0, -90)
ω in rad/s

Fig. 1.0.3