

# Control Systems

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### 1 Feedback circuits 1

**Abstract**—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/feedback/codes
```

#### 1 FEEDBACK CIRCUITS

1.0.1. Consider an op amp having a single pole open loop response  $G_o = 10^5$  and  $f_p = 10$  Hz. Let op amp be ideal connected in non-inverting terminal with a nominal low frequency of closed loop gain of 100 and wired as a unity gain buffer.

Find the frequency at which  $|GH| = 1$  and What is its corresponding phase margin

**Solution:** For a single-pole amplifier, open loop transfer function is

$$G(s) = \frac{G_o}{1 + \frac{s}{\omega_p}} \quad (1.0.1.1)$$

Given that  $f_p = 10$  Hz and  $G_o = 10^5$

$$G(s) = \frac{G_o}{1 + \frac{s}{2\pi f_p}} \Rightarrow \frac{10^5}{1 + \frac{s}{2\pi \cdot 10}} \quad (1.0.1.2)$$

So, the open-loop gain of the op amp is

$$G(s) = \frac{10^5}{1 + \frac{s}{2\pi \cdot 10}} \quad (1.0.1.3)$$

For a unity-gain buffer, the feedback factor is

$$H = 1 \quad (1.0.1.4)$$

Thus,

$$G(j\omega)H = \frac{10^5 \cdot 1}{1 + \frac{j\omega}{2\pi \cdot 10}} \quad (1.0.1.5)$$

To find the frequency at which  $|G(j\omega)H| = 1$ , we write

$$\left| \frac{10^5 \cdot 1}{1 + \frac{j\omega}{2\pi \cdot 10}} \right| = 1 \quad (1.0.1.6)$$

$$1 + \frac{\omega_1^2}{2\pi \cdot 10} = 10^{10} \quad (1.0.1.7)$$

Thus

$$\omega_1 = 6.283 \text{ Mrad/sec} \Rightarrow f_1 = \frac{\omega_1}{2\pi} = 1 \text{ MHz} \quad (1.0.1.8)$$

From definition of phase margin  $\alpha = 180^\circ + \phi$  where  $\phi$  is the phase of  $G(j\omega_1)H$

$$\phi = -\tan^{-1} \left( \frac{\omega_1}{2\pi \cdot 10} \right) \quad (1.0.1.9)$$

At  $\omega_1 = 2\pi \cdot 10^6 \text{ rad/sec}$

$$\phi = -\tan^{-1} (2\pi \cdot 10^6 \cdot 2\pi \cdot 10) \quad (1.0.1.10)$$

$$\Rightarrow \phi = -90^\circ (\text{approx}) \quad (1.0.1.11)$$

Therefore, the phase margin is

$$\alpha = 180^\circ + \phi \Rightarrow \alpha = 180^\circ - 90^\circ \Rightarrow \alpha = 90^\circ \quad (1.0.1.12)$$

**Hence for frequency  $f = 1 \text{ MHz}$  Hz,  $|GH| = 1$  and phase margin is  $90^\circ$**

1.0.2. The following is the code for bode plot of the given system

```
codes/ee18btech11012_1/ee18btech11012_1.
py
```

1.0.3. Verification using Bode plot

1.0.4. Realise the above system using a feedback circuit.

**Solution:** The transfer function of OPAMP is

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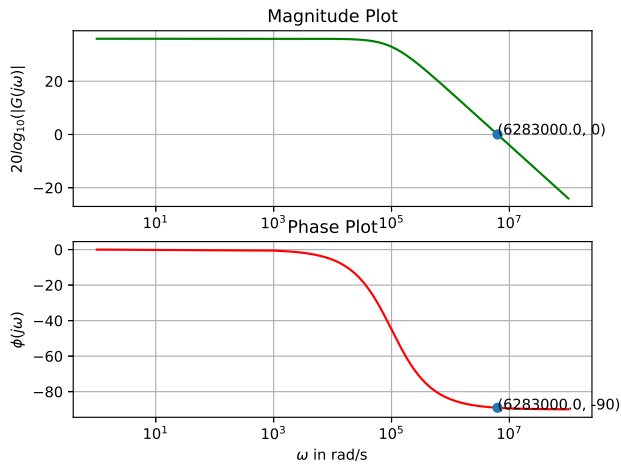


Fig. 1.0.3

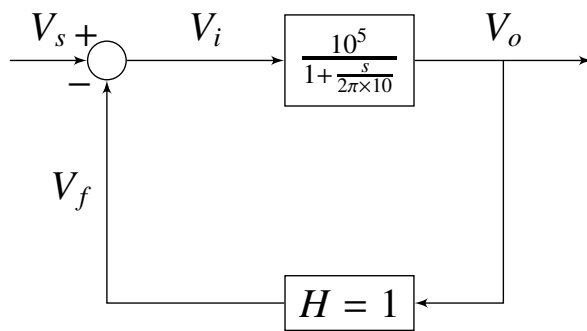


Fig. 1.0.4

$$G(s) = \frac{10^5}{(1 + \frac{s}{2\pi \times 10})} \quad (1.0.4.1)$$

1.0.5. For the feedback gain H

**Solution:**

Feedback gain H can be written as:

$$H = \frac{V_f}{V_o} = 1 \implies V_f = V_o \quad (1.0.5.1)$$

**Note:** This type of circuit containing Op-amp is called as "Voltage follower" or "Unity buffer". As the non-inverting input of the Op-amp is fed to the output of the system.

Which in turn makes the feedback factor (H=1). Here Output ( $V_o$ ) follows the input ( $V_f$ ) as shown in fig.

1.0.6. The closed loop transfer function of this system is

$$T = \frac{G(s)}{1 + G(s)} = \frac{10^5}{((10^5 + 1) + \frac{s}{2\pi \times 10})} \quad (1.0.6.1)$$

1.0.7. Feedback Circuit for this unity buffer system

is

**Solution:**

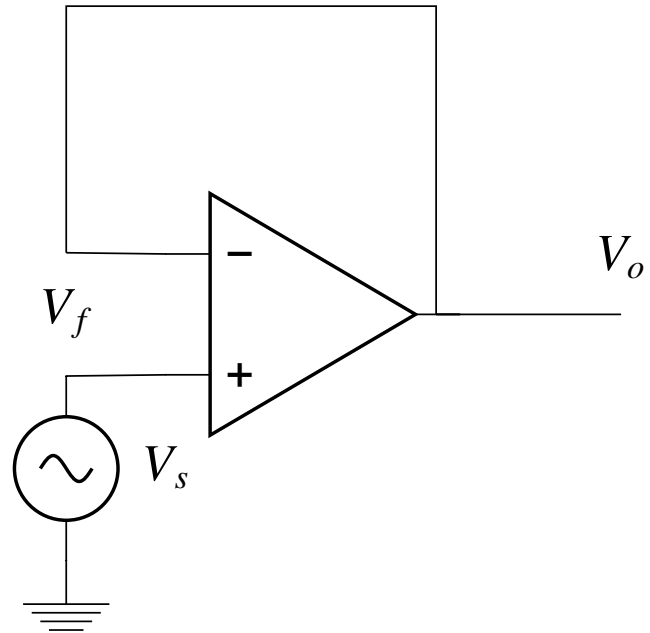


Fig. 1.0.7

1.0.8. Verification through Spice circuit

**Solution:** For H=1 the closed loop gain is

$$|T| \approx \frac{1}{H} = 1 \quad (1.0.8.1)$$

The following is the netlist file for spice simulation

```
spice/ee18btech11012/ee18btech11012.net
```

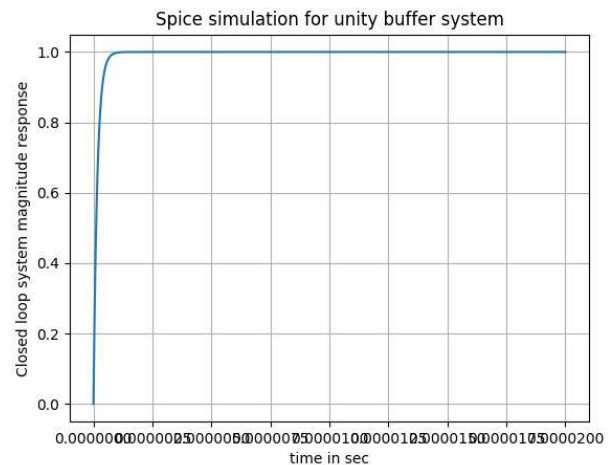


Fig. 1.0.8

1.0.9. The following python code plots the closed loop response versus time and the python plot is also shown below.

```
spice/ee18btech11012/
ee18btech11012_spiceresult1.py
```

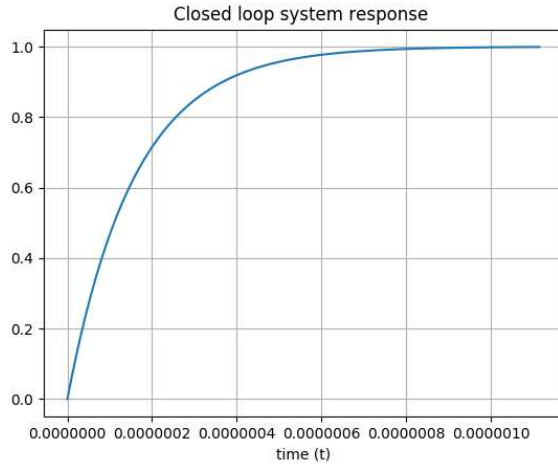


Fig. 1.0.9

1.0.10. Checking Unstability in the context of PM

**Solution:** A closed loop system is said to be unstable, if the phase margin (PM) of GH is negative

$$PM < 0^\circ \quad (1.0.10.1)$$

$$\Rightarrow \angle G(f)H(f) < -180^\circ \quad (1.0.10.2)$$

For the given GH where H=1

$$\angle G(f)H(f) = \angle G(f) \Rightarrow -\tan^{-1}\left(\frac{f}{10}\right) \quad (1.0.10.3)$$

At  $f = \infty$

$$\angle G(f) = -\tan^{-1} \infty = -90^\circ \quad (1.0.10.4)$$

So there won't exist any positive f where

$$\angle G(f) < -180^\circ$$

Hence, this system is always stable.