Physical Layer Design for a Narrow Band Communication System

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***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Specifications

1.0.1. 8-PSK Consider

$$\mathbf{y} = \mathbf{s} + \mathbf{n} \tag{1.0.1.1}$$

where $\mathbf{s} \in \{s_0,\ s_1,\ s_2,\ s_3, s_4, s_5, s_6, s_7\}$

$$s_0 = \begin{pmatrix} \sqrt{E_s} \\ 0 \end{pmatrix} \tag{1.0.1.2}$$

$$s_1 = \begin{pmatrix} \sqrt{\frac{E_s}{2}} \\ \sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.3}$$

$$s_2 = \begin{pmatrix} 0 \\ \sqrt{E_s} \end{pmatrix} \tag{1.0.1.4}$$

$$s_3 = \begin{pmatrix} -\sqrt{\frac{E_s}{2}} \\ \sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.5}$$

$$s_4 = \begin{pmatrix} -\sqrt{E_s} \\ 0 \end{pmatrix} \tag{1.0.1.6}$$

$$s_5 = \begin{pmatrix} -\sqrt{\frac{E_s}{2}} \\ -\sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.7}$$

$$s_6 = \begin{pmatrix} 0 \\ -\sqrt{E_s} \end{pmatrix} \tag{1.0.1.8}$$

$$s_7 = \begin{pmatrix} \sqrt{\frac{E_s}{2}} \\ -\sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.9}$$

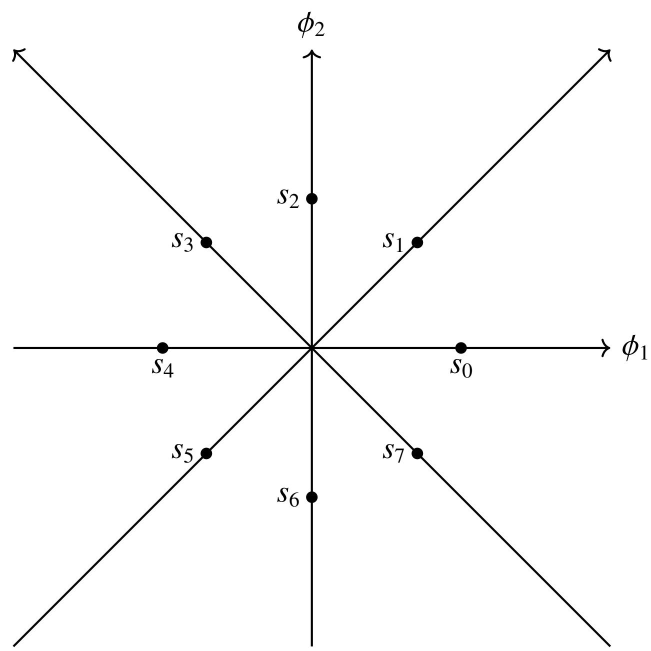

Fig. 1.0.1.1: Constellation diagram

1.0.2. Encoding
s_0 denote bits 000, s_1 denote bits 001, s_2 denote bits 011, s_3 denote bits 010, s_4 denote bits 110, s_5 denote bits 111, s_6 denote bits 101, s_7 denote bits 100.

1.0.3. Decoding
<u>Minimum distance Criterion</u>:

$$\hat{s} = min\,\|\mathbf{y} - \mathbf{s}\| \tag{1.0.3.1}$$

where $\mathbf{s} \in \mathrm{s}_0, s_1, s_2, \ldots.. s_M$

Symbol	Gray Code	Value
s_0	000	$\begin{pmatrix}1\\0\end{pmatrix}$
s_1	001	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_2	011	$\begin{pmatrix}0\\1\end{pmatrix}$
s_3	010	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_4	110	$\begin{pmatrix}-1\\0\end{pmatrix}$
s_5	111	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$
s_6	101	$\begin{pmatrix}0\\-1\end{pmatrix}$
s_7	100	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$

Fig. 1.0.2.1: Gray coding

Symbol	Gray Code	Inphase	Quadrature
s_0	000	1	0
s_1	001	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
s_2	011	0	1
s_3	010	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
s_4	110	-1	0
s_5	111	$\frac{-1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$
s_6	101	0	-1
s_7	100	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$

Fig. 1.0.2.2

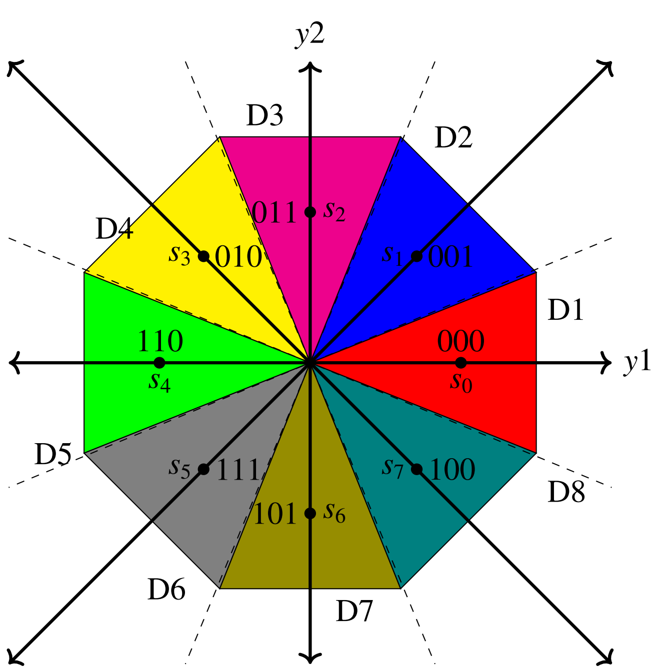

Fig. 1.0.3.1: decision regions

From eq.1.0.3.1,s_0 is chosen if

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_1\|^2 \tag{1.0.3.2}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_2\|^2 \tag{1.0.3.3}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_3\|^2 \tag{1.0.3.4}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_4\|^2 \tag{1.0.3.5}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_5\|^2 \tag{1.0.3.6}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_6\|^2 \tag{1.0.3.7}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_7\|^2 \tag{1.0.3.8}$$

Since $\|s_i\|^2 = E_s$,the above conditions can be simplified to obtain the region

$$(s_0 - s_1)^T y > 0 \tag{1.0.3.9}$$

$$(s_0 - s_2)^T y > 0 \tag{1.0.3.10}$$

$$(s_0 - s_3)^T y > 0 \tag{1.0.3.11}$$

$$(s_0 - s_4)^T y > 0 \tag{1.0.3.12}$$

$$(s_0 - s_5)^T y > 0 \tag{1.0.3.13}$$

$$(s_0 - s_6)^T y > 0 \tag{1.0.3.14}$$

$$(s_0 - s_7)^T y > 0 \tag{1.0.3.15}$$

Substituting the values of s_0,s_1,....s_7 in the above and eliminating $\sqrt{E_s}$,,the desired region is

$$\begin{pmatrix}(1 - \frac{1}{\sqrt{2}})\\ \frac{-1}{\sqrt{2}}\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.16}$$

$$\begin{pmatrix}1\\-1\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.17}$$

$$\begin{pmatrix}(1 + \frac{1}{\sqrt{2}})\\ \frac{-1}{\sqrt{2}}\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.18}$$

$$\begin{pmatrix}1\\0\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.19}$$

$$\begin{pmatrix} (1+\frac{1}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}} \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \tag{1.0.3.20}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \tag{1.0.3.21}$$

$$\begin{pmatrix} (1-\frac{1}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}} \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \tag{1.0.3.22}$$

yielding $y_2+(\sqrt{2}-1)y_1 > 0$, $y_2-(\sqrt{2}-1)y_1 < 0$. i.e.,Red region(D1) is detected at the receiver.

Similarly for all symbols their decision region and their respective inequalities are given below in the table shown.

Symbol	Decision region	Inequality	Matrix equation
s_0	D1	$y_2 + (\sqrt{2}-1)y_1 > 0$, $y_2 - (\sqrt{2}-1)y_1 < 0$	$\begin{bmatrix} \sqrt{2}-1 & 1 \\ \sqrt{2}-1 & -1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0$
s_1	D2	$y_2 - (\sqrt{2}+1)y_1 < 0$, $y_2 - (\sqrt{2}-1)y_1 > 0$	$\begin{bmatrix} \sqrt{2}+1 & -1 \\ -(\sqrt{2}-1) & 1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0$
s_2	D3	$y_2 - (\sqrt{2}+1)y_1 > 0$, $y_2 + (\sqrt{2}+1)y_1 > 0$	$\begin{bmatrix} -(\sqrt{2}+1) & -1 \\ \sqrt{2}+1 & 1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0$
s_3	D4	$y_2 + (\sqrt{2}-1)y_1 > 0$, $y_2 + (\sqrt{2}+1)y_1 < 0$	$\begin{bmatrix} \sqrt{2}-1 & 1 \\ -(\sqrt{2}+1) & -1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0$
s_4	D5	$y_2 + (\sqrt{2}-1)y_1 < 0$, $y_2 - (\sqrt{2}-1)y_1 > 0$	$\begin{bmatrix} -(\sqrt{2}-1) & -1 \\ -(\sqrt{2}-1) & 1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0$
s_5	D6	$y_2 - (\sqrt{2}+1)y_1 > 0$, $y_2 - (\sqrt{2}-1)y_1 < 0$	$\begin{bmatrix} -(\sqrt{2}+1 & 1 \\ \sqrt{2}-1 & -1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0$
s_6	D7	$y_2 - (\sqrt{2}+1)y_1 < 0$, $y_2 + (\sqrt{2}+1)y_1 < 0$	$\begin{bmatrix} \sqrt{2}+1 & -1 \\ -(\sqrt{2}+1) & -1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0$
s_7	D8	$y_2 + (\sqrt{2}-1)y_1 < 0$, $y_2 + (\sqrt{2}+1)y_1 > 0$	$\begin{bmatrix} -(\sqrt{2}-1) & -1 \\ \sqrt{2}+1 & 1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0$

Fig. 1.0.3.2: Decision regions and their inequalities

1.0.4. The following code has simulation of 8PSK.

```
codes/ee18btech11012.py
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