Physical Layer Design for a Narrow Band Communication System

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***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Specifications

1.0.1. 8-PSK Consider

$$\mathbf{y} = \mathbf{s} + \mathbf{n} \tag{1.0.1.1}$$

where $\mathbf{s} \in \{s_0,\ s_1,\ s_2,\ s_3, s_4, s_5, s_6, s_7\}$

$$s_0 = \begin{pmatrix} \sqrt{E_s} \\ 0 \end{pmatrix} \tag{1.0.1.2}$$

$$s_1 = \begin{pmatrix} \sqrt{\frac{E_s}{2}} \\ \sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.3}$$

$$s_2 = \begin{pmatrix} 0 \\ \sqrt{E_s} \end{pmatrix} \tag{1.0.1.4}$$

$$s_3 = \begin{pmatrix} -\sqrt{\frac{E_s}{2}} \\ \sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.5}$$

$$s_4 = \begin{pmatrix} -\sqrt{E_s} \\ 0 \end{pmatrix} \tag{1.0.1.6}$$

$$s_5 = \begin{pmatrix} -\sqrt{\frac{E_s}{2}} \\ -\sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.7}$$

$$s_6 = \begin{pmatrix} 0 \\ -\sqrt{E_s} \end{pmatrix} \tag{1.0.1.8}$$

$$s_7 = \begin{pmatrix} \sqrt{\frac{E_s}{2}} \\ -\sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.9}$$

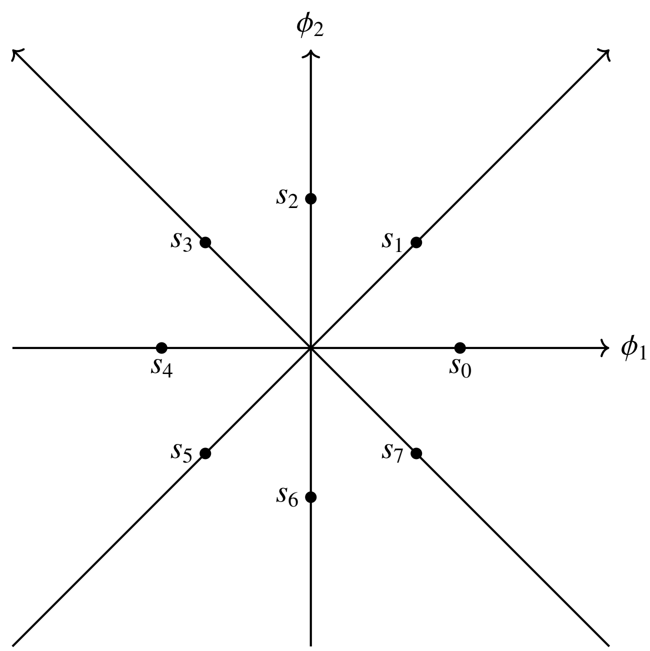

Fig. 1.0.1.1: Constellation diagram

1.0.2. Encoding
s_0 denote bits 000, s_1 denote bits 001, s_2 denote bits 011,s_3 denote bits 010,s_4 denote bits 110,s_5 denote bits 111,s_6 denote bits 101,s_7 denote bits 100.

1.0.3. Decoding
<u>**Minimum distance Criterion**</u>**:**

$$\hat{s} = min \left\|\mathbf{y} - \mathbf{s}\right\| \tag{1.0.3.1}$$

where $\mathbf{s} \in \mathrm{s}_0, s_1, s_2, ...s_M$

Symbol	Gray Code	Value
s_0	000	$\begin{pmatrix}1\\0\end{pmatrix}$
s_1	001	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_2	011	$\begin{pmatrix}0\\1\end{pmatrix}$
s_3	010	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$
s_4	110	$\begin{pmatrix}-1\\0\end{pmatrix}$
s_5	111	$\begin{pmatrix}\frac{-1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$
s_6	101	$\begin{pmatrix}0\\-1\end{pmatrix}$
s_7	100	$\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$

Fig. 1.0.2.1: Gray coding

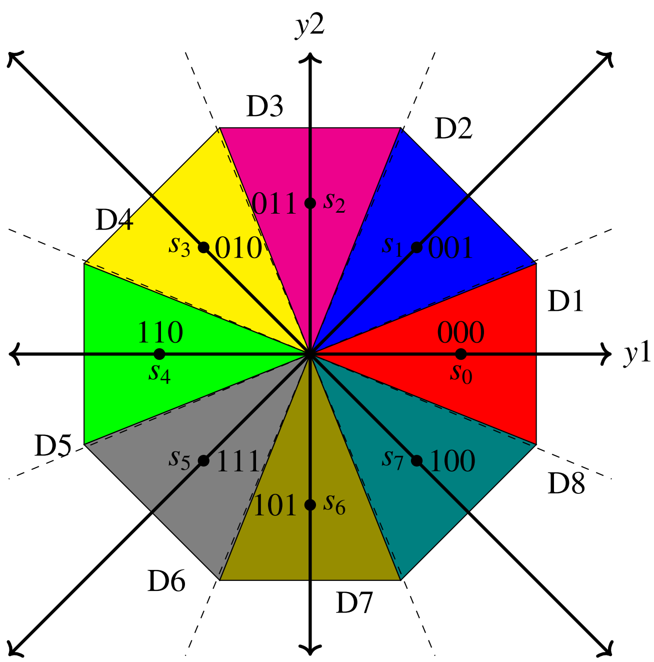

Fig. 1.0.3.1: decision regions

From eq.1.0.3.1,s_0 is chosen if

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_1\|^2 \tag{1.0.3.2}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_2\|^2 \tag{1.0.3.3}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_3\|^2 \tag{1.0.3.4}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_4\|^2 \tag{1.0.3.5}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_5\|^2 \tag{1.0.3.6}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_6\|^2 \tag{1.0.3.7}$$

$$\|\mathbf{y} - s_0\|^2 < \|\mathbf{y} - s_7\|^2 \tag{1.0.3.8}$$

Since $\|s_i\|^2 = E_s$,the above conditions can be simplified to obtain the region

$$(s_0 - s_1)^T y > 0 \tag{1.0.3.9}$$

$$(s_0 - s_2)^T y > 0 \tag{1.0.3.10}$$

$$(s_0 - s_3)^T y > 0 \tag{1.0.3.11}$$

$$(s_0 - s_4)^T y > 0 \tag{1.0.3.12}$$

$$(s_0 - s_5)^T y > 0 \tag{1.0.3.13}$$

$$(s_0 - s_6)^T y > 0 \tag{1.0.3.14}$$

$$(s_0 - s_7)^T y > 0 \tag{1.0.3.15}$$

Substituting the values of s_0,s_1,....s_7 in the above and eliminating $\sqrt{E_s}$,,the desired region is

$$\begin{pmatrix}(1 - \frac{1}{\sqrt{2}})\\ \frac{-1}{\sqrt{2}}\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.16}$$

$$\begin{pmatrix}1\\-1\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.17}$$

$$\begin{pmatrix}(1 + \frac{1}{\sqrt{2}})\\ \frac{-1}{\sqrt{2}}\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.18}$$

$$\begin{pmatrix}1\\0\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.19}$$

$$\begin{pmatrix}(1 + \frac{1}{\sqrt{2}})\\ \frac{1}{\sqrt{2}}\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.20}$$

$$\begin{pmatrix}1\\1\end{pmatrix}^T \begin{pmatrix}y_1\\y_2\end{pmatrix} > 0 \tag{1.0.3.21}$$

$$\begin{pmatrix} (1-\frac{1}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}} \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} > 0 \tag{1.0.3.22}$$

yielding $|y_2| < (\sqrt{2}-1)\mathrm{y}_1$
i.e.,Red region(D1) is detected at the receiver.
Similarly,
From eq:1.0.3.1
For detecting s_1, $y_2 - (\sqrt{2}+1)y_1 < 0$ and $y_2 - (\sqrt{2}-1)y_1 > 0$.
$\iff$ D2(blue) region is detected at receiver
For detecting s_2,$y_2 - (\sqrt{2}+1)y_1 > 0$ and $y_2 + (\sqrt{2}+1)y_1 > 0$
$\iff$ D3(pink) region at receiver.
For detecting s_3, $y_2 + (\sqrt{2}-1)y_1 > 0$ and $y_2 + (\sqrt{2}+1)y_1 < 0$.
$\iff$ D4(yellow) region at receiver.
For detecting s_4, $y_2 + (\sqrt{2}-1)y_1 < 0$ and $y_2 - (\sqrt{2}-1)y_1 > 0$.
$\iff$ D5(green) region at receiver.
For detecting s_5, $y_2 - (\sqrt{2}+1)y_1 > 0$ and $y_2 - (\sqrt{2}-1)y_1 < 0$.
$\iff$ D6(grey) region at receiver.
For detecting s_6, $y_2 - (\sqrt{2}+1)y_1 < 0$ and $y_2 + (\sqrt{2}+1)y_1 < 0$.
$\iff$ D7(olive) region at receiver.
For detecting s_7, $y_2 + (\sqrt{2}-1)y_1 < 0$ and $y_2 + (\sqrt{2}+1)y_1 > 0$.
$\iff$ D8(teal) region at receiver.

1.0.4. The following code has simulation of 8PSK.

```
codes/ee18btech11012.py
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