Physical Layer Design for a Narrow Band Communication System

G V V Sharma

***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Specifications

1.0.1. 8-PSK Consider

$$\mathbf{y} = \mathbf{s} + \mathbf{n} \tag{1.0.1.1}$$

where $\mathbf{s} \in \{s_0,\ s_1,\ s_2,\ s_3, s_4, s_5, s_6, s_7\}$

$$s_0 = \begin{pmatrix} \sqrt{E_s} \\ 0 \end{pmatrix} \tag{1.0.1.2}$$

$$s_1 = \begin{pmatrix} \sqrt{\frac{E_s}{2}} \\ \sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.3}$$

$$s_2 = \begin{pmatrix} 0 \\ \sqrt{E_s} \end{pmatrix} \tag{1.0.1.4}$$

$$s_3 = \begin{pmatrix} -\sqrt{\frac{E_s}{2}} \\ \sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.5}$$

$$s_4 = \begin{pmatrix} -\sqrt{E_s} \\ 0 \end{pmatrix} \tag{1.0.1.6}$$

$$s_5 = \begin{pmatrix} -\sqrt{\frac{E_s}{2}} \\ -\sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.7}$$

$$s_6 = \begin{pmatrix} 0 \\ -\sqrt{E_s} \end{pmatrix} \tag{1.0.1.8}$$

$$s_7 = \begin{pmatrix} \sqrt{\frac{E_s}{2}} \\ -\sqrt{\frac{E_s}{2}} \end{pmatrix} \tag{1.0.1.9}$$

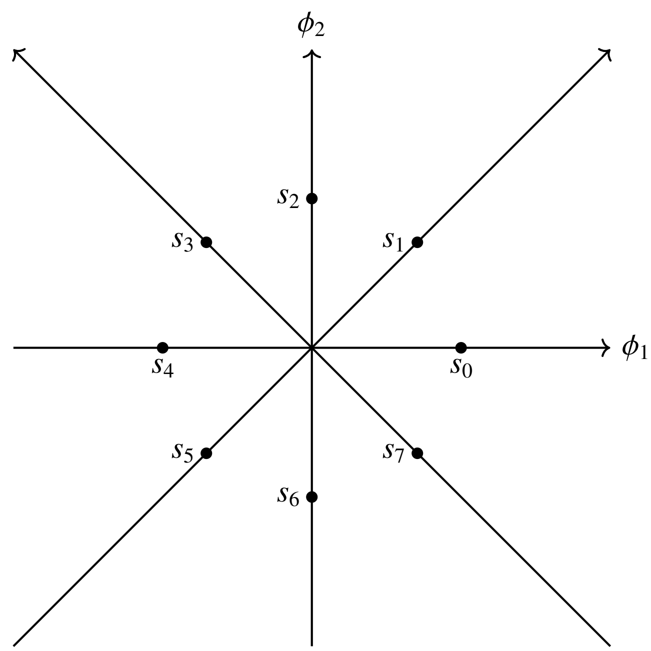

Fig. 1.0.1.1: Constellation diagram

1.0.2. Encoding
s_0 denote bits 000, s_1 denote bits 001, s_2 denote bits 011,s_3 denote bits 010,s_4 denote bits 110,s_5 denote bits 111,s_6 denote bits 101,s_7 denote bits 100.

1.0.3. Decoding
Let $\mathbf{r}$ be the received bits, $\mathbf{r} = [r_1, r_2, r_3]$.

$$r_1 = \begin{cases} 0, & \mathbf{y} \in D1 \cup D2 \cup D3 \cup D4 \Leftrightarrow \mathrm{y}_1(\sqrt{2}-1)+\mathrm{y}_2 > 0 \\ 1, & \mathbf{y} \in D5 \cup D6 \cup D7 \cup D8 \Leftrightarrow \mathrm{y}_1(\sqrt{2}-1)+\mathrm{y}_2 < 0 \end{cases} \tag{1.0.3.1}$$

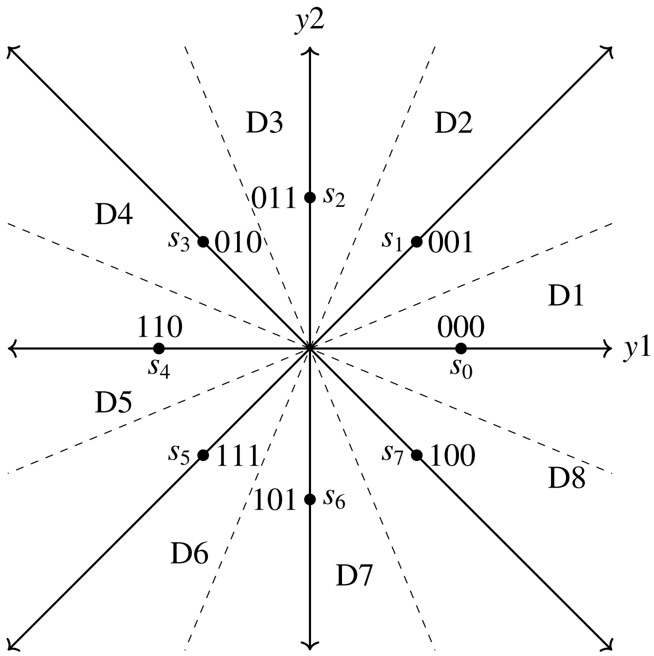

Fig. 1.0.3.1: decision regions

$$r_2 = \begin{cases} 0, & \mathbf{y} \in D2 \cup D1 \cup D8 \cup D7 \iff y_2 - (\sqrt{2}+1)y_1 < 0 \\ 1, & \mathbf{y} \in D3 \cup D4 \cup D5 \cup D6 \iff y_2 - (\sqrt{2}+1)y_1 > 0 \end{cases} \tag{1.0.3.2}$$

$$r_3 = \begin{cases} 0, & \mathbf{y} \in D4 \cup D5 \cup D1 \cup D8 \iff y_2 + (\sqrt{2}+1)y_1 < 0\ ,\ y_2 - (\sqrt{2}-1)y_1 > 0 \\ 1, & \mathbf{y} \in D2 \cup D3 \cup D6 \cup D7 \iff y_2 + (\sqrt{2}+1)y_1 > 0\ ,\ y_2 - (\sqrt{2}-1)y_1 < 0 \end{cases} \tag{1.0.3.3}$$

From eq.1.0.3.1,eq.1.0.3.2 and eq.1.0.3.3
For detecting s_0, $y_2 + (\sqrt{2}-1)y_1 > 0$ and $y_2 - (\sqrt{2}-1)y_1 < 0$.
For detecting s_1, $y_2 - (\sqrt{2}+1)y_1 < 0$ and $y_2 - (\sqrt{2}-1)y_1 > 0$.
For detecting s_2, $y_2 - (\sqrt{2}+1)y_1 > 0$ and $y_2 + (\sqrt{2}+1)y_1 > 0$.
For detecting s_3, $y_2 + (\sqrt{2}-1)y_1 > 0$ and $y_2 + (\sqrt{2}+1)y_1 < 0$.
For detecting s_4, $y_2 + (\sqrt{2}-1)y_1 < 0$ and $y_2 - (\sqrt{2}-1)y_1 > 0$.
For detecting s_5, $y_2 - (\sqrt{2}+1)y_1 > 0$ and $y_2 - (\sqrt{2}-1)y_1 < 0$.
For detecting s_6, $y_2 - (\sqrt{2}+1)y_1 < 0$ and $y_2 + (\sqrt{2}+1)y_1 < 0$.
For detecting s_7, $y_2 + (\sqrt{2}-1)y_1 < 0$ and $y_2 + (\sqrt{2}+1)y_1 > 0$.

1.0.4. The following code has simulation of 8PSK.

```
codes/8psk.py
```