Control Systems

D Lakshmi Sivani*

Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback circuits

1.0.1. Consider an op amp having a single pole open loop response $G_o = 10^5$ and $f_p = 10$ Hz.Let op amp be ideal connected in non-inverting terminal with a nominal low frequency of closed loop gain of 100 and wired as a unity gain buffer.

Find the frequency at which $|GH| = 1$ and What is its corresponding phase margin
Solution: For a single-pole amplifier, open loop transfer function is

$$G(s) = \frac{G_o}{1 + \frac{s}{\omega_p}} \tag{1.0.1.1}$$

Given that $f_p = 10$ Hz and $G_o = 10^5$

$$G(s) = \frac{G_o}{1 + \frac{s}{2\pi f_p}} \implies \frac{10^5}{1 + \frac{s}{2\pi.10}} \tag{1.0.1.2}$$

So,the open-loop gain of the op amp is

$$G(s) = \frac{10^5}{1 + \frac{s}{2\pi.10}} \tag{1.0.1.3}$$

For a unity-gain buffer,the feedback factor is

$$H = 1 \tag{1.0.1.4}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: ee18btech11012@iith.ac.in.

Thus,

$$G(j\omega)H = \frac{10^5.1}{1 + \frac{j\omega}{2\pi.10}} \tag{1.0.1.5}$$

To find the frequency at which $|G(j\omega)H| = 1$, we write

$$|\frac{10^5.1}{1 + \frac{j\omega}{2\pi.10}}| = 1 \tag{1.0.1.6}$$

$$1 + \frac{\omega_1^2}{2\pi.10} = 10^{10} \tag{1.0.1.7}$$

Thus

$$\omega_1 = 6.283 Mrad/sec \implies f_1 = \frac{\omega_1}{2\pi} = 1MHz \tag{1.0.1.8}$$

From definition of phase margin $\alpha = 180^\circ + \phi$ where ϕ is the phase of $G(j\omega_1)H$

$$\phi = -\tan^{-1}\left(\frac{\omega_1}{2\pi.10}\right) \tag{1.0.1.9}$$

At $\omega_1 = 2\pi.10^6 rad/sec$

$$\phi = -\tan^{-1}\left(2\pi.10^6 2\pi.10\right) \tag{1.0.1.10}$$

$$\implies \phi = -90^\circ(approx) \tag{1.0.1.11}$$

Therefore,the phase margin is

$$\alpha = 180^\circ + \phi \implies \alpha = 180^\circ - 90^\circ \implies \alpha = 90^\circ \tag{1.0.1.12}$$

Hence for frequency $f = 1MHz$ Hz,$|GH| = 1$ and phase margin is 90°

1.0.2. The following is the code for bode plot of the given system

```
codes/ee18btech11012_1/ee18btech11012_1.
    py
```

1.0.3. Verification using Bode plot

1.0.4. Realise the above system using a feedback circuit.
Solution: The transfer function of OPAMP is

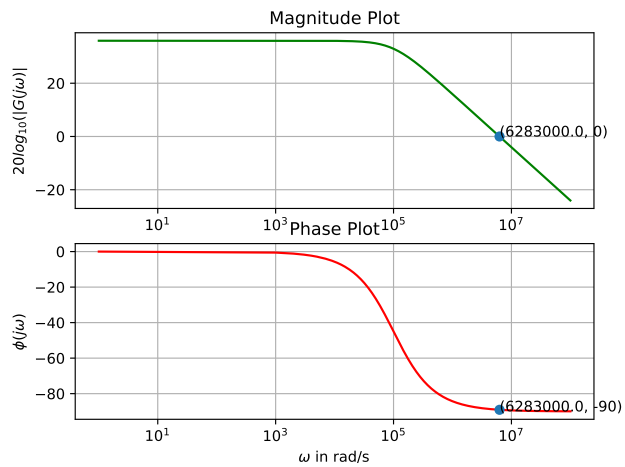

Fig. 1.0.3

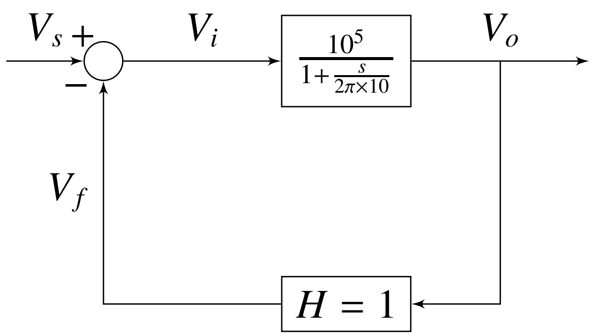

Fig. 1.0.4

$$G(s) = \frac{10^5}{(1 + \frac{s}{2\pi\times 10})} \tag{1.0.4.1}$$

1.0.5. For the feedback gain H
Solution:
Feedback gain H can be written as:

$$H = \frac{V_f}{V_o} = 1 \implies V_f = V_o \tag{1.0.5.1}$$

Note:This type of circuit containing Op-amp is called as ”Voltage follower” or ”Unity buffer”. As the non-inverting input of the Op-amp is fed to the output of the system.
Which inturn makes the feedback factor(H=1)
Here Output (V_o) follows the input(V_f) as shown in fig.

1.0.6. The closed loop transfer function of this system is

$$T = \frac{G(s)}{1 + G(s)} = \frac{10^5}{((10^5 + 1) + \frac{s}{2\pi\times 10})} \tag{1.0.6.1}$$

1.0.7. Feedback Circuit for this unity buffer system is
Solution:

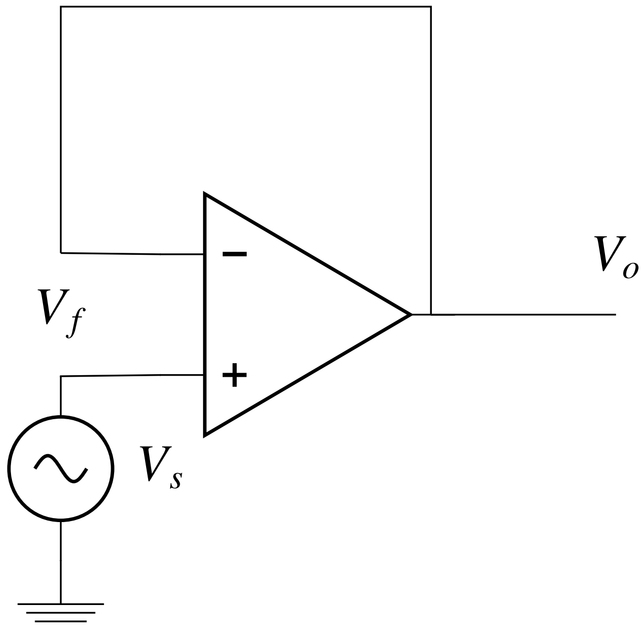

Fig. 1.0.7

1.0.8. Verification through Spice circuit
Solution: For H=1 the closed loop gain is

$$|T| \approx \frac{1}{H} = 1 \tag{1.0.8.1}$$

The following is the netlist file for spice simulation

spice/ee18btech11012/ee18btech11012.net

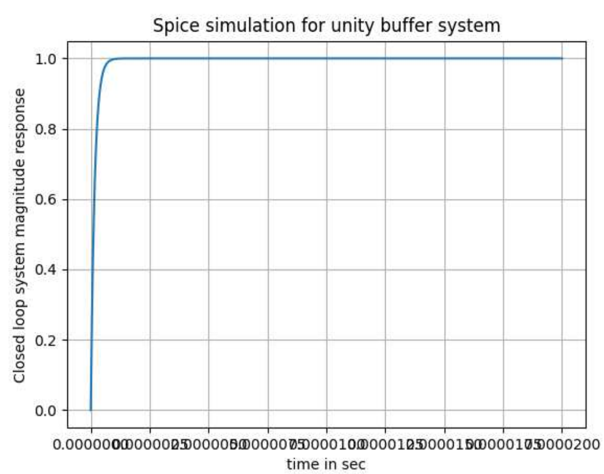

Fig. 1.0.8

1.0.9. The following python code plots the closed loop response verses time and the python plot is also shown below.

```
spice/ee18btech11012/
    ee18btech11012_spiceresult1.py
```

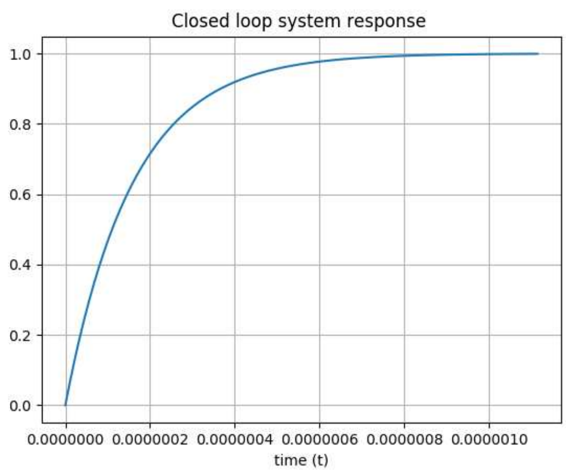

Fig. 1.0.9

1.0.10. Checking Unstability in the context of PM
Solution: A closed loop system is said to be unstable,if the phase margin(PM) of GH is negative

$$PM < 0^\circ \tag{1.0.10.1}$$

$$\implies \underline{/G(f)H(f)} < -180^\circ \tag{1.0.10.2}$$

For the given GH where H=1

$$\underline{/G(f)H(f)} = \underline{/G(f)} \implies -\tan^{-1}\left(\frac{f}{10}\right) \tag{1.0.10.3}$$

At $f = \infty$

$$\underline{/G(f)} = -\tan^{-1}\infty = -90^\circ \tag{1.0.10.4}$$

So there won't exist any positive f where $\underline{/G(f)} < -180^\circ$
Hence,this system is always stable.